Available online at www.sciencedirect.com

ScienceDirect

Solar Energy 112 (2015) 339–350

SOLAR ENERGY

www.elsevier.com/locate/solener

Solar eclipse monitoring for solar energy applications

Ibrahim Reda

National Renewable Energy Laboratory, 15013 Denver West Parkway, Golden, CO, USA

Received 7 August 2014; received in revised form 8 December 2014; accepted 11 December 2014
Available online 6 January 2015

Communicated by: Associate Editor David Renne

Abstract

In recent years, the interest in using solar energy as a major contributor to renewable energy applications has increased, and the focus to optimize the use of electrical energy based on demand and resources from different locations has strengthened. This article includes a procedure for implementing an algorithm to calculate the Moon's zenith angle with uncertainty of $\pm 0.001°$ and azimuth angle with uncertainty of $\pm 0.003°$. In conjunction with Solar Position Algorithm, the angular distance between the Sun and the Moon is used to develop a method to instantaneously monitor the partial or total solar eclipse occurrence for solar energy applications. This method can be used in many other applications for observers of the Sun and the Moon positions for applications limited to the stated uncertainty.

Keywords: Solar eclipse; Moon position; Solar position; Solar energy

1. Introduction

The interest in using solar energy as a major contributor to renewable energy applications has increased, and the focus to optimize the use of electrical energy based on demand and resources from different locations has strengthened. We thus need to understand the Moon's position with respect to the Sun. For example, during a solar eclipse, the Sun might be totally or partially shaded by the Moon at the site of interest, which can affect the irradiance level from the Sun's disk. Instantaneously predicting and monitoring a solar eclipse can provide solar energy users with instantaneous information about potential total or partial solar eclipse at different locations At least five solar eclipses occur yearly, and can last three hours or more, depending on the location. This rare occurrence might have an effect on estimating the solar energy as a resource.

This article includes a procedure for implementing an algorithm (described by Meeus (1998)) to calculate the Moon's zenith angle with uncertainty of $\pm 0.001°$ and azimuth angle with uncertainty of $\pm 0.003°$. The step-by-step format presented here simplifies the complicated steps Meeus describes to calculate the Moon's position, and focuses on the Moon instead of the planets and stars. It also introduces some changes to accommodate for solar radiation applications. These include changing the direction of measuring azimuth angle to start from north and eastward instead of from south and eastward, and the direction of measuring the observer's geographical longitude to be measured as positive eastward from Greenwich meridian instead of negative. In conjunction with the Solar Position Algorithm (SPA) that Reda and Andreas developed in 2004 (Reda and Andreas, 2004), the angular distance between the Sun and the Moon is used to develop a method to instantaneously monitor the partial or total

E-mail address: Ibrahim.Reda@nrel.gov

http://dx.doi.org/10.1016/j.solener.2014.12.010
0038-092X/

solar eclipse occurrence for solar energy and smart grid applications. This method can be used in many other applications for observers of the Sun and the Moon positions for applications limited to the stated uncertainty.

SPA has the details of calculating the solar position, so only the Moon position algorithm (MPA) is included in this report. When the solar position calculation is included in this report, the SPA report will be the source for the SPA calculation.

This article is used to calculate the Moon's position for solar radiation applications only. It is purely mathematical and not meant to teach astronomy or to describe the complex Moon rotation around the Earth. For more information about the astronomical nomenclature that is used throughout the report, review the definitions in the Astronomical Almanac (AA) or other astronomy references.

2. Moon position algorithm

2.1. Calculate the Julian and Julian Ephemeris Day, Century, and Millennium

The Julian date starts on January 1, in the year −4712 at 12:00:00 UT. The Julian Day (JD) is calculated using the Universal Time (UT) and the Julian Ephemeris Day (JDE) is calculated using the Terrestrial Time (TT). In the following steps, there is a 10-day gap between the Julian and Gregorian calendars where the Julian calendar ends on October 4, 1582 (JD = 2,299,160), and on the following day the Gregorian calendar starts on October 15, 1582.

2.1.1. Calculate the Julian Day

$$\mathrm{JD} = \mathrm{INT}(365.25 * (Y + 4716)) + \mathrm{INT}(30.6001 * (M + 1)) + D + B - 1524.5, \tag{1}$$

where INT is the integer of the calculated terms ($8.7 = 8$, $8.2 = 8$, and $-8.7 = -8$, etc.). Y is the year (2001, 2002, etc.). M is the month of the year (1 for January, etc.). If $M > 2$, then Y and M are not changed, but if $M = 1$ or 2, then $Y = Y - 1$ and $M = M + 12$. D is the day of the month with decimal time (e.g., for the second day of the month at 12:30:30 UT, $D = 2.521180556$). B is equal to 0, for the Julian calendar {i.e. by using $B = 0$ in Eq. (1), JD < 2,299,160}, and equal to ($2 - A + \mathrm{INT}$ ($A/4$)) for the Gregorian calendar {i.e. by using $B = 0$ in Eq. (1), and if $JD > 2{,}299{,}160$}; $A = \mathrm{INT}(\mathrm{Y}/100)$.

2.1.2. Calculate the Julian Ephemeris Day

Determine ΔT, which is the difference between the Earth's rotation time and TT. It is derived from observation only and reported yearly in the AA (Astronomical Almanac; US Naval Observatory).

$$\mathrm{JDE} = \mathrm{JD} + \frac{\Delta T}{86,400}. \tag{2}$$

2.1.3. Calculate the Julian Century and the Julian Ephemeris Century for the 2000 standard epoch

$$\mathrm{JC} = \frac{\mathrm{JD} - 2,451,545}{36,525}, \tag{3}$$

$$\mathrm{JCE} = \frac{\mathrm{JDE} - 2,451,545}{36,525}. \tag{4}$$

2.1.4. Calculate the Julian Ephemeris Millennium for the 2000 standard epoch

$$\mathrm{JME} = \frac{\mathrm{JCE}}{10}. \tag{5}$$

2.2. Calculate the Moon geocentric longitude, latitude, and distance between the centers of Earth and Moon (λ, β, and Δ)

"Geocentric" means that the Moon position is calculated with respect to Earth's center.

2.2.1. Calculate the Moon's mean longitude, L′ (in degrees)

$$L' = 218.3164477 + 481267.88123421 * T - 0.0015786 * T^2 + \frac{T^3}{538,841} - \frac{T^4}{65,194,000}, \tag{6}$$

where T is JCE from Eq. (4).

2.2.2. Calculate the mean elongation of the Moon, D (in degrees)

$$D = 297.8501921 + 445267.1114034 * T - 0.0018819 * T^2 + \frac{T^3}{545,868} - \frac{T^4}{113,065,000}. \tag{7}$$

2.2.3. Calculate the Sun's mean anomaly, M (in degrees)

$$M = 357.5291092 + 35999.0502909 * T - 0.0001536 * T^2 + \frac{T^3}{24,490,000}. \tag{8}$$

2.2.4. Calculate the Moon's mean anomaly, M′ (in degrees)

$$M' = 134.9633964 + 477198.8675055 * T + 0.0087414 * T^2 + \frac{T^3}{69699} - \frac{T^4}{14,712,000}. \tag{9}$$

2.2.5. Calculate the Moon's argument of latitude, F (in degrees)

$$F = 93.2720950 + 483202.0175233 * T - 0.0036539 * T^2 - \frac{T^3}{3,526,000} + \frac{T^4}{863,310,000}. \quad (10)$$

2.2.6. Use Table A1 to calculate the term l (in 0.000001 degrees)

$$l = \sum_{i=1}^{n} l_i * \sin(d_i * D + m_i * M + m'_i * M' + f_i * F), \quad (11)$$

where l_i, d_i, m_i, m'_i, and f_i are the ith term in columns l, d, m, m', and f in the table.

The terms in column m depend on the decreasing eccentricity of the Earth's orbit around the Sun; therefore, when the term in column $m = (1 \text{ or } -1)$ or $(2 \text{ or } -2)$, multiply l_i in Eq. (11) by E or E^2, respectively, where:

$$E = 1 - 0.002516 * T - 0.0000074 * T^2. \quad (12)$$

2.2.7. Use Table A1 to calculate the term r (in 0.001 km)

$$r = \sum_{i=1}^{n} r_i * \cos(d_i * D + m_i * M + m'_i * M' + f_i * F). \quad (13)$$

Similar to step 2.2.6, when $m = (1 \text{ or } -1)$ and $(2 \text{ or } -2)$, multiply r_i in Eq. (13) by E or E^2.

2.2.8. Use Table A2 to calculate the term b (in 0.000001 degrees)

$$b = \sum_{i=1}^{n} b_i * \sin(d_i * D + m_i * M + m'_i * M' + f_i F). \quad (14)$$

Similar to step 2.2.6, when $m = (1 \text{ or } -1)$ and $(2 \text{ or } -2)$, multiply b_i in Eq. (14) by E or E^2.

2.2.9. Calculate a_1

$$a_1 = 119.75 + 131.849 * T. \quad (15)$$

2.2.10. Calculate a_2

$$a_2 = 53.09 + 479264.29 * T. \quad (16)$$

2.2.11. Calculate a_3

$$a_3 = 313.45 + 481266.484 * T. \quad (17)$$

2.2.12. Calculate Δl

$$\Delta l = 3958 * \sin(a_1) + 1962 * \sin(L' - F) + 318 * \sin(a_2). \quad (18)$$

2.2.13. Calculate Δb

$$\Delta b = -2235 * \sin(L') + 382 * \sin(a_3) + 175 * (a_1 - F) + 175 * \sin(a_1 + F) + 127 * \sin(L' - M') - 115 * \sin(L' + M'). \quad (19)$$

2.2.14. Calculate the Moon's longitude, λ′ (in degrees)

$$\lambda' = L' + \frac{l + \Delta l}{1,000,000}. \quad (20)$$

2.2.15. Calculate the Moon's latitude, β (in degrees)

$$\beta = \frac{b + \Delta b}{1,000,000}. \quad (21)$$

2.2.16. Limit λ′ and β to the range of 0–360°

Limit λ' and β to the range of 0–360°.

2.2.17. Calculate the Moon's distance from the center of Earth, Δ (in kilometers)

$$\Delta = 385000.56 + \frac{r}{1000}. \quad (22)$$

2.3. Calculate the Moon's equatorial horizontal parallax, π

$$\pi = \frac{6378.14}{\Delta}. \quad (23)$$

2.4. Calculate the nutation in longitude and obliquity (Δψ and Δε)

2.4.1. Calculate the mean elongation of the Moon from the Sun, X_0 (in degrees)

$$X_0 = 297.85036 + 445267.111480 * \text{JCE} - 0.0019142 * \text{JCE}^2 - \frac{\text{JCE}^3}{189,474}. \quad (24)$$

2.4.2. Calculate the mean anomaly of the Sun (Earth), X_1 (in degrees)

$$X_1 = 357.52772 + 35999.050340 * \text{JCE} - 0.0001603 * \text{JCE}^2 + \frac{\text{JCE}^3}{300,000}. \quad (25)$$

2.4.3. Calculate the mean anomaly of the Moon, X_2 (in degrees)

$$X_2 = 134.96298 + 477198.867398 * \mathrm{JCE} + 0.0086972 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{56,250}. \tag{26}$$

2.4.4. Calculate the Moon's argument of latitude, X_3 (in degrees)

$$X_3 = 93.27191 + 483202.017538 * \mathrm{JCE} - 0.0036825 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{327,270}. \tag{27}$$

2.4.5. Calculate the longitude of the ascending node of the Moon's mean orbit on the ecliptic, measured from the mean equinox of the date, X_4 (in degrees)

$$X_4 = 125.04452 - 1934.136261 * \mathrm{JCE} + 0.0020708 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{450,000}. \tag{28}$$

2.4.6. For each row in Table A3, calculate the terms $\Delta\psi_i$ and $\Delta\varepsilon_i$ (in 0.0001of arc seconds)

$$\Delta\psi_i = (a_i + b_i * \mathrm{JCE}) * \sin\left(\sum_{j=1}^{4} X_j * Y_{i,j}\right), \tag{29}$$

$$\Delta\varepsilon_i = (c_i + d_i * \mathrm{JCE}) * \cos\left(\sum_{j=1}^{4} X_j * Y_{i,j}\right), \tag{30}$$

where a_i, b_i, c_i, and d_i are the values listed in the ith row and columns a, b, c, and d in Table A3. X_j is the jth X calculated by using Eqs. (15)–(19). $Y_{i,j}$ is the value listed in ith row and jth Y column in Table A3.

2.4.7. Calculate the nutation in longitude, $\Delta\psi$ (in degrees)

$$\Delta\psi = \frac{\sum_{i=1}^{n}\Delta\psi_i}{36,000,000}, \tag{31}$$

where n is the number of rows in Table A3 (n equals 63 rows in the table).

2.4.8. Calculate the nutation in obliquity, $\Delta\varepsilon$ (in degrees)

$$\Delta\varepsilon = \frac{\sum_{i=1}^{n}\Delta\varepsilon_i}{36,000,000}. \tag{32}$$

2.5. Calculate the true obliquity of the ecliptic, ε (in degrees)

2.5.1. Calculate the mean obliquity of the ecliptic, ε_0 (in arc seconds)

$$\varepsilon_0 = 84381.448 - 4680.93 * U - 1.55 * U^2 + 1999.25 * U^3 - 51.38 * U^4 - 249.67 * U^5 - 39.05 * U^6 + 7.12 * U^7 + 27.87 * U^8 + 5.79 * U^9 + 2.45 * U^{10}, \tag{33}$$

where U is $JME/10$.

2.5.2. Calculate the true obliquity of the ecliptic, ε (in degrees)

$$\varepsilon = \frac{\varepsilon_0}{3600} + \Delta\varepsilon. \tag{34}$$

2.6. Calculate the apparent Moon longitude, λ (in degrees)

$$\lambda = \lambda' + \Delta\psi. \tag{35}$$

2.7. Calculate the apparent sidereal time at Greenwich at any given time, v (in degrees)

2.7.1. Calculate the mean sidereal time at Greenwich, v_0 (in degrees)

$$v_0 = 280.46061837 + 360.98564736629 * (\mathrm{JD} - 2,451,545) + 0.000387933 * \mathrm{JC}^2 - \frac{\mathrm{JC}^3}{38,710,000}. \tag{36}$$

2.7.2. Calculate the apparent sidereal time at Greenwich, v (in degrees)

$$v = v_0 + \Delta\psi * \cos(\varepsilon). \tag{37}$$

2.7.3. Limit v to the range of 0–360°

Limit v to the range of 0–360°.

2.8. Calculate the Moon's geocentric right ascension, α (in degrees)

2.8.1. Calculate the Moon's right ascension, α (in radians)

$$\alpha = Arc\tan\ 2\left(\frac{\sin\lambda * \cos\varepsilon - \tan\beta * \sin\varepsilon}{\cos\lambda}\right), \tag{38}$$

where Arc *tan* 2 is an arctangent function that is applied to the numerator and the denominator (instead of the actual division) to maintain the correct quadrant of α, where α is in the rage of $-\pi$ to π.

2.8.2. Calculate α in degrees, then limit it to the range of 0–360°

Calculate α in degrees, then limit it to the range of 0–360°.

2.9. Calculate the Moon's geocentric declination, δ (in degrees)

$$\delta = Arc\sin(\sin\beta * \cos\varepsilon + \cos\beta * \sin\varepsilon * \sin\lambda), \tag{39}$$

where δ is positive or negative if the Sun is north or south of the celestial equator, respectively. Then change δ to degrees.

2.10. Calculate the observer local hour angle, H (in degrees)

$$H = v + \sigma - \alpha, \tag{40}$$

where σ is the observer geographical longitude, positive or negative for east or west of Greenwich, respectively.

Limit H to the range from 0° to 360° and note that it is measured westward from south in this algorithm.

2.11. Calculate the Moon's topocentric right ascension α′ (in degrees)

"Topocentric" means that the Moon's position is calculated with respect to the observer local position at the Earth's surface.

2.11.1. Calculate the term u (in radians)

$$u = Arc\tan(0.99664719 * \tan\varphi), \tag{41}$$

where ϕ is the observer's geographical latitude, positive or negative if north or south of the equator, respectively. The 0.99664719 number equals $(1 - f)$, where f is the Earth's flattening.

2.11.2. Calculate the term x

$$x = \cos u + \frac{E}{6,378,140} * \cos\varphi, \tag{42}$$

where E is the observer's elevation (in m). Note that x equals $\rho * \cos\phi'$ where ρ is the observer's distance to the center of the Earth, and ϕ' is the observer's geocentric latitude.

2.11.3. Calculate the term y

$$y = 0.99664719 * \sin u + \frac{E}{6,378,140} * \sin\varphi, \tag{43}$$

note that y equals $\rho * \sin\phi'$.

2.11.4. Calculate the parallax in the Moon's right ascension, Δα (in degrees)

$$\Delta\alpha = Arc\tan 2\left(\frac{-x * \sin\pi * \sin H}{\cos\delta - x * \sin\pi * \cos H}\right). \tag{44}$$

then change $\Delta\alpha$ to degrees.

2.11.5. Calculate the Moon's topocentric right ascension α′ (in degrees)

$$\alpha' = \alpha + \Delta\alpha. \tag{45}$$

2.11.6. Calculate the topocentric Moon's declination, δ′ (in degrees)

$$\delta' = Arc\tan 2\left(\frac{(\sin\delta - y * \sin\pi) * \cos\Delta\alpha}{\cos\delta - y * \sin\pi * \cos H}\right). \tag{46}$$

2.12. Calculate the topocentric local hour angle, H′ (in degrees)

$$H' = H - \Delta\alpha. \tag{47}$$

2.13. Calculate the Moon's topocentric zenith angle, θ_m (in degrees)

2.13.1. Calculate the topocentric elevation angle without atmospheric refraction correction, e_0 (in degrees)

$$e_0 = Arc\sin(\sin\varphi * \sin\delta' + \cos\varphi * \cos\delta' * \cos H'). \tag{48}$$

then change e_0 to degrees.

2.13.2. Calculate the atmospheric refraction correction, Δe (in degrees)

$$\Delta e = \frac{P}{1010} * \frac{283}{273 + T} * \frac{1.02}{60 * \tan\left(e_0 + \frac{10.3}{e_0 + 5.11}\right)}, \tag{49}$$

where P is the annual average local pressure (in millibars). T is the annual average local temperature (in °C). e_0 is in degrees. Calculate the tangent argument in degrees, then convert to radians.

2.13.3. Calculate the topocentric elevation angle, e (in degrees)

$$e = e_0 + \Delta e. \tag{50}$$

2.13.4. Calculate the topocentric zenith angle, θ (in degrees)

$$\theta_m = 90 - e. \tag{51}$$

2.14. Calculate the Moon's topocentric azimuth angle, Φ_m (in degrees)

2.14.1. Calculate the topocentric astronomers' azimuth angle, Γ (in degrees)

$$\Gamma = Arc\tan 2\left(\frac{\sin H'}{\cos H' * \sin\varphi - \tan\delta' * \cos\varphi}\right), \tag{52}$$

Change Γ to degrees, then limit it to the range of 0–360°. Γ is measured *westward* from *south*.

2.14.2. Calculate the topocentric azimuth angle, Φ_m for navigators and solar radiation users (in degrees)

$$\Phi_m = \Gamma + 180, \tag{53}$$

Limit Φ_m to the range from 0° to 360°. Φ_m is measured *eastward* from *north*.

3. Moon position algorithm validation

To evaluate the uncertainty of the MPA, arbitrary dates, January 17 and October 17, are chosen from each of the years 2004 to 2010, and 1981, at 0-h TT. Fig. 1 shows that the maximum difference between the AA and MPA is 0.00055° for the Moon's declination, Fig. 2 shows that the maximum difference is 0.00003° for the equatorial Moon parallax, and Fig. 3 shows that the maximum difference in the calculated zenith or azimuth angles is 0.0003° and 0.00075°, respectively. This implies that the MPA is well within the stated uncertainty of ±0.001° and ±0.003° in the zenith and azimuth angles, respectively.

4. Predicting and monitoring the solar eclipse occurrence

The full astronomical nomenclature for eclipse monitoring is beyond the scope of this report, so only the total and partial solar eclipse nomenclatures are used. In this section, the zenith and azimuth angles of the Sun and the Moon are calculated continuously using SPA described in Reda and Andreas (2004), and the Moon Position Algorithm described above. A copyrighted Solar and Moon Position Algorithm (SAMPA) software and calculator were developed by Andreas and Reda (2012), and then used to monitor the solar eclipse as follows:

4.1. Calculate the local observed, topocentric, angular distance between the Sun and Moon centers, E_{sm} (in degrees)

$$E_{ms} = \cos^{-1}[\cos\theta_s * \cos\theta_m + \sin\theta_s * \sin\theta_m * \cos(\phi_s - \phi_m)], \tag{54}$$

where θ_s and ϕ_s are the zenith and azimuth angles of the Sun, calculated using SPA, 2003.

4.2. Calculate the radius of the Sun's disk, r_s (in degrees)

$$r_s = \frac{959.63}{3600 * R_s}, \tag{55}$$

where R_s is the Sun's distance from the center of the Earth, in astronomical units (AUs). This distance is calculated in SPA (Reda and Andreas, 2004).

4.3. Calculate the radius of the Moon's disk, r_m (in degrees)

$$r_m = \frac{358,473,400 * (1 + \sin e * \sin\pi)}{3600 * \Delta}, \tag{56}$$

where e, π, and Δ are calculated in Section 2.

4.4. Set the boundary conditions for the solar eclipse

4.4.1. No eclipse

$$E_{ms} > (r_m + r_s),$$

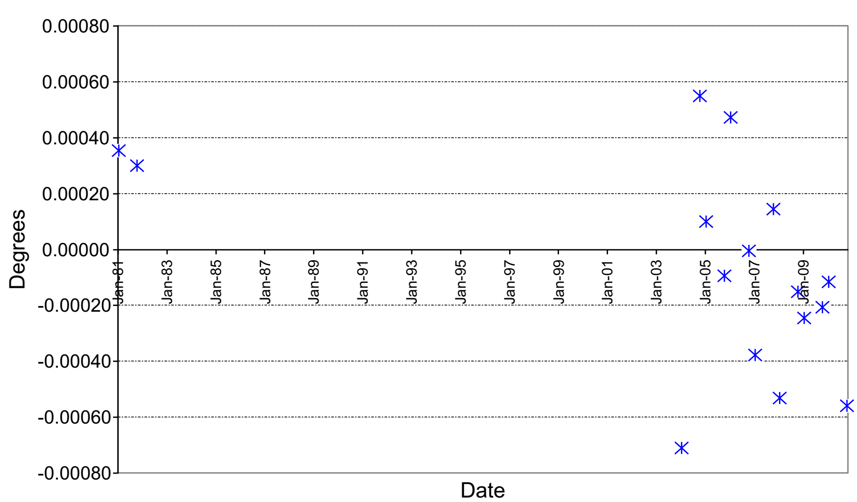

Fig. 1. Difference between the AA and MPA for the Moon's declination.

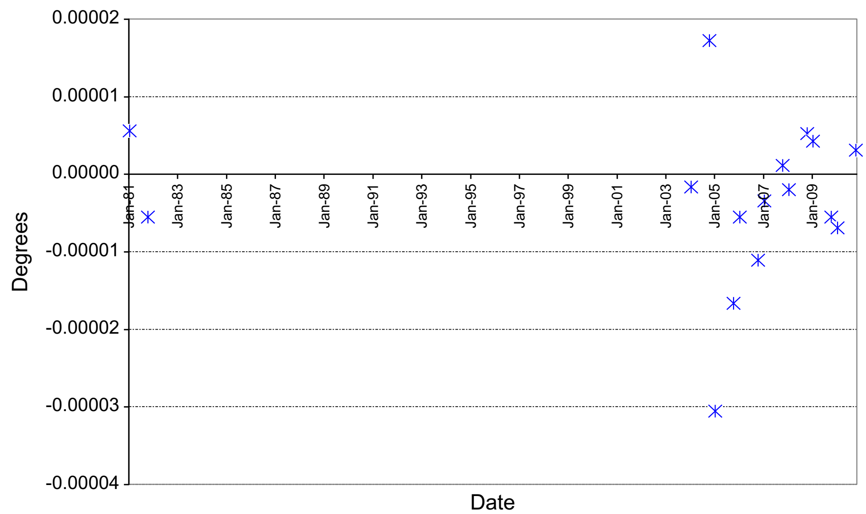

Fig. 2. Difference between the AA and MPA for the Moon's horizontal parallax.

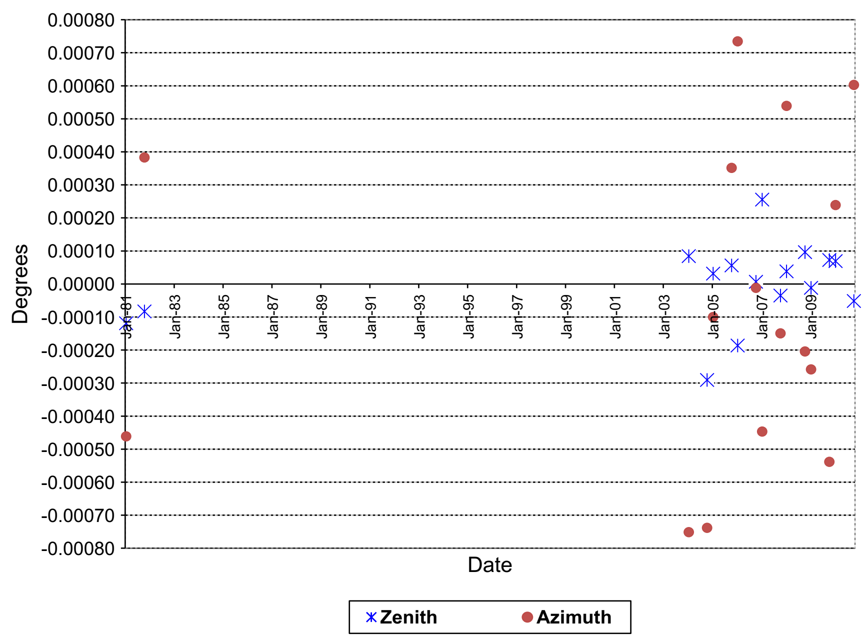

Fig. 3. Difference in the calculated zenith or azimuth angles using the differences in Figs. 1 and 2.

where r_m and r_s are the Sun and Moon radii.

4.4.2. Start and end of eclipse

$$E_{ms} = (r_m + r_s).$$

4.4.3. Solar eclipse

$$E_{ms} < (r_m + r_s).$$

4.4.4. Sun disk area during eclipse

If $E_{ms} \leqslant abs(r_m - r_s)$, it is a total eclipse where the Sun and Moon disks (circles) completely overlap; therefore, if $r_s > r_m$, the unshaded Sun area by the Moon will equal the area of the Sun disk minus the area of the Moon disk. Moreover, if $r_s \leqslant r_m$, the unshaded area of the Sun equals zero.

To monitor the solar eclipse, a criterion where E_{ms} equals $(r_m + r_s)$ is set at the beginning of the eclipse. At this moment the time is noted as T_{start} and then E_{ms} is recalculated every second for at least three hours. The distance E_{ms} can then be plotted against time to show the progress of the eclipse. From the plotted data, one might predict the time of a partial or total solar eclipse by calculating the time when minimum E_{ms} occurs, $T_{\min}$, then as the eclipse ends, the time when E_{ms} equals $(r_m + r_s)$, the time T_{end} is noted. The total duration for the eclipse occurrence will equal $T_{\text{end}} - T_{\text{start}}$. Fig. 4 shows E_{ms} versus time for the central solar eclipse on July 22, 2009 (see Table 1 for coordinates). Using this method, the minimum $E_{ms} = 0.0001°$, which is well within the uncertainty of calculating the Sun and Moon positions. To verify this method, E_{ms} is calculated for some historical total solar eclipses at different locations. Table 1 shows that $E_{ms} < 0.0011$, which is within the stated uncertainty of $\pm 0.003°$.

5. Estimating the solar irradiance during a solar eclipse

When a solar eclipse occurs, the Moon's disk will start shading the Sun's disk; the shaded area will change as time progresses; therefore, the unshaded area of the Sun disk is called the Sun's Unshaded Lune (SUL), which will also change with time. The percentage of the SUL, from the total Sun's disk area, is then calculated. The percentage area might then be multiplied by an estimated direct beam irradiance to calculate the irradiance during the eclipse. A spectacular phenomena occurs during the solar eclipse, when the spectral distribution of the irradiance from the Sun changes. The method described below illustrates how the irradiance might be estimated during the solar eclipse. Users might use other methods or models to achieve smaller uncertainty for such estimates.

5.1. Calculate the area of SUL, A_{SUL}

To calculate this area, draw two intersecting circles, with two different radii of the Sun and the Moon, r_s and r_m. An illustration is shown in Fig. 1.

$$A_{\mathrm{SUL}} = \pi * r_s^2 - A_i, \tag{57}$$

where A_i is the area of the Sun's disk that is shaded by the Moon. A step-by-step method to calculate A_{SUL} is described in Appendix B.

5.2. Calculate the percentage area of the SUL with respect to the area of the Sun's disk, $\%A_{SUL}$

$$\%A_{\mathrm{SUL}} = \frac{A_{\mathrm{SUL}} * 100}{\pi * r_s^2}. \tag{58}$$

5.3. Calculate the direct beam irradiance using the appropriate model for the required uncertainty, in W/m²

The Bird and Hulstrom simple model (Bird and Hulstrom, 1981) is used in this article as an illustration.

5.4. Calculate the irradiance (W/m²) during the eclipse, I_e

$$I_e = \frac{I * \%A_{\mathrm{SUL}}}{100}, \tag{59}$$

where I is the direct beam irradiance calculated by the model.

To evaluate the described method, the calculated irradiance is compared against the irradiance measured at the University of Oregon, Eugene, Solar Radiation Monitoring Laboratory. The irradiance was measured during the June 10, 2002 partial eclipse, using a pyrheliometer model NIP, manufactured by the Eppley Laboratory, Inc. Fig. 4 shows the difference between the measured irradiance and the calculated irradiance using the method described above. The figure shows that the difference between the measured irradiance at the University of Oregon and the calculated irradiance by SAMPA is about 8% when the partial eclipse starts, 4% at the maximum eclipse, and 6% as the eclipse ends. These differences are expected, because the measuring instruments (estimated $U_{95} = \pm 3\%$) and the model used above (estimated $U_{95} = \pm 5\%$) do not account for the significant change in the spectral distribution of the irradiance during the solar eclipse occurrence. In the future, with the advancement in pyrheliometer design and spectral measurement technology, advanced models might be used to improve the uncertainty in measuring such significant change in the spectral distribution during the eclipse occurrence.

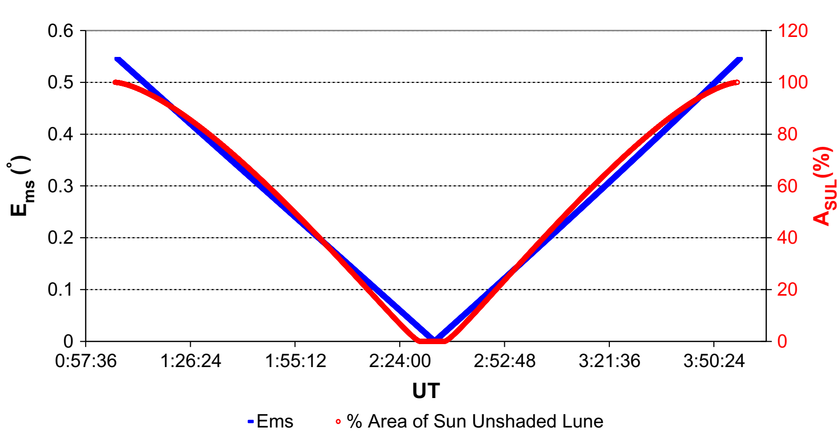

Fig. 4. Distance between the Sun and Moon centers and the percentage of the Sun unshade lune (SUL) for the July 22, 2009 solar eclipse.

Table 1
Historical solar eclipses versus SAMPA eclipse monitor, E_{ms}.

Historical eclipse dates	UT	Observer's longitude °	Observer's latitude °	SAMPA, $E°_{ms}$
7/22/2009	2:33:00	143.3617	24.6117	0.0001
8/1/2008	9:47:18	34.7417	81.1133	0.0002
3/29/2006	10:33:18	22.8867	29.6200	0.0005
4/8/2005	20:15:36	−123.4817	−15.7883	0.0011
12/4/2002	7:38:42	62.8383	−40.5283	0.0005
6/21/2001	11:57:48	0.9867	−11.5950	0.0003
2/4/1981	21:57:36	−145.9033	−45.8883	0.0004

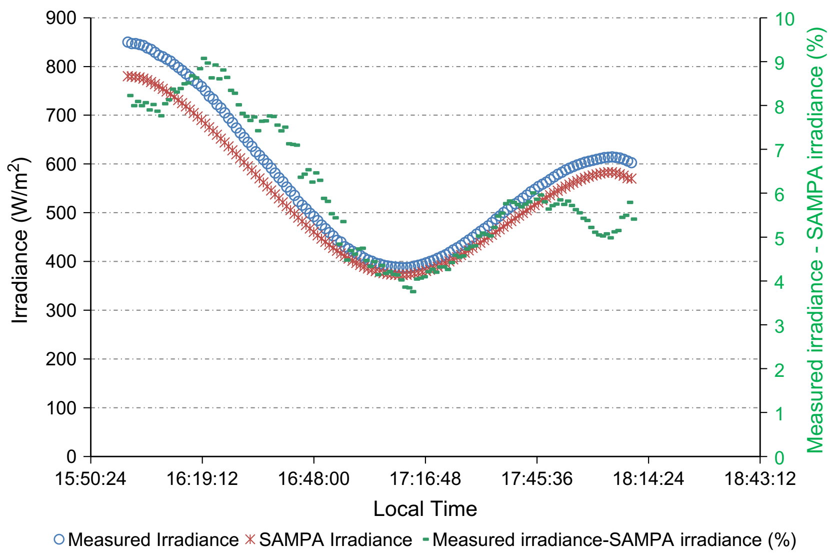

Fig. 5. Measured irradiance versus calculated irradiance using SAMPA during the June 10, 2002 partial solar eclipse.

6. Conclusions

The MPA achieves uncertainties of ±0.001° and ±0.003° in calculating the zenith and azimuth angles of the Moon (see Figs. 1 and 2). Using MPA in conjunction with the SPA (uncertainty of ±0.0003°) to monitor solar eclipses, is consistent with the historical eclipses to within the stated uncertainty of the MPA, see Table 1. Section 5 and Fig. 5 show that the direct beam irradiance from the Sun during a solar eclipse is estimated within 4–8% from measured irradiance that was collected during the eclipse of June 10, 2002. This implies that solar energy users might be able, with the current technology, to use this information to manage the grid's solar resources during eclipses to within the model's limitations. Improved uncertainties might be achieved by developing advanced models that include the change of the spectral distribution during the spectacular solar eclipse. Fig. 4 also shows that the partial solar eclipse of June 10, 2002 in Eugene, Oregon, lasted longer than two hours, which might have an effect on estimating the solar energy when used as a resource.

Acknowledgments

I would like to thank the Quality Management and Assurance office, NREL Metrology Laboratory, and DOE-ARM and Solar programs for providing the funds for this publication. I extend special appreciation to my wife Mary Alice and my daughter Lenah for their inspiration about the Moon's influence on romance, legends, and art. I also thank the staff from the University of Oregon for providing the measured data during the solar eclipse in Oregon, USA.

Appendix A. Tables

Tables A1–A3.

Appendix B. Sun and Moon lunes

Note that some symbols used in this appendix are independent from those used in the main report.

Table A1
Moon's periodic terms for longitude and distance.

d	M	m'	f	l	R
0	0	1	0	6,288,774	−20,905,355
2	0	−1	0	1,274,027	−3,699,111
2	0	0	0	658,314	−2,955,968
0	0	2	0	213,618	−569,925
0	1	0	0	−185,116	48,888
0	0	0	2	−114,332	−3149
2	0	−2	0	58,793	246,158
2	−1	−1	0	57,066	−152,138
2	0	1	0	53,322	−170,733
2	−1	0	0	45,758	−204,586
0	1	−1	0	−40,923	−129,620
1	0	0	0	−34,720	108,743
0	1	1	0	−30,383	104,755
2	0	0	−2	15,327	10,321
0	0	1	2	−12,528	
0	0	1	−2	10,980	79,661
4	0	−1	0	10,675	−34,782
0	0	3	0	10,034	−23,210
4	0	−2	0	8548	−21,636
2	1	−1	0	−7888	24,208
2	1	0	0	−6766	30,824
1	0	−1	0	−5163	−8379
1	1	0	0	4987	−16,675
2	−1	1	0	4036	−12,831
2	0	2	0	3994	−10,445
4	0	0	0	3861	−11,650
2	0	−3	0	3665	14,403
0	1	−2	0	−2689	−7003
2	0	−1	2	−2602	
2	−1	−2	0	2390	10,056
1	0	1	0	−2348	6322
2	−2	0	0	2236	−9884
0	1	2	0	−2120	5751
0	2	0	0	−2069	
2	−2	−1	0	2048	−4950
2	0	1	−2	−1773	4130
2	0	0	2	−1595	
4	−1	−1	0	1215	−3958
0	0	2	2	−1110	
3	0	−1	0	−892	3258
2	1	1	0	−810	2616
4	−1	−2	0	759	−1897
0	2	−1	0	−713	−2117
2	2	−1	0	−700	2354
2	1	−2	0	691	
2	−1	0	−2	596	
4	0	1	0	549	−1423
0	0	4	0	537	−1117
4	−1	0	0	520	−1571
1	0	−2	0	−487	−1739
2	1	0	−2	−399	
0	0	2	−2	−381	−4421
1	1	1	0	351	
3	0	−2	0	−340	
4	0	−3	0	330	
2	−1	2	0	327	
0	2	1	0	−323	1165
1	1	−1	0	299	
2	0	3	0	294	
2	0	−1	−2		8752

Table A2
Periodic terms for the Moon's latitude.

D	m	m'	f	b
0	0	0	1	5,128,122
0	0	1	1	280,602
0	0	1	−1	277,693
2	0	0	−1	173,237
2	0	−1	1	55,413
2	0	−1	−1	46,271
2	0	0	1	32,573
0	0	2	1	17,198
2	0	1	−1	9266
0	0	2	−1	8822
2	−1	0	−1	8216
2	0	−2	−1	4324
2	0	1	1	4200
2	1	0	−1	−3359
2	−1	−1	1	2463
2	−1	0	1	2211
2	−1	−1	−1	2065
0	1	−1	−1	−1870
4	0	−1	−1	1828
0	1	0	1	−1794
0	0	0	3	−1749
0	1	−1	1	−1565
1	0	0	1	−1491
0	1	1	1	−1475
0	1	1	−1	−1410
0	1	0	−1	−1344
1	0	0	−1	−1335
0	0	3	1	1107
4	0	0	−1	1021
4	0	−1	1	833
0	0	1	−3	777
4	0	−2	1	671
2	0	0	−3	607
2	0	2	−1	596
2	−1	1	−1	491
2	0	−2	1	−451
0	0	3	−1	439
2	0	2	1	422
2	0	−3	−1	421
2	1	−1	1	−366
2	1	0	1	−351
4	0	0	1	331
2	−1	1	1	315
2	−2	0	−1	302
0	0	1	3	−283
2	1	1	−1	−229
1	1	0	−1	223
1	1	0	1	223
0	1	−2	−1	−220
2	1	−1	−1	−220
1	0	1	1	−185
2	−1	−2	−1	181
0	1	2	1	−177
4	0	−2	−1	176
4	−1	−1	−1	166
1	0	1	−1	−164
4	0	1	−1	132
1	0	−1	−1	−119
4	−1	0	−1	115
2	−2	0	1	107

Table A3
Periodic terms for the nutation in longitude and obliquity.

$Y0$	Coefficients for sin terms				Coefficients for $\Delta\psi$		Coefficients for $\Delta\varepsilon$	
	$Y1$	$Y2$	$Y3$	$Y4$	a	b	c	d
0	0	0	0	1	−171,996	−174.2	92,025	8.9
−2	0	0	2	2	−13,187	−1.6	5736	−3.1
0	0	0	2	2	−2274	−0.2	977	−0.5
0	0	0	0	2	2062	0.2	−895	0.5
0	1	0	0	0	1426	−3.4	54	−0.1
0	0	1	0	0	712	0.1	−7	
−2	1	0	2	2	−517	1.2	224	−0.6
0	0	0	2	1	−386	−0.4	200	
0	0	1	2	2	−301		129	−0.1
−2	−1	0	2	2	217	−0.5	−95	0.3
−2	0	1	0	0	−158			
−2	0	0	2	1	129	0.1	−70	
0	0	−1	2	2	123		−53	
2	0	0	0	0	63			
0	0	1	0	1	63	0.1	−33	
2	0	−1	2	2	−59		26	
0	0	−1	0	1	−58	−0.1	32	
0	0	1	2	1	−51		27	
−2	0	2	0	0	48			
0	0	−2	2	1	46		−24	
2	0	0	2	2	−38		16	
0	0	2	2	2	−31		13	
0	0	2	0	0	29			
−2	0	1	2	2	29		−12	
0	0	0	2	0	26			
−2	0	0	2	0	−22			
0	0	−1	2	1	21		−10	
0	2	0	0	0	17	−0.1		
2	0	−1	0	1	16		−8	
−2	2	0	2	2	−16	0.1	7	
0	1	0	0	1	−15		9	
−2	0	1	0	1	−13		7	
0	−1	0	0	1	−12		6	
0	0	2	−2	0	11			
2	0	−1	2	1	−10		5	
2	0	1	2	2	−8		3	
0	1	0	2	2	7		−3	
−2	1	1	0	0	−7			
0	−1	0	2	2	−7		3	
2	0	0	2	1	−7		3	
2	0	1	0	0	6			
−2	0	2	2	2	6		−3	
−2	0	1	2	1	6		−3	
2	0	−2	0	1	−6		3	
2	0	0	0	1	−6		3	
0	−1	1	0	0	5			
−2	−1	0	2	1	−5		3	
−2	0	0	0	1	−5		3	
0	0	2	2	1	−5		3	
−2	0	2	0	1	4			
−2	1	0	2	1	4			
0	0	1	−2	0	4			
−1	0	1	0	0	−4			
−2	1	0	0	0	−4			
1	0	0	0	0	−4			
0	0	1	2	0	3			
0	0	−2	2	2	−3			
−1	−1	1	0	0	−3			
0	1	1	0	0	−3			
0	−1	1	2	2	−3			
2	−1	−1	2	2	−3			
0	0	3	2	2	−3			
2	−1	0	2	2	−3			

B.1. Calculating the areas of the lunes when two circles with different diameters intersect

The following steps are intended for calculating the area of the Sun Unshaded Lune, A_{SUL}, during solar eclipses. The Sun and Moon radii are not equal and change with the day of the year. Also, during the solar eclipse, as the Moon starts to shade the Sun disk, the intersection area changes with time. In Fig. B1, the circles with centers C_s and C_m represent the Sun and Moon disks, respectively. Refer to the figure to calculate A_{SUL}, bounded by sector aebq.

B.2. Calculate the areas of triangles T_s (bounded by abC_s) and T_m (bounded by abC_m), then calculate A_{SUL}

1. Use the procedure described in this article to calculate the distance between the Sun and Moon centers, E_{ms}.
2. Write the following equation:

$$E_{ms} = m + s, \tag{B1}$$

where m is the distance cC_m, s is the distance cC_s.

Note that m and s are the heights of the two triangles.

3. Using the Pythagorean theorem:

$$h^2 = r_s^2 - s^2 = r_m^2 + m^2, \tag{B2}$$

where r_s and r_m are the Sun and Moon radii, calculated using the procedure described in this report. h is half the base of the two triangles, T_s and T_m.

Therefore:

$$r_s^2 - s^2 = r_m^2 - m^2. \tag{B3}$$

4. Solve Eqs. (B1) and (B3) with two unknowns to calculate s and m:

$$s = \frac{E_{ms}^2 + r_s^2 - r_m^2}{2 * E_{ms}}, \tag{B4}$$

and

$$m = \frac{E_{ms}^2 - r_s^2 + r_m^2}{2 * E_{ms}}. \tag{B5}$$

5. Use Eqs. (B2) and (B4) to calculate h:

$$h = \frac{\sqrt{4 * E_{ms}^2 * r_s^2 - (E_{ms}^2 + r_s^2 - r_m^2)^2}}{2 * E_{ms}}. \tag{B6}$$

6. Calculate the area of triangle abC_s, T_s:

$$T_s = h * s. \tag{B7}$$

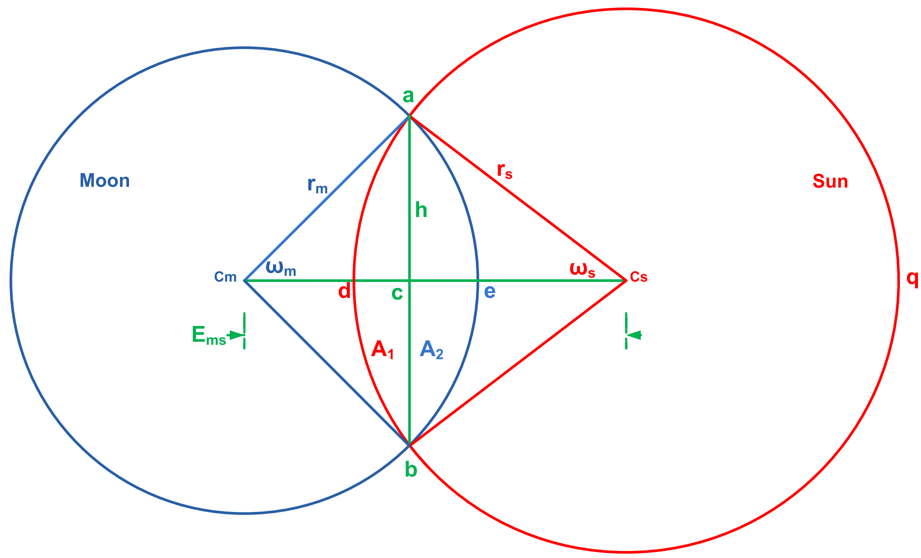

Fig. B1. Intersecting circles to calculate the areas of the Sun and Moon lunes.

7. Calculate the area of triangle abC_m, T_m:

$$T_m = h * m. \tag{B8}$$

8. Calculate the area of sector $adbC_s$ in the Sun's circle, A_s:

$$A_s = \pi * r_s^2 * \frac{2 * \omega_s}{360} = r_s^2 * \cos^{-1}\frac{s}{r_s}, \tag{B9}$$

where ω_s is half the central angle of sector $adbC_s$ in the Sun's circle.

9. Calculate the area of section abd in the Sun's circle, A_1:

$$A_1 = A_s - T_s. \tag{B10}$$

10. Similar to Eq. (B9), calculate the area of sector $aebC_m$ in the Moon's circle, A_m,

$$A_m = r_m^2 * \cos^{-1}\frac{m}{r_m}. \tag{B11}$$

11. Calculate the area of section abe in the Moon's circle, A_2:

$$A_2 = A_m - T_m. \tag{B12}$$

12. Calculate the area of the Sun's circle shaded by the Moon's circle, A_i:

$$A_i = A_1 + A_2. \tag{B13}$$

13. Calculate the Sun's Unshaded Lune, A_{SUL}:

$$A_{SUL} = \pi * r_s^2 - A_i. \tag{B14}$$

References

Andreas, A., Reda, I., 2012. Solar and Moon Position Algorithm (SAMPA). National Renewable Energy Laboratory, Golden, CO, USA, <http://www.nrel.gov/midc/sampa/>.

Bird, R.E., Hulstrom, R.L., 1981. Simplified Clear Sky Model for Direct and Diffuse Insolation on Horizontal Surfaces, Technical Report No. SERI/TR-642-761. Solar Energy Research Institute, Golden, CO, USA.

Meeus, J., 1998. Astronomical Algorithms, 2nd ed. Willmann-Bell, Inc., Richmond, VA, USA.

Reda, I., Andreas, A., 2004. Solar position algorithm for solar radiation applications. Solar Energy 76 (5), 577–589. http://dx.doi.org/10.1016/j.solener.2003.12.003, NREL Report No. JA-560-35518.

The Astronomical Almanac <http://asa.usno.navy.mil/>.

The U.S. Naval Observatory. Washington, DC, USA <http://www.usno.navy.mil/> (accessed 24.07.14).