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ERROR IN DERIVATION OF COMPTON SCATTERING FORMULA

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ABSTRACT. Arthur H. Compton published his light photon scattering theory in 1922. He derived a remarkably elegant formula which now bears his name, the Compton scattering formula: $\Delta\lambda = \lambda_c(1 - \cos\theta)$. It was derived basically from energy and momentum conservation considerations for collision of x-ray and gamma-ray photons with electrons within the atoms of light elements. Due possibly to the sterling reputation of Compton as a physicist, his theory was readily accepted. But there is a critical flaw in the derivation of the Compton formula that should render the formula dubious. In the derivation, Compton assumed the scattering electron to be initially at rest. The original experiment of Compton used carbon graphite as the scattering target. The ionization energy of carbon is about 11.3eV and this is also the kinetic energy of the least bound electrons in the carbon atom. For the scattering angle of 10°, the energy lost to the x-ray photon which ended up as the recoil energy of the scattered electron was around 9.04eV. This shows that the initial kinetic energy of the scattering electron is not insignificant and should not be ignored. This unjustified assumption in the derivation makes the generality of Compton scattering formula now dubious.

1. INTRODUCTION

Arthur Holly Compton was an American physicist with a sterling reputation. He was a key figure in the Manhattan Project that developed the first nuclear weapons. In 1922, he published a paper [1] that proposed a photon theory of light to explain the scattering of x-rays and gamma-rays when they strike light elements. He found that for various scattering angles(0-180°), the scattered rays consist of rays with the unmodified original wavelength of the incident ray and modified rays with an increased wavelength. He obtained a simply elegant formula which gives the increased in wavelength of the modified rays with scattering angle :

$$\Delta\lambda = \lambda' - \lambda = \lambda_c(1 - \cos\theta) \tag{1}$$

$\lambda_c = \dfrac{h}{mc}$, λ being the wavelength of the original ray, h the planck constant, m the rest mass of the electron, c the light speed and θ the

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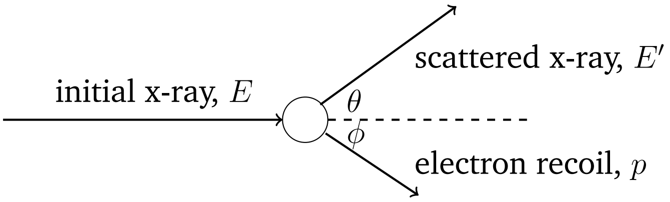

FIGURE 1. Compton scattering of a x-ray photon by an electron; E, E' are the photon energies, p the recoil momentum of the electron. The electron is assumed to be at rest initially.

angle the scattered ray makes with the initial direction of the ray. The constant λ_c is now named the Compton wavelength with the value 2.426 x 10^{-12}m.

Compton published the experiment he conducted for scattering of x-rays by graphite. The x-ray he used was the Molybdenum K-alpha rays of wavelength 0.711Å provided by an x-ray tube. The scattered wavelengths were measured by a Bragg spectrometer. He provided three data points for scattering angles of 30°,45° and 135°. The result was shown to be in good agreement with his derived formula (1). His theory was readily accepted. There was no known opposition to his theory nor any doubt about the experiment which verified his formula. The ready acceptance of his photon theory may in part be due to his sterling reputation as a physicist.

2. WHY THE COMPTON SCATTERING FORMULA IS DUBIOUS

The current interpretation of Compton scattering is that of a 'collision' of light particles, the photons, with the orbital electrons of atoms. It is treated as a collision between two classical particles. Figure (1) illustrates a typical situation. Compton introduced his light quantum hypothesis which regards electromagnetic radiations to be light particles:

(1) The energy E of a photon of light of frequency ν is: $E = h\nu$, h being the Planck constant.
(2) Light has momentum. The relation between the photon energy E and momentum p is: $E = pc$, c being the constant speed of light.

Compton's derivation of his formula was just simply the application of the conservation of energy and momentum to the collision based on the above hypotheses. He employed relativistic mechanics. The energy and momentum of an electron would be related by the formula: $E^2 = m_0^2c^4 + p^2c^2$. There was one more assumption that Compton made that enabled him to come up with his remarkably elegant formula which now bears his name. He assumed a scattering electron that was initially at rest. It is this assumption which renders his formula dubious.

> *The assumption that the scattering electron was initially at rest is unjustified.*

It will be shown why the scattering electron cannot be assumed to be initially at rest.

The x-ray that Compton used in his experiment had a wavelength of 0.711Å. The energy of the incident photon is given by $h\nu$, or 17.4KeV. For a scattering angle of 10°, the value of $\Delta\lambda/\lambda$ is 0.00052 giving the change in photon energy to be 9.04eV. This is also the recoil energy acquired by the scattering electron. The Compton experiment used carbon graphite as the target scattering element. Carbon has an ionization energy of about 11.3eV, the energy required to remove a least tightly bound electron from the atom. This 11.3eV is also the initial kinetic energy of the electron. It is clear that this initial kinetic energy of 11.3eV cannot be ignored when energy conservation consideration gives the recoil energy of the electron acquired from the incident photon to be only 9.0eV. A value assumed to be insignificant come out to be greater than some predicted values of the theory. For the scattering angle of 45°, the recoil energy is 174.3eV. The 11.3eV is still 6.5% of the recoil energy and this may not be considered insignificant and be ignored.

Table 1 shows the calculated scattering data for x-rays of 0.0711nm and 0.3nm. The recoil energy values for 0.3nm x-rays are even lower than that for 0.0711nm suggesting the formula derived based on assuming an electron initially at rest may not be generally applicable.

λ(nm)	E(eV)	Scattering angle°	$\Delta\lambda/\lambda$	ΔE(eV)
0.0711	17,438	10	0.00052	9.04
0.0711	17,438	45	0.01	174.3
0.0711	17,438	90	0.034	595.0
0.0711	17,438	135	0.058	1015.7
0.3	4,132	10	0.00012	0.51
0.3	4,132	45	0.0023	9.8
0.3	4,132	90	0.008	33.4
0.3	4,132	135	0.013	57.1

TABLE 1. Calculated Compton scattering data for various incident x-ray wavelengths and scattering angles.

3. COMPTON SCATTERING IS NOT WELL VERIFIED

The Compton scattering formula has been with us for almost a full hundred years. It has become ubiquitous in atomic physics. It would not be strange if students of physics believe that Compton scattering should have been rigorously tested and therefore may be accepted unquestionably. The facts seem to suggest otherwise. As the author has shown in an earlier paper [3], the Compton scattering formula is not well verified. Since the original experiment of Compton in 1922, no one has repeated the Compton experiment. The original three data points published by Compton for scattering angles of 30° ,45° and 135° remain as the only confirmed data available to date.

Others may disagree pointing to the many modern experiments [4], [5] routinely done in universities around the world with gamma-rays that seem to have verified Compton scattering to a very high degree of accuracy. If this were the truth, it would be a very strong vindication for the Compton scattering theory. Not just so, it would also be a strong vindication of special relativity as Compton scattering was derived base on relativistic mechanics; it would also be a not-too-happy ending for Newtonian mechanics.

But these modern gamma-ray experiments do not constitute any manner of experimental verification for Compton scattering as they do not directly verify the wavelength Compton formula (1). They are designed to verify an intermediate energy relation formula:

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{mc^2}(1 - \cos\theta) \tag{2}$$

Equation (2) is the formula relating the photon energies before and after scattering. Even if these experiments are valid and acceptable, they still would not constitute a verification of the Compton scattering formula (1) which relates the wavelengths. Equation (1) may be derived from (2), but only through introducing the relation $E = h\nu$.

These gamma-ray experiments employ modern energy sensors to measure the energy of the scattered gamma-ray pulses for various angles. These are commercial detectors. They may be NaI scintillation detector or a semiconductor germanium detector. These detectors basically converts the electrical interaction signals gamma-rays have with matter. From these electrical pulses, an energy figure for the gamma-ray pulse is derived. But these detectors needs calibration to map electrical voltages to energy values. The calibration parameters used are all embedded in the software supplied for the detectors. Such detectors are not *'impartial'* in measuring the energy pulses of the gamma-rays. They may be calibrated to be consistent with the relativistic energy scale using :$KE = m_0c^2(\gamma - 1)$ or the classical formula: $KE = \frac{1}{2}mv^2$. The result of experiments would now be dependent on how the commercial manufacturers calibrate their instruments! This situation, of course, cannot be accepted.

4. CONCLUSION

The Compton scattering formula:$\Delta\lambda = \lambda_c(1-\cos\theta)$ was derived by Arthur H. Compton in 1922. It is remarkably elegant and shows the change in the scattered wavelengths to be dependent only on scattering angles. But the simplicity of the formula was achieved only through introducing a simplistic assumption that the scattering electron is initially at rest. It has been shown that such an assumption is unjustified. Though elegant, the Compton scattering formula is now dubious. A proper verification of the formula requires experiments for which the related wavelengths of the scattered x-rays or gamma-rays are directly measured. To date, no such experiments have been done.

REFERENCES

[1] Compton, Arthur H. (May 1923). A Quantum Theory of the Scattering of X-Rays by Light Elements. Physical Review. 21 (5): 483–502.
[2] Compton, Arthur H. The spectrum of scattered X-rays. The Physical Review, 1923, 22 (5).
[3] Chan Rasjid Kah Chew. Compton Scattering Not Well Verified. https://vixra.org/abs/1908.0393
[4] Priyanga Ganesan, Rudrajyoti Palit. "Measurement of Electron Mass using Compton Scattering". DOI: 10.13140/RG.2.1.2781.9280.
[5] Simon Lacoste-Julien, Mathieu Plamondon. "Compton Scattering: Light reveals its particle nature". Lab Report, Department of Physics, McGill University, 2002.

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