Relativity of Simultaneity, the Global Positioning System and Some Questions About Gravity

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We rarely stop to think how much our conceptions of the physical world are colored by the theories we use to describe it. In the minds of most physicists the idea of rotating and falling clocks communicating with one another by light signals will immediately evoke features of Newton's Theory of Gravitation and Einstein's Special and General Theories of Relativity. While acknowledging the incalculable value of these theories, we here reconsider the phenomena mentioned above, trying at every step to distinguish *physical* facts from *theoretical* facts; i.e., to distinguish concrete *reality* from *abstraction* based on reality; and to distinguish what we know from what we assume. As of course must be the case, the resulting new perspective agrees with the theories mentioned throughout the range over which they have been tested. But it also turns up a couple blindspots where feasible experiments would reveal which perspective is closer to the truth.

1 Introduction

Leaving most gravity issues for later, we begin with the roots of Einstein's Special Theory of Relativity (SR). [1] A curious and well known fact about SR is that the equations at its core, the Lorentz transformations, were first derived by Lorentz [2] – whose conceptions of space and time differed from those of Einstein. Lorentz assumed the existence of a "luminiferous ether." Einstein did not. Yet the respective theories of these authors yield the same predictions for physical phenomena. There are good reasons that Einstein's interpretation has turned out to prevail in the course of time. And I do not intend to revive Lorentz's views of the ether. Nevertheless, it is worthwhile to reconsider some of the ideas that motivated Einstein to give the Lorentz transformations the interpretation that he did. In particular, we shall look critically at what Einstein called "the most important, and also the most controversial theorem" of SR: the relativity of simultaneity. [3]

It is essential that we remain alert to the thought-coloring effect of theoretical knowledge. A way to do this is to be perpetually engaged in assessing the distinction between *raw physical facts* and the abstractions that remain after distillation and mixing by theory. Since this is a recurring theme of this paper, in §2 I present an example to clarify my intent.

In §3 we set up a base inertial system much like Einstein's "stationary system." In §4 we add a second inertial system and address the significance of the speed of light in any attempt to relate the one system to the other. With the benefit of hindsight, we here also consider how the ideas of Reichenbach [4] (from the mid 1920's) bear on the way Einstein set up SR. Reichenbach emphasized the interdependence between the relativity of simultaneity and *one-way* light propagation assumptions. Einstein, of course, assumed that every inertial observer has the right to regard the one-way speed of light as being equal to the two-way speed of light (second postulate). Thus, one of Einstein's recurring themes was the idea that everybody has a right to think of themselves as being *at rest*; that this is the most satisfactory point of view from which to sort out the "laws of nature." In §5 we take a look at how far Einstein took the idea of "self rest."

In §6 we introduce a uniformly rotating frame of reference centered on our original base system. Thus we come to the Sagnac effect and comments by various authors concerning how rotation affects the problem of clock synchronization.

If our rotating frame were a sphere, in §7 we temporarily suspend its rotation, but instead endow it with mass. Finally, we consider the sphere having both rotation and mass, and discuss how the combination of these effects applies to the problem of clock synchronization in the Global Positioning System (GPS).

Having up to this point not strayed too far beyond the bounds of standard ideas, §8 is where we begin to suggest some new ones. In the interest of improving palatability, these new ideas are presented as being the hypotheses that a team of imaginary space explorers might come up with to explain their *first* encounter with a large gravitating body.

Coming back to the real world in §9, the highest priority idea that the space explorers conceive to test their hypotheses is discussed in more detail, because it is the most clear-cut and most feasible way to decide between them. The experiment would be a test of gravitational *interior* solutions. In this case the prediction of our space explorers deviates not only from General Relativity, [5] but from Newton's Theory of Gravitation. [6] Since their first experience with gravity is entirely different from Newton's and Einstein's experiences, we should not be too surprised to find that our space explorers would devise a profoundly different hypothesis.

In §10 additional hypothesis-testing ideas are discussed in terms of experiments and observations that have already been carried out from Earth: the Shapiro time-delay test [7] and the Vessot-Levine falling clock experiment. [8] It is predicted that a new variation of the latter experiment might yield deviations from the predictions of General Relativity (GR).

In §11 we discuss the falling clock experiment as being conducted in a frame of reference undergoing uniform linear acceleration to show that the new prediction actually agrees more exactly with Einstein's original statement of his Equivalence Principle.

Though some of the predictions of the new hypothesis deviate from Newton's and Einstein's theories, in §12 it is shown that it is nevertheless consistent with the operation of the GPS and other well known consequences of GR.

In §13 we point out further implications of the new hypothesis. We conclude by emphasizing that, even if physicists trained in the Newton-Einstein tradition have reasons to doubt the new hypothesis, the experiments proposed to test it are still worthwhile. Their results would still contribute significantly to our empirical knowledge of gravity.

2 Facts and Abstractions

There are different kinds of facts. Some are direct reflections of physical reality, what may be called *raw physical* facts. And some are derived from theories *about* physical reality. It is often the case that physical facts are perceptible by children or animals. Whereas theoretical facts require a more sophisticated level of thought to invent or comprehend. Since theories *about* physical reality are a step or more removed from physical reality itself, the latter kinds of facts are clearly *abstractions*. Of course there is nothing wrong with abstractions; they are quite necessary. But they are inevitably colored with preconceptions and tacit assumptions. An abstraction can and should be argued about; it is open to debate. Whereas, unless one subscribes to a school of philosophy in which physical reality itself is doubted, raw physical facts are not open to debate. An example follows.

Suppose we have, in the far reaches of outer space, a long hollow cylinder being propelled along its axial direction by a rocket engine. Inside the cylinder a test object is released from the top (leading end). Observations reveal the raw physical fact: *the distance between the released object and the bottom (floor) of the cylinder diminishes at an accelerated rate*. (Except to acknowledge that they exist, this paper is not the least bit concerned with the opinions of those who would debate this kind of fact.) Since accelerometers attached to the rocket give positive readings that correspond exactly to the observed motion, and an accelerometer attached to the falling object gives a zero reading, we may conclude further that the reason the distance between object and floor diminishes is because the rocket accelerates while the object moves only uniformly. The explanation for why the distance diminishes is so clearly correlated with observable physical facts that this phenomenon can reasonably be claimed to be understood.

Now suppose we have a similar cylinder oriented vertically on Earth's surface but there is no rocket. The distance between the released object and the floor diminishes at an accelerated rate. Accelerometers attached to the cylinder give positive readings which correspond exactly to the observed motion, and an accelerometer attached to the released object gives a zero reading. The physical facts are nearly the same as before. But in this case the reasons are not so obvious. The usual description of this circumstance is that the released object accelerates downward; the usual description of Earth and its surrounding space is that they are essentially *static* things. The accelerometer readings are *anti*-correlated with these descriptions. Therefore the facts do not yield so clearcut an explanation as in the previous scenario. What are we missing?

In our attempts to correlate the facts, the cylinder on Earth scenario is commonly discussed in terms of gravitational attraction, gravitational fields, the warpage of spacetime, geodesic vs non-geodesic motion, the Equivalence Principle, force mediation by gravitons, and so on. Each of these concepts is part of one little story or another whose purpose it is to make sense of the facts. Though useful as "talking points," the explanations these ideas provide for the facts, being uncertain and incomplete, are best regarded as tentative. Depending on which idea is involved, we will even find different assessments as to whether or not the accelerometers are reliable indicators of motion. More experiments and perhaps more ideas are needed to decide among the possibilities.

To conclude then, in the cylinder on Earth scenario, either we are misinterpreting some relevant facts, we are missing some relevant facts, or both. We know the distance between the released object and the floor diminishes and that accelerometers behave as described above, but *we don't know exactly why*. We cannot reasonably claim to understand this phenomenon as we do the rocket in space phenomenon. So we acknowledge what few physical facts we have and assiduously refrain from giving our abstractions the same status.

3 Base Reference Frame

Our base system is an inertial reference frame (IRF), which we will sometimes also refer to as an inertial system, or simply, reference frame. It should be emphasized at the outset that an IRF is rather artificial. There is no good reason to think a strictly inertial system of finite extent would ever be found in Nature. Our conception and construction of such a thing is guided by simplicity. Many physical problems are clearly simplified by assuming that phenomena are being observed from the vantage point of an idealized inertial system.

Imagine in empty space a massless grid. Suppose the

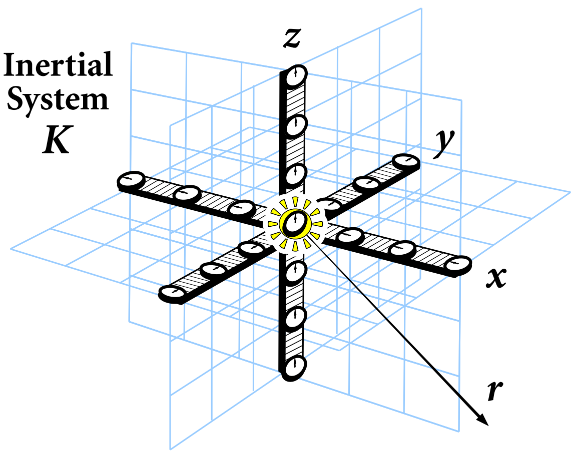

Fig. 1: Base reference system. Clocks are preset to time r/c and start ticking upon receipt of flash from center.

lengths between intersections are known. Identically constructed clocks at the intersections are synchronized by a flash of light from the origin. (See Figure 1.) The idea is essentially the same as that used by Taylor and Wheeler, in their book, *Spacetime Physics*. [9] Each clock is preset to the time $t = r/c$, where r is the radial distance and c is the speed of light. The clocks are all programmed to begin ticking at the precise moment of receiving the signal. At the moment the flash is emitted, the clock at the origin, whose preset time $= 0$, also begins ticking. Since we make no pretense of representing a physically realistic system (absence of asymmetries and inhomogeneities) we are justified to assume that the speed of light is exactly c in all directions. Therefore, it is reasonable to assert that, by the above procedure, all clocks in this system have been synchronized with one another. Let's call this system, K.

Since this synchronization procedure is equivalent to the various procedures prescribed by Einstein we will refer to it (them) as the *Einstein* or *standard* procedure, whose result is Einstein or standard "synchrony."

4 Uniform Velocity

4.1 Two Postulates

Static emptiness is easy to conceive and problem-free. Things suddenly become complicated and controversial, however, by adding a second reference system, say K', that moves uniformly with the speed v with respect to K. As is well known, the root of the problem is the speed of light and its importance for synchronizing clocks in a given frame. Within our base system K the problem of "distant simultaneity" is simple and not really a problem at all because in the course of building it up we had not yet any reason to even imagine asymmetries such as anisotropic light propagation. Yes, we "stipulated" that light propagation would be isotropic; but it was surely a natural stipulation, since our construction was motivated primarily by simplicity. The question is, is it justifiable to think we can have a *plurality* of IRF's constructed this way when each one has a unique uniform velocity with respect to all the others? Einstein recognized that a positive answer to this question required a reconciliation between two seemingly incompatible postulates, and that this reconciliation was to be achieved by replacing the Galilean transformation equations with the Lorentz transformations. (See Box 1.) The two postulates are:

> **1.** The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion.
>
> **2.** Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body. [10]

Without these postulates, or perhaps more accurately, without the Lorentz transformations, the speed v itself would already be problematical because each frame needs its own set of synchronized clocks to measure it. That's why Einstein was careful to build his theory by first defining the concepts of time and simultaneity before discussing the relative velocity of inertial systems. In order for observers in both K and K' to measure the speed v as having the same magnitude they also have to "measure" the speed of *light* as having the same magnitude. The only way to arrange this is to agree by "stipulation" that clocks will be set so as to make it appear true. Clocks "synchronized" so as to make it appear true in one frame will then appear to be "*de*synchronized" when viewed from the other frame. Specifically, in order to satisfy both postulates for both frames, it turns out that clocks in K' would have to appear desynchronized for observers in K, and vice versa.

4.2 Desynchronization and Anisotropy

The physical reason for the desynchronization is that, from the point of view of one system, e.g., K, the speed of light in the other system (K') is not isotropic. Desynchronization and light speed anisotropy are interdependent and inextricable. They are both always attributable to all inertial systems except one's own. This might be called the "back side" of the Relativity coin. The "front side," which gets most of the attention, asserts the *isotropy* of light for *all* inertial systems. The cross-frame desynchronization resulting from Einstein's synchronization procedure is typically thought of as a result of this "isotropy for everyone" idea, rather than as a result of the exclusive "isotropy only for me" idea that I've presented.

UNIFORM VELOCITY TRANSFORMATIONS (for motion along x-axis)

Galilean Transformations

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

The above transformations cannot be generally valid because they fail to account for the finite speed of light and the effect of velocity on length and time standards.

Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}}$$

These transformations satisfactorily account for the finite speed of light and they accommodate (predict) length contraction and time dilation. To Lorentz the time t represents the "true time" of clocks at rest in the "preferred" ether frame; and x represents the corresponding "true" uncontracted lengths. To Einstein, no frame is "truer" than any other.

Box 1.

There's no argument here about observable physical facts; but there is a difference in perspective, in philosophical outlook, if you will. Isotropy for everyone perhaps sounds more democratic, and it gives rise to the pleasant sounding expression, "relativity of simultaneity." But it is not often enough pointed out, in my opinion, that satisfying Einstein's postulates by adjusting clocks this way only "works" because, as viewed from one's own reference frame the speed of light with respect to *moving* bodies does *not* equal c. You can't have the speed of light being isotropic in one's own frame unless you regard it, at least sometimes, as being anisotropic in other frames.

Obvious though this may be, I hesitate to call it trivial. So let's look at a few examples. In the course of deriving the Lorentz transformations, Einstein describes the progression

Step in Einstein's derivation of the Lorentz transformations

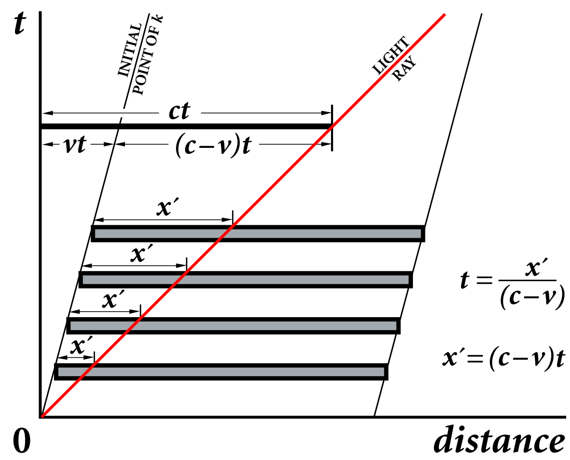

Fig. 2: When viewed from K the speed of light in k is not c. In the direction of increasing x along the x-axis, it is $(c - v)$. Isotropic light propagation is only assumed for one's own frame of reference; which means it has to appear as being anisotropic for all other frames of reference.

of a light ray in a stationary system (K) with respect to a point in the moving system (k). (See Figure 2.) Time as measured in K is referred to as t; the ray is emitted from the origin when $t = 0$, and x' is the distance the ray travels in the time t. Einstein writes: "The ray moves relatively to the initial point of k, when measured in the stationary system, with the velocity $c - v$, so that $x'/(c - v) = t$." [11]

Minkowski diagrams that depict a pair of IRF's having a relative speed $= v$ are a more common context showing the anisotropy of light speed for other inertial systems. Each system, of course regards itself as being just like our initial system K, in which light speed is patently isotropic. In the frame that is typically reserved for ourselves, the space and time axes are visibly perpendicular to each other and light speed is represented by 45° light cones. But with respect to the tilted worldline of the other IRF the speed of light is $c \pm v$; the magnitude of tilt corresponds to the magnitude of v. Figure 3 is a Minkowski diagram showing two reference frames with a relative speed $v = 0.6c$. For this large a velocity the effects of desynchronization are quite pronounced. By studying the space and time data marked on the figure one can deduce the diagram's symmetry and reciprocity. Both systems regard the other as being out of sync and as having slowed clocks and shortened rods. It would of course be possible to draw this diagram from the point of view of the "moving" frame, as a mirror image in which the latter frame is now straight up and perpendicular to the time line and the asymmetries apply to the other frame. But such a re-draw isn't really necessary. Although the frame that gets the 90° axes seems superficially to be "preferred," strictly speaking, both frames in this view are already quite "equivalent."

We have equations and a diagram. Now let's consider a simple *physical* example. In my left hand I have a laser pointer that I shine on my outstretched right hand. Suppose the speed of the beam with respect to my left hand is c. Now I move my right hand toward the pointer at the speed v. Shall we insist that even when my right hand is so moving, the speed of light with respect to it hasn't changed, that it is still c? It's *possible* to set up a Minkowski diagram representing my left and right hand. But why should I bother invoking a new set of clocks conforming to it? So that I can "find" the speed of light to be c with respect to these clocks? Democratic though it may be, the symmetry of the Minkowski diagram strikes one as being rather more stipulated and contrived than physical. SR is all about *arranging* (by stipulation and contrivance) a set of clocks that *make* the second postulate appear true. As we will see momentarily, even Einstein endorsed the view that it is no less correct to say that the speed of light with respect to my right hand may indeed be regarded as $c + v$. If it's no less correct, then why doesn't this approach get "equal time"?

4.3 Lorentz and Reichenbach

Although most relativists don't think of it quite this way, it is certainly true that Lorentz did. Lorentz knew that his equations accommodated the *appearance* of symmetry, while also allowing the idea of a preferred ether frame with respect to which light propagation is *really* (not just stipulated as being) isotropic. To Lorentz, his transformation equations were an expression of how Nature camouflages the existence of the ether. It is worthwhile to consider one of his last statements on the matter (from 1927):

> A physicist of the old school says, "I prefer the time that is measured by a clock that is stationary in the ether, and I consider this as the true time, though I admit that I cannot make out which of the two times is the right one, that of A or that of B." The relativist, however, maintains that there cannot be the least question of one time being better than the other.
>
> Of course this is a subject that we might discuss for a long time. Let me say only this: All our theories help us to form pictures, or images, of the world around us, and we try to do this in such a way that the phenomena may be coordinated as well as possible, and that we may see clearly the way in which they are connected. Now in forming these images we can use the notions of space and time that have always been familiar to us, and which I, for my part, consider as perfectly clear and, moreover, as distinct from one another. My notion of time is so definite that I clearly distinguish in my picture what is simultaneous and what is not. [12]

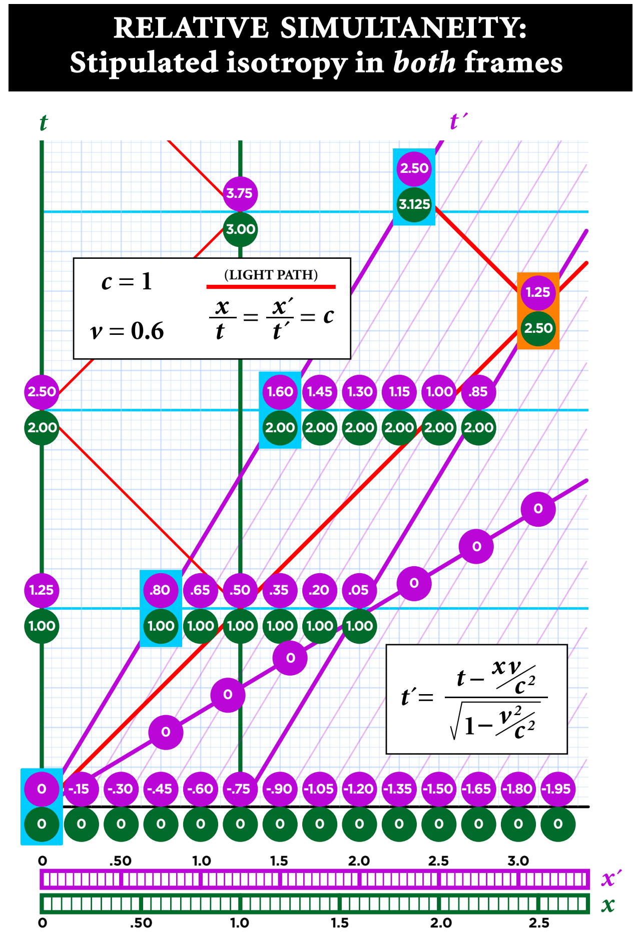

Fig. 3: Symmetry and reciprocity of Special Relativity. By comparing clocks at key events, we see that the speed of light = c in both frames, so that all relativistic effects will be attributed to the *other* frame, no matter which one we think of as being "at rest."

Remember, we are not advocating a return to the "old school." Rather, the idea is to point out that the path taken by the "new school" may not be the only viable one. Indeed, it may be that, by insisting on the theoretical value of the relativity of simultaneity, other alternatives are being overlooked.

In this spirit, note that in the mid 1920's H. Reichenbach [13] proposed a method whereby any of Lorentz's "untrue" frames can be explicitly represented as such, by introducing a new "synchrony parameter," having a magnitude $0 < \epsilon < 1$. The idea is a generalization of Einstein's clock synchronization prescription, as follows. Suppose A and B are two fixed points in K, t_A is the time of emission of a signal from A, t_B is the time at which the signal is received by B and is immediately reflected back to A, and $t_{A'}$ is the time at A when the reflected signal returns:

$$t_B = \epsilon(t_{A'} - t_A)\,. \qquad (1)$$

Einstein did not include or discuss the range of possible ϵ-values; in place of ϵ, Einstein simply stipulated that its value is $\frac{1}{2}$. This is the standard approach. If observers in every inertial system adopt this convention, they can justify the assumption that light speed is isotropic for each observer with respect to himself. The obvious advantage of this approach is its stark simplicity. If it's simple and it works, one can understand why many practitioners would want to just leave it at that. Neither Einstein nor most relativists ever concern themselves with non-standard ϵ-values. Besides Reichenbach and a few others, a recent exception is Selleri, [14] who has introduced a "synchronization gauge," e_1, whose purpose is essentially the same as ϵ. Since we are here concerned more with the *idea* than with its rigorous application, we simplify the discussion by referring to the idea in terms of ϵ only.

Suppose that the uniformly moving reference frame K' is the one to which $\epsilon \neq \frac{1}{2}$ applies. We may thus think of ourselves as being "at rest" in K', so that with respect to ourselves light propagates anisotropically. Adopting a non-standard value for ϵ as applying to our own IRF is tantamount to singling out some *other* reference frame (e.g., K) in which the value is $\frac{1}{2}$ and light propagation is isotropic. This possibility is depicted in the "ϵ-synchrony" spacetime diagram in Figure 4, which adopts the same velocity ($v = 0.6c$) as that used in Figure 3, and so represents exactly the same physical circumstance.

Although the clocks in the different (purple and green) frames have different rates, we can see that they are not desynchronized at $t = t' = 0$; the time given by both clock sets is independent of *distance*. Also we see that the light paths are not at 45°. Even though this scheme exhibits anisotropic light propagation (for the purple frame) and non-reciprocal velocities, all clock comparisons made from the origin of the isotropic frame (world line with blue boxes) and all back and forth light speed measurements agree with SR. Such a scheme is thus equally in accord with all empirical evidence. Of course, Solar System inhabitants have practical reasons for not adopting it. Many of us instead take for granted that the $\epsilon = \frac{1}{2}$ scheme is the "universally" most appropriate one. But perhaps there is a circumstance somewhere in the Universe in which this $\epsilon \neq \frac{1}{2}$ scheme actually does make more sense.

4.4 Geometrical Approach

The emphasis on light speed anisotropy in the above is likely to be frowned upon by most relativists. To a particular "school" of relativists even slow clocks and shortened rulers

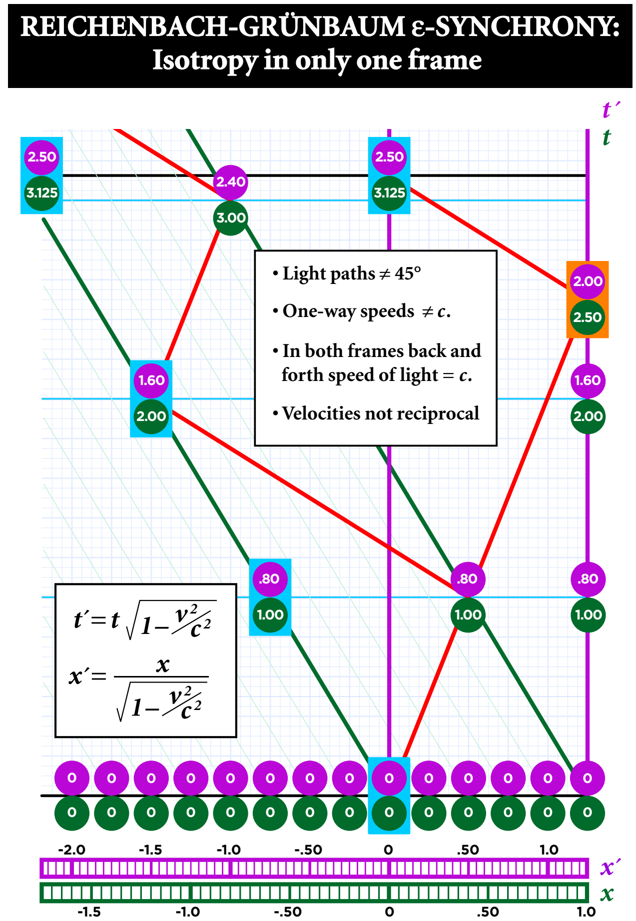

Fig. 4: Spacetime diagram whose physical circumstances are exactly the same as in Figure 3 ($v/c = 0.6$). Even though the ϵ-synchrony scheme is not usually practical, "it could not be called false."

are antithetical concepts. It is appropriate to acknowledge this here.

Within the context of standard physics, as Denker has pointed out, "There are two ways of formulating the essential ideas of special relativity." [15] Denker laments that the most popular approach – especially as found in most introductory texts – is what he calls the "contraction/dilation approach that alleges that rulers contract and clocks run more slowly when moving relative to the observer." This is the approach by which SR was born; it is the approach of physicists whom I will refer to as "Einsteinians." Since I shall also argue that the Einsteinian approach may itself be excessively abstract, the alternative approach mentioned by Denker will be seen as being even more so. In fact, I think it is a perfect example of how the "Einsteinian style" of reasoning might be expected to evolve; it is the natural outcome of what Einstein started. The latter "evolved" approach emphasizes the abstract geometrical relationships of relativistic (Minkowski) spacetime, and is espoused, to one degree or another, by authors such as

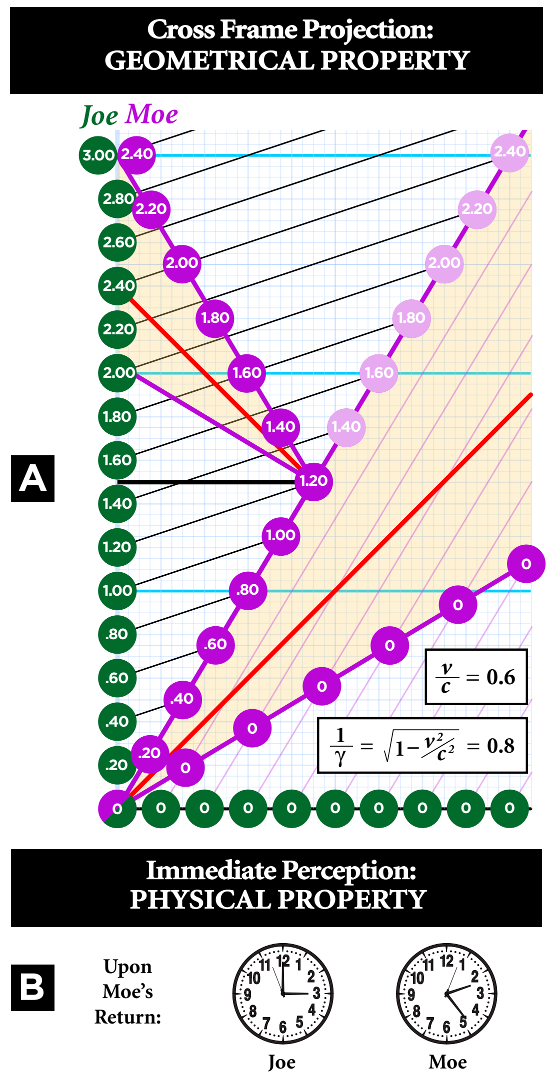

Fig. 5: Due to their relative velocity, a projection of Joe's time onto Moe's is oblique. Symmetry is disrupted when Moe's velocity changes; the projections no longer line up.

Wheeler and Cuiffolini, [16] Ohanian and Ruffini, [17] Carroll, [18] Hartle, et al, [19] and of course, by Denker himself.

It will be useful to discuss one example. Denker presents the classic twin problem, starring Joe, who stays at home and Moe, who takes a high-speed journey and returns. Denker insists,

> Let's be clear: Yes, at the end of the journey, Moe is younger than Joe (assuming they started out with equal ages). No, this does not prove that Moe's clock ran slower. A far simpler explanation is that Moe took a less time-consuming path.

If Δt is the final time on Joe's clock and $\Delta\tau$ is the final time on Moe's clock, then, Denker concludes:

> $\Delta t/\Delta\tau$ merely describes the projection from Moe's frame onto Joe's frame. ...The ratio $\Delta t/\Delta\tau$ is a property of the projection, not a property of anybody's clock.

Figure 5A shows one way of projecting Joe's time onto Moe's. Figure 5B shows what looks to me like a physical clock property. Notice that the hands of the clocks do not line up. Moe's clock shows a smaller time than Joe's clock. (Are we really *not* to think of this as a property of the clocks?) Denker argues that this does not mean that Moe's clock ran more slowly.

After Moe's journey the side-by-side clock comparison indicates that "Moe took a less time-consuming path, not that his clock ran slower." To Denker this argument is not only acceptable, it is purported to be "far simpler, more powerful, elegant and consistent." The time difference is a "property of the projection, not a property of anybody's clock." A properly functioning clock returns from a journey with a smaller elapsed time than the stay at home clock. Denker purports to have provided a *simpler* explanation than that the moving clock ran slower. Will it make more sense if I repeat it again? My repetition is borne of incredulity. I'm sorry, but I seem to have been deprived of the side of the brain needed to appreciate this style of reasoning. Unfortunately, we will encounter it again.

4.5 Conventions and Stipulations

Assuming that the anisotropy and corresponding metric effects (length contraction and time dilation) apply *in one's own* frame of reference entails many complications. The assumption implies the need to identify the "preferred" frame in which clocks have maximum rates and rods have maximum lengths. This would in turn imply the need to keep track of any pertinent third frames wherein the metric effects are also due to motion with respect to the preferred frame. Therefore reciprocity would be lost; symmetry would obtain only with respect to the preferred frame. These characteristics can be clearly seen in terms of Figures 3 and 4. The desynchronized clocks of Figure 3 provide the simplifying advantage of *making* the speed relative. Both frames find v to have the same magnitude. Whereas Figure 4 shows that, if a moving observer uses the same operations to measure his own velocity, the magnitude comes out greater than that measured by the resting observer. Therefore (by design) the $\epsilon \neq \frac{1}{2}$ scheme is such that one's "true" speed is the speed as observed from the "true" rest frame.

Given the obvious complexity of the above approach, it is understandable that one may sense some *superiority* to the scheme resulting from adoption of Einstein's postulates, whereby symmetry, reciprocity and simplicity are built into the foundation. This "simplicity argument" is often invoked (if not explicitly, then tacitly) for adopting the standard $\epsilon = \frac{1}{2}$

and agreeing to let all inertial observers do the same. But the simplicity argument does not mean the $\epsilon \neq \frac{1}{2}$ scheme is incorrect. At the same time he proposed his ϵ-synchrony idea, Reichenbach argued that it "could not be called false." [20] Within a year of its publication, *Reichenbach's analysis was endorsed by Einstein*: "special care has been taken to ferret out clearly what in the relativistic definition of simultaneity is a logically arbitrary decree and what in it is a hypothesis, i.e., an assumption about the constitution of nature." [21] In spite of Einstein's endorsement, very little attention was paid to Reichenbach's work till Grünbaum [22] revived it in the mid 1950's.

Since then, the relativity of simultaneity has been a subject of lively debate. Einstein's assertion decades earlier that this was the most controversial aspect of SR seems no less true today. This is not the place to hash out the many arguments that have been put forth. For that, consult Max Jammer's 2006 book, *Concepts of Simultaneity* [23]. I should mention, however, a few key contributions. The debate is divided, roughly, into so called non-conventionalists and conventionalists. The former contend, essentially, that the temporal and light propagation relationships between IRF's according to SR are ultimately more than conventions, definitions, stipulations and assumptions. Corresponding to this view is the inclination to think of Einstein's second postulate as being physically true, and that experiments purporting to be tests of light speed isotropy are as claimed. The "geometrists" mentioned in §4.4 would evidently be classified as especially hard core non-conventionalists.

The conventionalists, by contrast, contend that the temporal and light propagation relationships between IRF's are indeed only convenient definitions, stipulations and assumptions. However useful the conventions may be, conventionalists claim it is impossible to prove that one IRF should be preferred over others or to prove in which direction the anisotropy axis with respect to any particular frame actually lies. But this view clearly allows that some such anisotropy axis may exist, or even that some such anisotropy axis *inevitably* exists for most reference frames, however "disguised" it may be. According to this view (which is similar to that of Lorentz) experiments that purport to establish the "isotropy of space" are actually *illusions* which are implicitly based on non-trivial conventions.

4.6 Central Logical Unclarity?

SR was published in 1905. Over a century later the status of the core of SR is still a subject of debate. In 1970 Winnie showed that transformation equations could be derived based on Reichenbach's ϵ-synchrony idea; and that these

> ϵ-Lorentz transformations, for arbitrary choices of ϵ, are simply *one-one translatable* into consequences of the standard Lorentz transformations, by taking initial clock-settings into account. This result thus demonstrates that different choices of ϵ result in *kinematically equivalent* versions of the Special Theory, and this is precisely the claim made by the thesis of the conventionality of simultineity. [24]

In 1977 Salmon analyzed various performed and proposed experiments whose purpose was to measure the one-way speed of light. He showed that they all involve non-trivial conventional elements and so failed (or would fail) to measure light's *actual* one-way speed. Thus Winnie's results were further reinforced. One of the consequences stressed in Salmon's work is a point raised in §4.1; i.e., that reciprocity of *any* one-way velocity hinges on (or goes with) reciprocity of *light* velocity. In other words, unless clocks are desynchronized exactly as in Figure 3 (i.e., under the assumption that $\epsilon = \frac{1}{2}$) the speed of light and the speed of *any* material body will not be symmetrical (will not exhibit reciprocity) as between one frame and another (as shown in Figure 4). Since this reciprocity has not the status of a physical fact, but is a matter of convention, Salmon ponders where that leaves us:

> The status of the one-way speed of light...lies at the heart of our analysis of the concept of any one-way velocity whatever. How deeply can we claim to understand the physical world if this concept suffers central logical unclarity? [25]

One of the most influential recent arguments *against* the conventionality thesis is a paper by Malament [26]. Later commentators have acknowledged Malament's main argument that only standard synchrony singles out the simplest Minkowski space (perpendicular simultaneity surfaces) but many are not convinced that this means the physical world should necessarily conform to it. Janis [27] for example, has argued that Malament's analysis boils down to a restatement of the simplicity argument: However true the analysis may be, it by no means proves that space is isotropic in one's own frame or that the $\epsilon \neq \frac{1}{2}$ scheme is incorrect.

My impression is that most critical physicists and philosophers acknowledge that the various attempts, with thought or real experiments, to prove that space is actually isotropic, contain non-trivial conventions. Curiously, both sides of the argument have appealed to Einstein's work to defend their positions. The non-conventionalists cite the second postulate itself and the many instances of Einstein's stating or defending it. And the conventionalists cite Einstein's need to *define* or *stipulate* the meaning of simultaneity as conforming to the light postulate by convention. Doing so results in a simpler theory. But it does not change the fact that to measure the one-way speed of light a set of spatially separated synchronized clocks is needed. You can't know for a physical fact that your clocks are actually synchronized unless you know the one-way speed of light. If one of these things is physically unascertainable, then so is the other. This is the case. So in a roundabout way Einstein's choice to stipulate his way out of the logical circle is a concession that a measurement of the one-way speed of light in inertial systems is impossible; only the two-way speed is measureable. (And this speed comes out equal to c for every permissible value of ϵ.)

Einstein's position on the matter is not as clear or consistent as it could have been. As late as 1949 Einstein writes of his light speed postulate as being "based on experience." [28] In that context he did not point out that this included only the average two-way speed or point out the conventions behind this "experience." To my knowledge, Einstein never *explicitly* expressed the possibility, inherent in his own work and as brought out by Reichenbach, that light propagation might actually be anisotropic with respect to oneself. So one can see how both conventionalists and non-conventionalists might try to adopt Einstein as one of their own. After weighing the evidence on this question, Max Jammer wrote: "Summing up, it seems that, as far as the concept of distant simultaneity is concerned, Einstein can be classified as a conventionalist, who however sometimes made statements not wholly consistent with the position." [29]

"How deeply can we claim to understand the physical world if [the concept of one-way velocity] suffers central logical unclarity?" Most of the "conventionalism" debate has been carried out by "philosophers" of science or by the more philosophically-minded physicists. One still finds little discussion of the matter in text books; and one still finds alleged "Tests of the isotropy of the speed of light..." [30] by competent experimental physicists. Many physicists, no doubt, think such tests have established "space isotropy" to a high degree of accuracy. Meanwhile, conventionalists call these tests "illusions." [31] If he were alive would Einstein help to settle the debate? Perhaps. But all we have to go on is what he left us with. I think the record shows that Einstein's deep interest in simplifying and symmetricalizing *theories* overrode his interest in being critical of his relativity of simultaneity. In one of his last public talks, Einstein was quoted as saying,

> "The laws of physics should be simple." Someone in the audience asked, "But what if they are not simple?" [to which Einstein answered] "Then I would not be interested in them." [32]

Theoretical and mathematical simplicity are recurring themes in Einstein's career. Writing in 1933, he asserted that, "our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas." [33] Given the priority Einstein put on mathematical simplicity, it is not at all surprising that he seems always to have assumed $\epsilon = \frac{1}{2}$. Any scheme that explicitly attaches importance to anisotropic light propagation is clearly at odds with Einstein's view of the world, and is even less congenial toward the geometrists who have taken Einstein's quest for abstract simplicity to even greater extremes.

4.7 Foreshadowing the Importance of Non-Inertial Reference Frames

The relativity of simultaneity and light speed anisotropy are inextricably linked. But notice that they have a crucial difference in character. Light speed anisotropy is an inevitable physical fact with respect to most everybody. Light speed anisotropy an *inevitable physical fact with respect to most everybody?* To back up this assertion I appeal to the real physical world. As a brief preview of the subject of §6, we have the fact that light signals sent in opposite directions around a rotating body (e.g., Earth) take unequal times to return to their starting point; i.e., the speed of light with respect to *rotating* observers is not isotropic. *This fact can be ascertained without use of a clock* and therefore does not depend on any non-trivial stipulations or conventions. If one objects that facts gleaned from non-inertial systems are inadmissible in the context of SR, I would point out that we are not, I hope, as government-sanctioned lawyers and judges abiding by the formalities of a court of law. Our investigation is not to be mind-cuffed by legalities. Rather, we are as *private detectives* whose mission is to *figure out what's really going on*. If in the *general* case (non-inertial) system we find anisotropy to be a fact, then, by what logic should this anisotropy be overruled by an abstract contrivance whose purpose is to make it appear to go away in every *special* case (inertial) system? Relativistic physics is the only context I know of in which special, local "laws" have the status to squelch global physical reality.

It may not be possible to determine the axis (or axes) of anisotropy. But isn't it more reasonable to accept this uncertainty than to insist that light is really physically isotropic with respect to, for example, each of the thousands of changing inertial systems attached to every bee in a hovering swarm? Strict adherence to SR, of course, permits (or even requires) one to imagine this multitude of "equivalent" frames. This is a mathematically consistent *theoretical* fact. On the other hand, *relativity of simultaneity is not a physical fact*. It is an abstract idea imprinted (or not) on one's conception of reality. You could argue that the relativity of simultaneity derives from light speed anisotropy; light speed anisotropy explains why clocks in other IRF's are desynchronized. But turning the argument around makes no sense: desynchronized clocks do not explain why light propagation is anisotropic. As noted above, this is demonstrated in the case of rotation, where the anisotropy can be observed as a physical fact without use of any clocks. Physical facts do not derive from abstract ideas.

One must nevertheless acknowledge the value of the simplicity argument. Both theoretical and experimental physics have benefitted from Einstein's theory. I don't mean to suggest that we should stop assuming that light speed is isotropic when adopting this assumption harmlessly simplifies our work. But I would suggest that we should strive to become more *conscious* of the difference between physical reality and abstract theory; and to be more candid and straightforward about what is a physical fact and what is not. The *abstract law* is that the speed of light equals c for everybody. *The physical fact is that it does not.* The "central logical unclarity" concerning velocity is not going to go away if we keep obscuring the complicated physical world with our simple laws.

5 Did Einstein Ever Move?

The title of this section may seem like a silly question having an obvious answer. If we take Einstein at his word, however, it would appear that Einstein himself may well have answered, "no." As we have seen, for Einstein, simplicity in the "laws of nature" was a very high priority. Alongside simplicity is the concept of symmetry, which is not far from the concept of relativity, the impossibility of ascertaining absolute motion and, ultimately, perpetual "self rest."

Perhaps the best source for examples is Einstein's book, *Relativity, the Special and General Theory*. In our first example, Einstein refers to the experience of a passenger on a moving train when the brakes are applied. He writes:

> It is certainly true that the observer in the railway carriage experiences a jerk forwards as a result of the application of the brake, and that he recognizes in this the non-uniformity of motion (retardation) of the carriage. But he is compelled by nobody to refer this jerk to a "real" acceleration (retardation) of the carriage. He might also interpret his experience thus: "My body of reference (the carriage) remains permanently at rest. With reference to it, however, there exists (during the period of application of the brakes) a gravitational field which is directed forwards and which is variable with respect to time. Under the influence of this field, the embankment together with the earth moves non-uniformly in such a manner that their original velocity in the backwards direction is continuously reduced." [34]

Proceeding immediately to our second example,

> ... we shall imagine K' to be in the form of a plane circular disc, which rotates uniformly in its own plane about its center. An observer who is sitting eccentrically on the disk K' is sensible of a force which acts outwards in a radial direction, and which would be interpreted as an effect of inertia (centrifugal force) by an observer who was at rest with respect to the original reference-body K. But the observer on the disc may regard his disc as a reference-body which is "at rest"; on the basis of the general principle of relativity he is justified in doing this. The force acting on himself, and in fact on all other bodies which are at rest relative to the disc, he regards as the effect of a gravitational field. Nevertheless, the space-distribution of the gravitational field is of the kind that would not be possible on Newton's theory of gravitation. But since the observer believes in the general theory of relativity, this does not disturb him. [35]

Thirty years later (in 1953) Einstein defended the point of view presented in our second example in the course of explaining the Equivalence Principle to a correspondent. Einstein writes: "It [i.e., the relationship between acceleration and gravitation] is the same in the case of the rotation of the coordinate system: there is *de facto* no reason to trace centrifugal effects back to a 'real' rotation." [36]

Einstein's motivation for these pronouncements was to either justify or apply what he called the *General Principle of Relativity*. In his Nobel Award lecture, Einstein stated this principle as follows:

> The conclusion is obvious that any arbitrarily moved frame of reference is equivalent to any other for the formulation of the laws of Nature, that there are thus no physically preferred states of motion at all in respect of regions of finite extension (general relativity principle). [37]

One of the contemporary objections to Einstein's approach will be mentioned momentarily. First, however, note that these and similar remarks found in Einstein's work represent not only the meaning that he attached to General Relativity, but key parts of his strategy for building a wider theory. Thus, in his 1949 *Autobiographical Notes*, Einstein spoke highly of "the plane of insight achieved by the general principle of relativity," as a route to not only GR but as a worthy guide for going beyond GR. Subsequently he stated the criteria he hoped to meet by his exploration: "It is clear that in general one will judge a theory to be the more nearly perfect the simpler a 'structure' it postulates and the broader the group is concerning which the field equations are invariant." [38] Einstein's intent was thus to keep building on his theories using the same basic strategy, in hopes that he might "succeed in expanding the group once more, analogous to the step which led from special relativity to general relativity." [39]

It is almost a cliché that Einstein persisted for decades, yet failed to reach his goal. His path was solitary and divorced from the developments taking place in quantum theory. Those who worked on GR (whose numbers were not very great prior to the 1960's) for the most part focussed on Einstein's field equations and had little use for the general *principle* of relativity. In more recent years Ohanian and Ruffini, for example, have explicitly rejected the principle, writing: "tidal effects allow us to measure an absolute difference between the gravitational force and the pseudo-force found in accelerating reference frames. It is therefore false to speak of a general relativity of motion." [40] A more thorough critique of the general principle of relativity and its relation to Einstein's Equivalence Principle is given by Friedman. [41] (Note also that one can find arguments more sympathetic with the Einsteinian view. See, for example, Brown [42].)

I don't suppose many people would ascribe the effects of slamming their brakes or executing pirouettes to transient gravitational fields and insist that all the while they remained at rest. Einstein failed to reap any additional fruit by adhering to his cherished symmetry / simplicity principles. But he seems to have succeeded in reinforcing the idea that the degree of *symmetry* in a theory is an indication of how "perfect" it is.

What about the *opposite* approach? [43] Perhaps the result of reinforcing the above strategy has been in some ways *detrimental* to physics. Gravitation might be a case in which theoretical symmetry / simplicity principles are being adhered to when physical reality is providing clues that it would be better to let go. Maybe gravity is a more *a*symmetrical phenomenon than our favorite theories would have us believe. I would advise, for what follows and in general, to remain alert to this possibility.

6 Uniformly Rotating Reference Frame

I have yet to see any problem, however complicated, which, when you looked at it in the right way, did not become still more complicated.

– Poul Anderson [44]

6.1 Background

It is well known that Einstein appealed to certain theoretical and physical properties of rotating bodies as a kind of stepping stone from SR to GR. In particular, Einstein inferred that the effect of rotation on uniformly rotating length standards suggests the need for non-Euclidean geometry to properly account for the phenomenon. The idea was then extended to gravitational fields. But uniform rotation also involves consequences that certainly do not require non-Euclidean geometry, consequences that have been a source of controversy since the early days of Relativity.

The controversy arises for two key reasons: 1) The speed of light is demonstrably anisotropic with respect to rotating observers. The prevailing view is that any such observers may regard themselves as being instantaneously at rest in *local* inertial frames in which light speed is once again isotropic. But a few authors have emphasized that this view is only rather awkwardly reconcilable with the *non-local* physical anisotropy. 2) Because of the anisotropy mentioned in (1) and because of the range of speeds found on a rotating body, there arises the question as to how exactly, or whether it's possible at all, to synchronize clocks on the rotating body. The answers given by the prevailing view appear once again to some authors as being needlessly "awkward." (This is not the word used by these authors. But I think they would agree with my characterization. Perhaps the best source for an extended discussion of the problem is the book, *Relativity in Rotating Frames*. [45])

Much ink and paper have been consumed in the course of discussing these issues. Here we will of course strive for brevity, but the questions are of such importance that I hope to also make the facts of the matter quite clear. This will sometimes involve the opinions and the points of view of workers in the field. (See also Appendices A and B.)

One of the reasons rotation has been so extensively studied is that it is relatively easy to do so. Not only do we live on a conveniently rotating body and have the need to travel and communicate around it, since its invention, the *wheel* has evolved in many ways to facilitate probing. And wheels are usually easy enough to fit in a typical physics laboratory. In the latter case, we have the advantage of being observers at rest with respect to the rotation axis.

6.2 Basic Facts

Suppose now that our laboratory surrounds the origin of our original system K and that we'll be observing a rotating body whose axis passes through that origin. To be more concrete, imagine a thin material disk of radius r rotating uniformly with an angular velocity ω. Suppose the axis of rotation corresponds to the z-axis. Now imagine that a continuous concave cylindrical mirror is mounted on the disk's rim so as to facilitate light propagation around the circumference.

Light signals are sent around the disk, one with and one against the direction of rotation. As judged by a clock in K, the time taken for the signal sent in the direction of rotation to return to the source is

$$T_{\uparrow} = \frac{2\pi r}{c - r\omega} = \frac{L}{c - v}\,, \tag{2}$$

and for the signal sent against the direction of rotation the time to return to the source is

$$T_{\downarrow} = \frac{2\pi r}{c + r\omega} = \frac{L}{c + v}\,. \tag{3}$$

The difference in time is

$$\Delta T = T_{\uparrow} - T_{\downarrow} = \frac{4\pi r^2 \omega}{c^2 - r^2\omega^2} = \frac{2Lv}{c^2(1 - v^2/c^2)}\,. \tag{4}$$

A clock located at the emission / reception point on the disk has a lower frequency, f_r than a clock in K by the factor

$$\frac{f_r}{f_K} = \frac{1}{\gamma} = \sqrt{1 - \frac{r^2\omega^2}{c^2}} = \sqrt{1 - \frac{v^2}{c^2}}\,. \tag{5}$$

So the reception times as judged by disk-observers using such a clock will be correspondingly smaller. Above and in Figure 6 we have used T and L to denote times and lengths as measured with rods and clocks in K. Using t to denote times measured with clocks in the rotating frame, we thus have

$$t_\uparrow = \frac{L\sqrt{1 - v^2/c^2}}{c - v}\,, \tag{6}$$

$$t_\downarrow = \frac{L\sqrt{1 - v^2/c^2}}{c + v}\,, \tag{7}$$

and

$$\Delta t = t_\uparrow - t_\downarrow = \frac{2Lv}{c^2\sqrt{1 - v^2/c^2}}\,. \tag{8}$$

(We will address the question of lengths measured by rods in the rotating frame in §6.4.) The first order times and the time difference given by Equations 6-8 have been observed. This experimental work began about 100 years ago [46]. One of the earliest experimenters was Sagnac [47], whose name is often associated with the observed effect. Sagnac measured the time difference (the first order part of Equation 4) as a phase shift in counter-rotating light beams using a rotating interferometer. Since then, his observations have often been corroborated in laboratories and around our planet Earth.

The second order clock slowing effect given by Equation 5 and implied by the factor $1/\sqrt{1 - v^2/c^2}$ in Equation 8, is often neglected in the context of the Sagnac effect because its contribution to the result is typically very small. Nevertheless, various independent experiments and the Global Positioning System have proven its reality and importance.

6.3 Rotation Plus Translation

The disk as a whole may also have a linear velocity (e.g., in a direction along its rotation plane). We can thus *imagine* that the rotating body is involved in two kinds of motion, each with its own consequences for the setting of clocks and timing of light signals. But the purely linear motion can be neglected because any consequences of this motion, as we saw in §4, are impossible to determine. They are camouflaged as per the Lorentzian interpretation of his transformation equations. With respect to an observer at the center (what we may call the "lab frame") where $r\omega = 0$, it would therefore be most natural to adopt the simplest possibility by convention; i.e., to assume light propagation is isotropic.

The effects of rotational motion, on the other hand, are not camouflaged this way. No convention could make its consequences disappear. What makes it possible to see the effect on light propagation in this case is that a light signal sent in one direction (with or against the rotation) closes back on itself while maintaining the "withness" or "againstness" the whole time. This makes it possible for one observer with only one

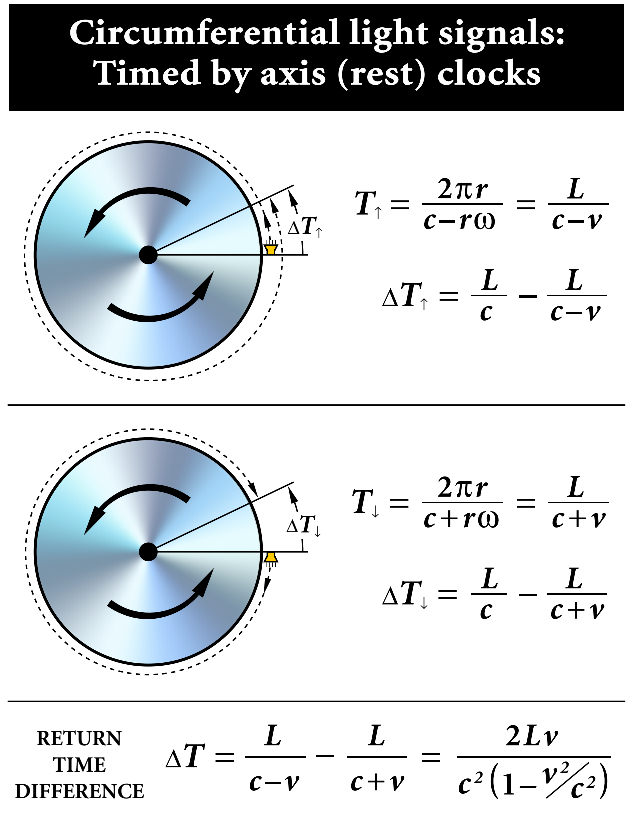

Fig. 6: Anisotropic light propagation with respect to rotating disk; basis for the Sagnac effect.

clock to measure the duration of the trip, because the signal returns to its starting point.

6.4 Interferometer Within an Interferometer

Now consider the time average of signals sent in both directions:

$$\langle T\rangle_{\uparrow\downarrow} = \frac{1}{2}\left[\frac{2\pi r}{c - r\omega} + \frac{2\pi r}{c + r\omega}\right] = \frac{L}{c}\frac{1}{(1 - v^2/c^2)}\,. \tag{9}$$

This is the time as measured in K. We've already mentioned that for disk observers this time would be smaller by the factor $\sqrt{1 - v^2/c^2}$ due to the slowing of their clocks. With respect to rotating clocks we thus have

$$\langle t\rangle_{\uparrow\downarrow} = \frac{L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}\,. \tag{10}$$

We must now address the question whether the motion has any effect on *length* measurements. The rotating body as a whole can be regarded as a kind of "interferometer," whose "phase shifts" could be so large, in principle, that they could be timed by clocks or even witnessed by human observers

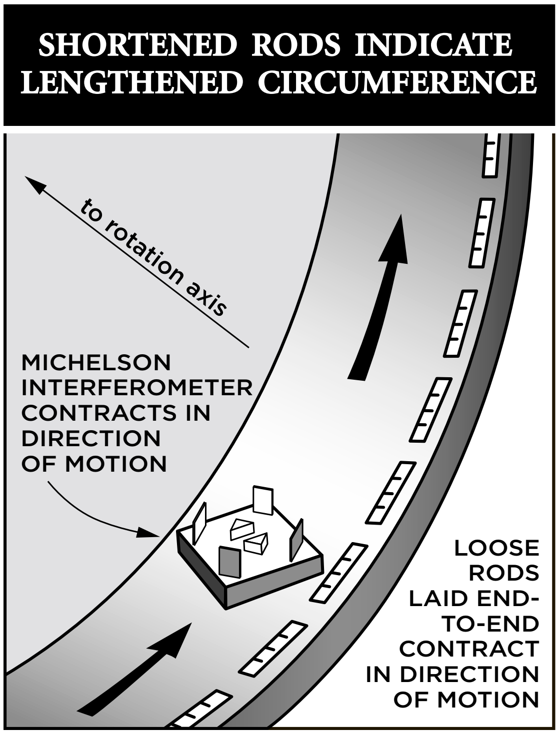

Fig. 7: Uniformly rotating bodies undergo changes in length in the direction of the velocity.

who discern a difference in arrival times of signals emitted in opposite directions. Now a different kind of instrument, known as a Michelson interferometer, whose dimensions may be quite small compared to the rotating body, can provide information on the length question. This device is very sensitive to changes in length of light paths in perpendicular directions. As is well known, if light propagation is anisotropic, the path of a light ray going back and forth, reflected from the ends of a *rigid* rod, say, in the direction of the anisotropy ("ether wind") is longer by $1/\sqrt{1-v^2/c^2}$ than it is when the rod is oriented in the perpendicular direction. The light path would be longer because the ray spends more time traveling in the direction in which it needs to *catch up* to its destination $(c-v)$ than it does traveling in the opposite direction, in which case the ray and its destination are moving *toward* each other $(c+v)$. The average time for these two legs is greater by $1/\sqrt{1-v^2/c^2}$ than the time needed to travel along the same rigid rod when it is oriented in the perpendicular direction.

Since rotating observers have already proven that light speed is anisotropic relative to themselves using a Sagnac interferometer, they might also expect the effect to be detectable using a Michelson interferometer. As emphasized by Klauber [48], two independent experiments using Michelson-type instruments have recently been conducted. In both cases the interferometers were sensitive enough to detect such a difference in path length if it were caused by the rotation of the Earth. [49] [50]. The results were null. This evidently means that the back and forth path lengths in the perpendicular directions are *the same*. Just as with the original Michelson-Morely experiment, the most reasonable explanation for this result (first proposed by Fitzgerald and Lorentz in 1892 [51]) may be that what we spoke of as "rigid" rods are not really rigid. The lengths of physical bodies, such as the Michelson interferometer, are evidently contracted by the factor $\sqrt{1-v^2/c^2}$ in the direction of motion. (See Figure 7.)

6.5 Local, Global Agreement

It is important to make a distinction between a device like the Michelson interferometer and the rotating body as a whole. The length contraction implied by the null result may clearly affect small bodies loosely attached to the rotating circumference. But these effects are independent of what happens to the circumference as a whole. The latter question has also been a lively matter of debate. Aside from any "centrifugal" effects caused by the rotational acceleration, the rotating body's outer portions will also be subject to "relativistic" stresses due solely to *velocity*. We need not enter into that debate. It suffices for our purposes to regard the Michelson interferometer much as a pair of small crossed measuring rods laid out along the circumference. We are concerned with measurements of the circumference made by observers in K, which is $2\pi r$. And with measurements of the circumference made by observers who rotate. Evidently, they will measure it to be *longer* by $1/\sqrt{1-v^2/c^2}$ because their rods are *shorter* by $\sqrt{1-v^2/c^2}$.

With this in mind, consider again light rays completing circuits around the disk in opposite directions. Or better, consider a single ray that completes one circuit in one direction and is then reflected back to return to its source in the opposite direction. We can rearange Equation 10 (whose time refers to a rotating clock) to give the average speed of the light ray:

$$\frac{L}{\langle t\rangle_{\uparrow\downarrow}} = c\sqrt{1-v^2/c^2}\,. \tag{11}$$

But L is the circumference measured by lab rods. The evidence discussed above indicates that the circumference measured with rotating rods comes out longer than that measured by lab rods:

$$l = \frac{L}{\sqrt{1-v^2/c^2}}\,. \tag{12}$$

Therefore, rotating observers would find the average back and forth speed

$$\frac{l}{\langle t\rangle_{\uparrow\downarrow}} = c_{\uparrow\downarrow} = c\,. \tag{13}$$

The Michelson interferometer doesn't tell us the speed of light. It tells us that the length / time ratio is the same no matter which direction the apparatus is oriented. That's the nature of a “null” result. By contrast, Equation 13 refers to actual speed measurements, made with rulers and a clock. Furthermore, it need not be a back and forth average, but yields also definite one-way speeds. Bearing in mind the relationship between L, T, l, and t, the average *one-way* speeds corresponding to the back and forth average are given by

$$\frac{l}{t_\uparrow} = \frac{c-v}{1-v^2/c^2} \quad \text{and} \quad \frac{l}{t_\downarrow} = \frac{c+v}{1-v^2/c^2}\,. \tag{14}$$

The speeds are *faster* than those measured in K because the path lengths are judged to be longer and the travel times shorter.

These speed measurements break down to a length measurement and a time measurement. The length is measured either with many rods laid end to end or with one rod placed multiple times from one position to the next. Whereas the time (between emission and reception) is measured with only *one clock* at one location. It would clearly be useful for a resident of the rotating body to synchronize all clocks around the rim so that any given *pair* of *adjacent* clocks would yield the same results. This is simply achieved by synchronizing all clocks by a flash of light from the rotation axis. In this way the *local* (clock-pair) speed measurements become consistent with the *global* (one-clock) speed measurements.

6.6 Time Gap

This is where we find some controversy. Although the synchronization procedure just mentioned is so reasonable and practical that it is, in effect, the same as that actually used by the GPS, opinions vary as to how to “regard” the end result. Many authors prefer to think of the clocks synchronized this way as actually being *de*synchronized. Why? Short answer: Einstein and his second postulate again. Long answer:

The Einstein method of synchronization is supposed to work for any two closely spaced clocks that are at rest with respect to each other. If the speed of light with respect to such clocks is actually equal to c in both directions, then the method does of course work. What’s more, even if the speed of light is *actually* anisotropic you can *get away* with supposing (or stipulating) that the method works *if the light path doesn’t close on itself*. In other words, the method works if you send a synchronization signal and merely *assume* that the time at the receiving clock is r/c as in our original grid. What happens if we adopt the same procedure for a circumferential segment of our rotating body? It becomes a *problem* when you find that, as you work your way around from one segment to the next, upon returning to the starting clock, the times don’t agree. For a straight line light path you can never find out if the time for the synchronization signal to arrive is *really* l/c or not. But on a rotating body you can’t avoid discovering that when the signal returns to its starting point it’s off by $\approx lv/c^2$. The last clock in the chain is behind or

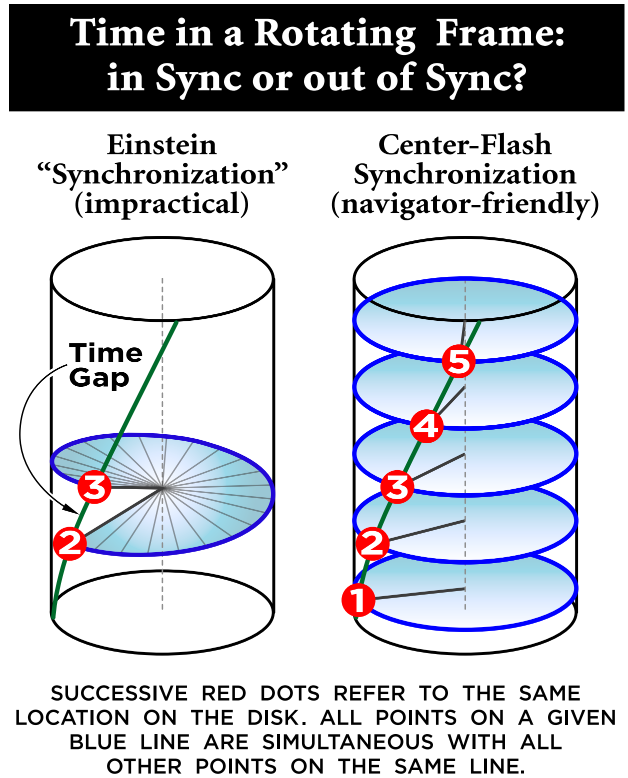

Fig. 8: Tubular spacetime diagram for rotation: World line is green helix. Using Einstein synchronization, the line of simultaneity is also a helix; it does not close on itself and so yields two different times for a point in spacetime that should have just one time. Center-flash synchronization does not have such problems.

ahead by about this much, depending on whether the procedure is conducted with or against the direction of the rotation. Taking the average, as judged by rotating observers using rotating clocks and rods, the difference is exactly lv/c^2. Using lab clocks and rods the average difference is one half of Equation 4: $Lv/c^2(1-v^2/c^2)$. This is known as the “time gap” (or time *lag*). (See Figure 8.)

Since a synchronization time gap is totally impractical and it’s so easy to eliminate it by the center-flash synchronization method, why isn’t it simply accepted that this latter method is *the* way to synchronize clocks on a rotating body? The “problem” is that by using center-flash synchronized clocks, the speed of light comes out being anisotropic and not equal to c. Some people don’t like that, or at least they seem to not like it, because they call the clocks *de*synchronized and insist that the speed of light is still *locally* equal to c. (See Appendix A.) This approach strikes me as making the postulated constancy and isotropy of light speed such lofty inviolable *absolutes*, that any real world measurement that fails to maintain their sanctity implicitly becomes less worthy, no matter how practical it may be.

6.7 Digression: Map vs. Territory

Before following up on this point, it will be useful to reconsider the "fact vs abstraction" discussion of §2. For reasons that need not presently concern us, I was once given the following advice by a well known astrophysicist: *"You must believe Newton's equations because your life depends on them every time you cross a bridge or fly in an airplane."* Is it really so? With a little reflection one sees that the answer is no. A logical error like the one committed here is sometimes called, *the fallacy of misplaced concreteness*. Since powered flight (birds) and bridges were around long before Newton was born, we see that the confusion is between the *physical properties* that make these things possible and the *abstract laws* used to express their interrelationships. Just as we don't eat restaurant menus, we don't fly in or tread over the equations. Due to repeated use and a reputation for reliability, the equations have come to be regarded as being real (concrete) as concrete, real as aluminum or feathers or muscle. But they're not. When the law becomes as real as the physical reality, insensibly we forget that our job is to twist theories to suit facts; not to twist facts to suit theories. It bears repeating: The *abstract law* is that the speed of light is c for everybody; *the physical fact is that it is not*.

6.8 Dramatization by Amplification

To more dramatically illustrate the reality of the Sagnac effect, imagine that the disk is much larger and rotates much faster than the Earth. Suppose then, at a given moment, we send a green beam in the direction of rotation and an orange beam against the direction of the rotation. It is not hard to imagine that further details of the setup could be arranged so that anybody – even a child – could *see* the time gap with their own eyes and with their own sense of time. The orange beam returns to the observer before the green beam. In our actual experience the effects require sensitive instruments to detect. In this scaled-up example we have only, so to speak, *amplified* the effects to the level of gross human perception. The anisotropic light speed and the time gap then become starkly evident physical properties.

6.9 Einstein and Others on Rotation

Now let's return to our disk, whose physical and abstract properties, we've begun to see, have sometimes been confused. Going back to the source helps to clarify our perspective. Einstein wrote:

> Thus on our circular disk, or to make the case more general, in every gravitational field, a clock will go more quickly or less quickly, according to the position in which the clock is situated (at rest). For this reason it is not possible to obtain a reasonable definition of time with the aid of clocks which are arranged at rest with respect to the body of reference. A similar difficulty presents itself when we attempt to apply our earlier definition of simultaneity in such a case, but I do not wish to go any further into this question. [52]

One might quibble with exactly what Einstein meant by "not possible," "reasonable," and "difficulty." But the truth is that physicists, engineers and technicians have succeeded in doing the "not possible." They have built a system around our massive rotating Earth in which time and position are quite reasonably defined. The end product provides a physically robust spacetime grid on and near Earth's surface that is accurate down to an extremely fine scale. Note further that improvements to the system are limited only by our technological ability, not by the abstract theoretical "difficulties" Einstein was referring to.

And yet, as late as the 1960-70's one could still find in the literature statements to the effect that synchronizing clocks on a rotating body is "impossible." The most extreme "impossibility statement" I know of is from Zeldovich and Novikov, who wrote not only that "it is impossible to synchronize time on a rotating body," but that "the very concept of synchronization does not exist." [53] (Some stupendously misplaced concrete here, we observe.)

Other authors who have at least allowed rotating observers to *conceive the idea* of synchronizing their clocks include Born [54], Landau & Lifschitz [55] and Rosser [56]. Still these authors, following Einstein, have claimed it is not possible to actually do so. The "problems" for these authors are essentially the same as those mentioned by Einstein: the rate differences due to radial distance differences and the time gap resulting from the Einstein synchronization method. It turns out that the rate difference problem is actually quite easy to solve. As we will see in the next section, the solution has been implemented in practice in the Global Positioning System. The time gap "problem" is even more of an *imagined* problem than a real problem. It arises if one insists on using Einstein's synchronization method so as to maintain that the speed of light will equal c with respect to an adjacent pair of clocks on the circumference.

Note that the time gap has two different manifestations: As stated above, the gap can be measured using *one* clock to find the difference in times taken for co-rotating or counter-rotating light beams to return to their source. This doesn't cause any real problems and it is consistent with the center-flash synchronization method for any two adjacent clocks. The problematic manifestation is a result of Einstein synchronization in which case the gap would be the difference in time setting that one clock would need to have *with itself*.

I think it is safe to assume that one of the reasons Einstein did "not wish to go any further into this question," is that he was loathe to contemplate a scheme in which light speed did not equal c, and yet he, too, found the time gap to be objectionable (awkward?). Many authors nevertheless advocate Einstein-synchrony because if one ignores the need to synchronize all the way around the circumference, it allows one to maintain that light speed is always *locally* equal to c. This is sometimes thought of in terms of imaginary "Locally Comoving Inertial Frames" (LCIF's). These are essentially the inertial frames attached to clocks released from rotation onto infinitesimal tangents. At the moment of release, one such clock suddenly finds itself in a bona fide inertial frame so that observers in it are supposed to synchronize their clocks according to the Einstein prescription, and thereby arrange it so that light speed suddenly becomes isotropically equal to c. One of the questionable aspects of this approach is the sheer multitude of frames that must be invoked in the course of working one's way around the rotating disk.

Superficially, this sidesteps the time gap problem, but it is much more complicated than a simple center-flash procedure and it denies rotating observers a global set of synchronized clocks. Is it really so important to have the speed of light "locally" equal to c? Does the possibility of implementing this procedure mean that the speed of light *actually does* locally equal c? Or are the stipulators further enabling the fallacy of misplaced concreteness? That is, are Einstein's postulates and his interpretation of the Lorentz transformations taking undue precedence over the *physical fact* that, at least *globally*, the speed of light with respect to rotating observers clearly does not equal c? If the speed does not equal c in the global (general, non-inertial) case, then why insist that it does equal c in the local (special, inertial) case? The soundness of the standard logic is not at all obvious.

Having similar misgivings as my own concerning this "traditional (LCIF) approach" to the Sagnac effect, R. D. Klauber has commented:

> The plethora of possible settings for the same clock in a rotating frame results from insisting on "desynchronization" of clocks in order to keep the (one way) speed of light locally c everywhere. And thus, one is in the position of choosing whichever value for time one needs in a given experiment in order to get the answer one insists one must have (i.e., invariant, isotropic local light speed). One can only then ask if this is really physics or not. [57]

It appears we have three choices: 1) You can arrange to have *isotropic and constant light speed* with respect to adjacent clocks if you're willing to accept an objectionable time gap. 2) You can "avoid" the time gap and arrange to have *isotropic and constant light speed* with respect to adjacent clocks by invoking a multitude of local inertial frames, which denies the possibility of a set of globally synchronized clocks. Or 3) you can simply accept the measurement made using one clock indicating *anisotropic non-c light speed* with no time gap – which is the same as measurements made locally when clocks are center-flash synchronized.

Clocks in the GPS are synchronized by the latter approach. It is practical and reasonable. What is *not* practical nor reasonable is the set of desynchronized clocks that you get by following Einstein's prescription – whether the ambiguous local only variation or the unambiguous but troublesome time gap variation. The "impossible to synchronize" stance is thus clearly not that of a sailor or pilot or detective who would persist at solving the problem till he found a scheme that works. It would seem rather to be that of a stubborn "philosopher" who would prefer not to disrupt his foundational precepts.

6.10 Interlude and Assessment

In Appendix A and B you'll find further examples from the literature that reinforce the above impressions on the Sagnac effect and the problem of rotation in SR. In the next subsection an imagined scenario is presented whose purpose is to illustrate one final point concerning all this. The point is the limited nature of SR, especially with regard to its applicability to the problems raised by the phenomenon of rotation. Ultimately, a more comprehensive theory is needed to satisfactorily answer the questions that rotation draws attention to. (On this point, Einstein and I agree.) I would recommend reading §6.11 and §6.12 so as to fully absorb this point. But readers eager to see where I'm going with all this may wish to skip ahead to Sections 7 and 8.

In any case, in the interest of keeping our bearings straight, the reader is reminded here that we are not out to "refute" SR. The theory is mathematically consistent and I make no pretense of falsifying it with my arguments. The main purpose has been rather to show that the empirical facts that are often associated with SR need not be looked at through orthodox Special Relativistic glasses. It is never too late to *question* the cornerstones of the prevailing theoretical edifice – especially if those cornerstones are more abstract than physical. And especially if their apparent validity depends on them keeping to their local corners.

6.11 Hypothetical Dilemma

Imagine an astronomically huge circular ring that rotates with respect to the axis perpendicular to its center with the speed $r\omega = \frac{1}{2}c$. Suppose the ring is inhabited by intelligent beings who have synchronized their clocks by a flash from the center. The ring's radius is $r \approx 2.3 \times 10^{15}$ meters, which makes its centripetal acceleration an Earthlike $9.8\,\mathrm{m/s^2}$. Now suppose the inhabitants need to take an expedition to quell a distant emergency – a problem that requires not only urgency, but a lot of personnel.

The plan is to launch 360 ships onto the same tangential line directed to the distant trouble. At every 1° interval along the outer periphery, a ship is prepared with crew and two clocks, whose purpose will be specified momentarily. The first ship to be launched is designated #1, the last, #360, with corresponding numbers in between.

For many many years events on the ring have been coordinated and recorded on the basis of the center-flash clock synchronization scheme. For this is the only way the inhabitants could keep a continuous (no time gap) set of clocks which both individually and in coordination with one or more additional clocks, would reflect the physical fact that the average speed of light signals relayed around the circumference is $\approx (c + r\omega)$ against the direction of rotation and $\approx (c - r\omega)$ with the direction of rotation. The proportions of the system are so extreme, that, in the fast direction, a signal would return to its starting point in ≈ 1 year and in the slow direction the signal would return in ≈ 3 years.

Note: Although length contraction and time dilation would be important effects to include for certain contexts, for ours they would play a small role. In the interest of simplicity, we therefore sacrifice some accuracy by omitting these second order effects from the following discussion.

The 1° spatial intervals correspond to propagation times ≈ 1 day or ≈ 3 days, depending on direction. Following the initial launch of ship #1, over the course of three years, each ship is released onto the tangent.

As to the reason for having two clocks on each ship: One of the hoped for scientific byproducts of the voyage is that it might help determine which of two hypotheses is more accurate with regard to light propagation speeds. As per the recommendation of Dr. Sellauber, one clock on each ship is to remain continuous with the time it kept while back on the ring. Using these unadjusted original clocks, Sellauber has argued that it may be best to maintain the impression of anisotropic light propagation from one ship to the next. These times would thus still be measured as

$$t_\uparrow \approx \frac{L}{c - v} \qquad \text{and} \qquad t_\downarrow \approx \frac{L}{c + v}, \tag{15}$$

where $L \approx 2\pi r$, $v \approx r\omega$, $t_\uparrow$ is the time light would take to go in the forward direction, from ship #360 to ship #1, and $t_\downarrow$ is the time light would take to go in the backward direction, from ship #1 to ship #360.

As per the recommendation of Dr. Ashstein, on the other hand, each ship should also be equipped with a clock that reflects the fact that the tangential train of ships now comprises an inertial system. Being in uniform motion, the fleet should regard light propagation as being isotropic. Using ship #1 as the Master clock, the second clock on all of the trailing ships is accordingly reset. Only on ship #1 do both clocks show the same time, t_1. The propagation times can be expressed as in Eq. (15), except that the velocity, v drops out, so the time is supposed to be the same in either direction.

If n is the number of ships along the line of communication and l is the distance between any two adjacent ships, then the propagation time (in either direction) would be given by

$$t_{(n-1)} = \frac{l(n-1)}{c}\,. \tag{16}$$

If n is thought of as the number of ships connecting a given ship with ship #1, the initial synchronization signal propagating backwards from ship #1 yields the time $t_1 + l(n-1)/c$. Each second clock (except for the one on ship #1) is thus reset to show this time and so abide by the idea of stipulated isotropic light propagation. As we should expect, this procedure results in a discrepancy with the original clocks:

$$\Delta t_{(n-1)} \approx \frac{lv(n-1)}{c^2} \tag{17}$$

This amounts to about 18 hours between any two adjacent clocks along the train. So for ship #360, the difference will be about $3/4$ of a year.

Now here's the problem: An engineer on board ship #180 has discovered quite conclusively that a design flaw in the Freem Thruster Containment Module (FTCM) will cause a fatal malfunction in every ship unless it is corrected. The correction would only take a few minutes; the problem is informing everyone about it. To make matters worse, the transmission antenna of ship #180, which needs to be focussed in one direction at a time in order to maximize signal strength, has been badly damaged by cosmic debris collisions. It is only good for one more transmission. Unfortunately, for budgetary reasons, only ships #1, #180 and #360 were equipped with transmitters powerful enough to reach the whole train. Signals from other ships' transmitters dissipate too rapidly to remain coherent to receivers beyond the nearest adjacent ships. This rules out the possibility of having ship #181, e.g., transmit a signal forward to ship #179 for a relay up to ship #1. The time of FTCM failure was predicted with 99.99% confidence to be 180.5 days (± 5 minutes) as given by the fleet's original clocks. Given the above equipment limitations, it is therefore impossible to warn the whole fleet in time. Damage control will require some hard decisions.

Recall that the mission is of some urgency in the first place, but even more important is that enough personnel arrive to accomplish the goal. If more than half the fleet were to perish, the mission would become increasingly difficult with each additional lost ship. If the speed of light is now really isotropic, as suggested by the second set of clocks, then, whether a signal is sent forward or backward, it would reach only 2/3 of the ships in that direction (i.e., 1/3 of the total fleet) before the rest were obliterated by Freem radiation. Sending warning signals forward would thus save some time, but leave alive at most 1/3 of the fleet.

According to Sellauber the situation might not be quite so dire. According to his hypothesis, light propagation is not likely to be any more isotropic now than it was back on the ring. Resetting the clocks didn't change that. If he is right, then sending the signal forward would probably only save 1/3 of the forward ships (i.e., only 1/6 of the fleet). But sending the signal backward would probably save the whole rear half. The mission would be delayed a bit, but they'd have ample personnel to get the job done. Sellauber is compelled to point out a caveat (implied by the use of words, "likely," "might" and "probably," above): There is no guarantee that light propagation was actually isotropic with respect to their original rotation axis. (That's why the one-way speeds given above were stated as averages.) If light propagation was anisotropic even with respect to their original rotation axis, the present dilemma could possibly be diminished or exacerbated, depending on the direction. But since both the magnitude and the direction of this possible anisotropy is unknown, its effect on the present situation would be random and so should not influence their decision.

Though not conclusive, another clue is pointed out by Sellauber, which nevertheless implies that the anisotropy found as they rotated on their ring would persist in the same sense along their tangential train. While attached to the rotating ring, the Cosmic Background Radiation (CBR) revealed a spectral red / blue dipole whose magnitude remained constant, but whose direction changed so that the blueshift was always on a tangent in their direction of rotation. Correspondingly, with respect to the axis of rotation, there was no dipole at all. After being launched onto one of these tangents, the dipole has remained constant (blue) in the direction of their motion.

Since we are contemplating this whole scenario, essentially, from the point of view of rest with respect to the original rotation axis (from which) the red / blue shifts reveal no dipole) and we have followed every important detail, it should be obvious that sending the signal backward is the right choice. From the point of view of the crew of ship #180, the decision is similarly based on this *remembrance* of the motional history of the fleet and the consistency with the CBR red / blue shifts. But suppose all record of this history, in their minds and data records, was somehow wiped out so that they don't even know they are "heading towards" their troubled destination. Suppose further that their view of the microwave background radiation has been obscured. They would thus find themselves in a string of ships whose direction in indiscernible. Having no clues as to their state of motion (aside from that it is uniform) they reasonably assume that they are at rest. In this case they would certainly be forgiven for setting their clocks based on the assumption that light propagates isotropically.

If the bad news from the engineer on ship #180 was discovered under these circumstances, they could hardly do better than to flip a coin to decide which direction to transmit the warning. But with an unobscured view of the cosmos and a non-amnesiatic consciousness that considers history and any other possible clue, a better than random decision can be made. With so many lives at stake, it would surely be an act of horrible negligence to decide by flipping a coin or to deliberately send the signal forward, when experience and awareness of one's surroundings provide such strong evidence that more lives would be saved by sending the signal backward. This scenario thus implies not only that light propagation cannot in general really be isotropic with respect to uniformly moving bodies, but that it may be possible, in some circumstances to make a reasoned guess as to the direction of the anisotropy. Dogmatically insisting that light propagation must be isotropic with respect to oneself or other uniformly moving bodies (IRF's) just because one such frame is "related" to all others by the Lorentz transformations, could cause fatal damage.

6.12 What Does it All Mean?

Any statement claiming or implying that light propagates with a constant speed equal to c, if it does not at the same time point out that this is just an average back and forth speed, is plainly false in most every case. If correctly determining the direction of light's anisotropic axis becomes a matter of life and death, any right-thinking physicist will surely consider all clues, not only in accelerating reference frames, but in uniformly moving frames as well.

If, after being released onto a tangent, you choose to *define* away the anisotropy, if, while moving uniformly you ignore any clues gathered from your history and your global, non-local surroundings, if you still choose to define light propagation as being isotropic, it may cost your life. Does light go faster toward the east or west; north or south; up or down? How does the CBR fit into the picture? How might huge concentrations of matter affect light propagation? One had better think clearly and thoroughly examine all the evidence because the Universe cares not a whit about petty human stipulations nor "elegant" geometrical principles.

Similarly, any claim or implication that the relativity of simultaneity provides some kind of profound insight into the nature of space and time which neglects to clearly point out that this abstract idea derives from the physical fact of anisotropic light propagation is also sorely lacking in truth. The purpose of the scenario in the previous subsection and this section as a whole, is to distinguish as clearly as possible what we know from what we don't know, and to emphasize that stipulations are a poor substitute for knowledge – especially if we tend to forget the difference.

The domain of SR is not broad enough to be of any use for solving the above dilemma. Without bringing in clues from cosmology, no certainty can be attached to the fleet crew's decision to send the signal backward. One needs only to point out that the tangent-traveling fleet could well be *at rest* with respect to the rotation axis of *another* huge ring, a mirror image of their own, we might say, upon which clocks were also synchronized by the center-flash method. Launching a tangential fleet from this second ring back "toward" the original ring would put these tangentially launched (Ring #2) ships *at rest* with respect to the original ring. Is light propagation isotropic with respect to this train of ships or with respect to the original train heading in the opposite direction? If it's possible to find out, it would only be because our Universe is much more complicated than Minkowski space.

It is an intriguing theoretical fact that the phenomenon of *rotation* appears as a sort of stepping stone or guide post. Rotation seems to define a zone beyond which SR does not really go without causing suspicious awkwardness or raising questions whose answers require a more comprehensive theory. It is well known that Einstein had a similar regard for rotation, as it helped him to conceive GR. As we recall from §5, Einstein's conception of gravity, which included the idea of *static* gravitational fields, inspired him to regard any observer, even those in non-inertial systems, as being in a state of *rest*. In later sections we will take the opposite approach. We will adopt the rotation analogy as inspiration for a model of gravity in which *motion* plays the more significant role. For now, the most important thing to bear in mind is that the simplification of theoretical principles brought about by accepting the symmetric, isotropic conceptions of SR comes at the cost of obscuring asymmetries and anisotropies that are likely to exist – and in the case of rotation have been proven to exist – in the real physical world.

7 Simplified GPS

The conflict between Einstein synchronization and the desire to have a continuous definition of time around the circumference is just one of the reasons given (or implied) for the "impossibility" of finding a "reasonable definition of time" on a rotating body. Another reason is that the frequency of a rotating clock depends on its distance, r, from the axis. Even though a given axially concentric circle of clocks can be synchronized by the center-flash method, they will not tick at the same rate as other circles of clocks and so would quickly grow ever more out of synch with clocks at different radii.

The solution to this problem is called "syntonization" – adjustment which makes the frequency of one oscillator the same as another. Knowing how rotation, with its varying r-dependent speeds, affects the rates of identical clocks, it becomes a simple matter, in principle (and these days not even too difficult in practice) to adjust clocks' frequencies so they are all the same – even though their "natural" frequencies would be different.

In the GPS syntonization comes into play not only for the effect of orbital motion and rotation, but also for the effect of gravity. Instead of immediately combining these effects, let's first consider gravity by itself. Accordingly, let's imagine a massive non-rotating sphere centered on the grid we started with in K. Imagine as well, that the sphere has an array of very tall vertical poles planted on the surface. By "very tall" we mean extending to some astronomical distance. At each one of many regular intervals along the poles we have mounted an accelerometer, a clock, a trio of measuring rods

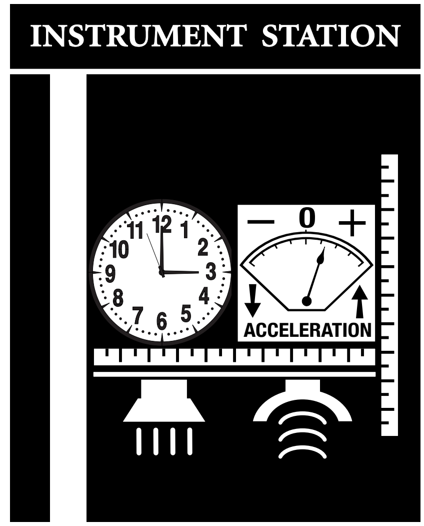

Fig. 9: The essential basics.

– mounted as cartesian axes – as well as devices to send and receive light signals. We will refer to such places as "Instrument Stations," (IS's). (See Figure 9.) In practice, of course, such assemblies of instruments are carried on board satellites. But orbital motion adds a level of complexity which for the moment we'll neglect. The GPS comprises 24 satellites orbiting at a distance of about four Earth surface radii $r = 4R$. So let's imagine 24 poles whose IS's include one each at $r = 4R$. As predicted by GR and as data from our poles would confirm, even clocks that appear not to be moving, have rates that depend on distance from the center of the sphere. Following is a scheme for completing the setup of a simplified GPS on our idealized sphere. First we'll syntonize, then synchronize, then syntonize again.

Syntonization has been achieved in the real GPS by either equipping the satellite clocks with frequency synthesizers that can be controlled from the ground, or by pre-programming them to run at a rate that would compensate for the effects of gravity and orbital motion. The rate that would need to be compensated for gravity alone would be

$$\frac{f_r}{f_\infty} = \sqrt{1 - \frac{2GM}{rc^2}}\,, \tag{18}$$

where f_∞ is the rate of a clock at infinity, G is Newton's constant and M is the sphere's mass.

Since most inhabitants of the sphere live on or near its surface, it is deemed most practical to have the rates of all other clocks adjusted to match those located on the surface. This idea comes directly from the real GPS design. Due to deviations from sphericity, slightly eccentric satellite orbits, etc., implementing the idea is not as simple in practice as it is for our idealized sphere. But the principle is the same. Let us therefore imagine that the clocks on our tall poles – especially those at $r = 4R$ – have been pre-programmed to tick at the rate which matches the surface clocks. This completes the first syntonization.

Synchronizing the GPS clocks, in practice, involves calculating and compensating for rotation and orbital motion. But the idea is to end up with an array of clocks whose settings are the same as what you'd get by the center-flash method. As we recall from §3, this is essentially the same as the method described by Taylor and Wheeler. [9] Imagine that this is achieved for our sphere with an actual center-flash, as though there were no matter obstructing the signal from $r = 0$. Assuming that the radial distance r (as that appearing in Eq. 18) is an accurate measure of distance, we'd want all clocks to be pre-set to the time $t = r/c$ and to start ticking as soon as the flash arrives.

Various factors make the real world syntonization / synchronization scheme a little more complicated than the above. Only one of these factors is of fundamental importance for our purpose. That is the fact that the r in Eq. 18 and that used to compute the propagation time, refers to a "coordinate" distance, not a "proper" distance. The importance of the distinction and the magnitude of the difference will be discussed in more detail later. For now, note that in the Earth's GPS the difference is so small as to involve a ranging error of only a few millimeters.

The last major part of the System is the user, i.e., receiver-bearing observers who want to know where they are, what time it is and how fast (and in what direction) they're going. For details on how all these parts function so as to give the observers what they want, consult [58]. Here it will be sufficient to say, first, that the clocks built into most receivers are less accurate than the clocks on the satellites (poles). The System's accuracy depends on the fact that every location on the sphere is in view of at least four IS's (satellites). With timing and ranging signals from four different directions, by using a sophisticated calculation algorithm, a receiver can have its clock adjusted to the correct GPS-time and have its three dimensional location determined to within a few meters, centimeters or millimeters, depending (crudely speaking) on how expensive the unit is.

Assuming that the syntonization needed for gravity alone has been achieved, having begun with a non-rotating sphere, we see that it is simple, in principle, to set up a system of synchronized clocks. Replicating the actual GPS requires one more step: to set everything in motion. After the center-flash signal has been received *Synchronization* would require no further adjustment. But extra syntonization factors will be needed to account for the rate change arising from change in motional state. So let's imagine that the extra factors needed for rotation and orbiting are designed to kick in at a given moment when the "planet" begins to rotate and all the IS's at $r = 4R$ are launched onto circular orbits. The result would be, in essence, what we've got on and around our planet Earth.

So as to make the sequel logically consistent, let us now go back to the state of our sphere-with-poles prior to rotation and prior to syntonization. In other words, we're back to our non-rotating sphere whose array of Instrument Stations contains clocks that now all tick at their natural frequencies.

8 Gravity Virgins

> *Does the totality of the observable interactions compel us to adopt [the] standard interpretation, or might the same pattern of experiences be explainable within some other, possibly quite different, conceptual framework?*
>
> – Kevin Brown [59]

The foregoing contains many clues. Those who are satisfied with the status quo may well ask, clues to what? What's the mystery? But those who feel some discontent with our present state of knowledge of space, time, matter and gravity, will be eager to try looking at it all from a new angle. To make it easier for the mind to accommodate that angle, the idea is presented as the experience of "space explorers" who come to *discover the effects of gravity for the first time*. How else could we be assured of sufficiently minimizing our accumulated metaphysical baggage so as to get a truly fresh look at the problem?

8.1 A Different World

Imagine a civilization advanced enough for space travel, yet ignorant of gravity. This civilization has evolved far from any large masses – perhaps as an experiment conducted by an even more advanced civilization – living inside a huge self-sustaining Rotating Cylindrical space station. Let's call members of this civilization, *RC's*. The RC's "ground" is the inner wall of the Cylinder. They have conducted many experiments involving light, clocks and motion. Motion is "sacred" to the RC's because they realize that if their Cylinder did not rotate they would not survive.

The two devices that have most benefitted the RC's concepts of motion are accelerometers and clocks. While spending most of their time happily pressed against the Cylinder's inner wall, they have also thoroughly explored the Cylinder's immense bracing, which spans across the rotation axis, where the acceleration and velocity are both zero. The RC's have conducted Sagnac-like experiments with light and understand that clocks on the inner wall are slowed while those at rest with respect to the axis have a maximum rate. They understand that it's not acceleration, but velocity that causes clocks to slow down.

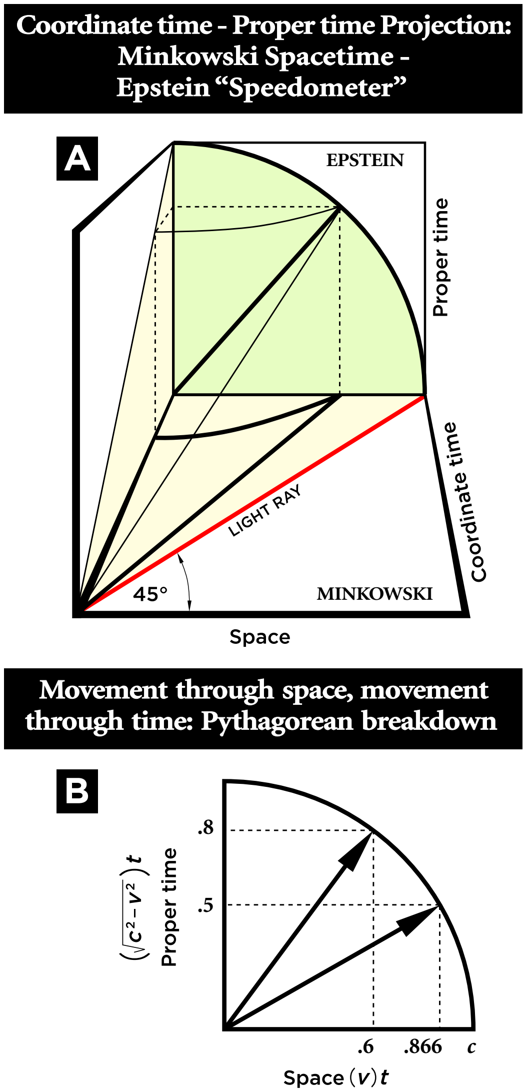

Fig. 10: Space-propertime diagrams, adapted from L. C. Epstein, *Relativity Visualized*.

8.2 Odometer

Having full access to the rotation axis, both inside and (its imaginary extension) outside their cylinder, the RC's have even developed a "mileage" tracking system based on the effect velocity has on clock rate. The rotation axis has proved itself to be a most reliable standard of rest. Thus any movement with respect to a stationary point on the axis results in a slowed clock and a correspondingly reduced *elapsed time*.

The RC's basic odometer relation is

$$L = c\sqrt{T_{\odot}^2 - T^2}\,, \tag{19}$$

where L is the distance travelled, $T_{\odot}$ is the time given by an axis clock and T is the time given by a clock that moves during the same $T_{\odot}$-time with a constant speed. This equation is perfectly reliable for simple situations, such as any point attached to the Cylinder's bracing or wall. For more complicated motion paths the RC's have developed more sophisticated equations and algorithms that are built into all vehicles. With very sensitive gyroscopes, accelerometers and clocks, the RC's have devised a tracking system giving them accurate measurements of distance travelled with respect to a point on the axis based entirely on the elapsed time of a moving clock. The RC's and their vehicles usually carry a second clock that can be reset to match the local time. But the factory installed original clock is left untouched so that "total mileage" is a readily available datum.

How different is this world from ours, with its clearly defined standard of rest, where *accelerometers and clocks are absolutely reliable indicators of motion!* To the RC's it is a fact of everyday life that motion is absolute. For example, they have no reason to doubt the implications borne of their Sagnac-like experiments, that the speed of light is isotropic only with respect to the rotation axis. This is the basis of their odometer relation and their general reckoning. Their kinematic accounting system is thus quite like the ϵ-Lorentz transformations discussed in §4. For the RC's $\epsilon = \frac{1}{2}$ only for the rotation axis and for all moving bodies the $\epsilon \neq \frac{1}{2}$ values depend on velocity, seemingly *absolute* velocity with respect to the axis. Therefore, when they take an excursion from the Cylinder, either by tangential launch or by rockets, it would never occur to the RC's to "resynchronize" their clocks for the purpose of *making* the speed of light equal c. For uniform motion their excursions can be charted essentially as in Figure 4. For excursions involving more complicated maneuvers, the effects can be charted as a *series* of similar figures, each one having the appropriate ϵ value. The RC's know well that the back and forth speed of light – when measured with their co-moving rods and clocks – is always equal to c and that it is therefore a meaningful universal constant. But the one-way speed is generally equal to something else.

Traveling in the neighborhood of their Cylinder, the RC's not only measure the speed of light to be anisotropic, they also find lengths in the rest frame to be "expanded" and clocks in the rest frame to be ticking *faster* than their own. This is all to be expected and is perfectly consistent with empirical results of Einsteinian SR. The superficial differences are only to do with the different synchrony parameter values. Though the RC's may be able to appreciate the computational convenience of Einstein's preferred "$\epsilon = \frac{1}{2}$ for everybody" approach, it is so far removed from their *physical experience* that the idea is only rarely discussed – such discussions being found mostly in obscure philosophical journals.

8.3 Speedometer

The RC's basic odometer – the model used to represent motion of the various parts of their rotating cylinder – may be simply depicted graphically. Not surprisingly, the odometer

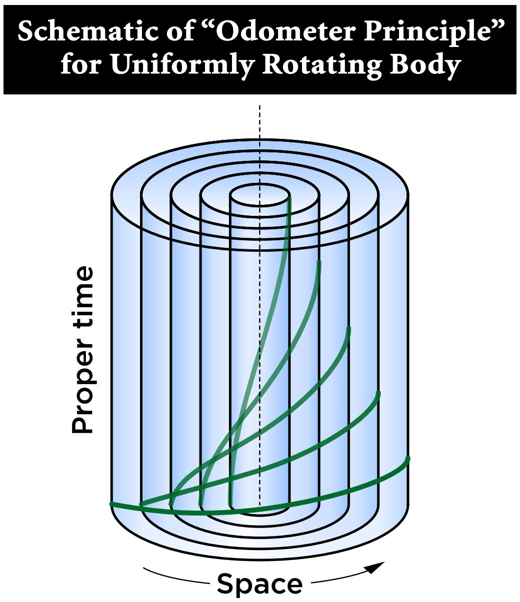

Fig. 11: Identical length worldlines, showing that different radial distances in the same rotating system travel proportionally different distances through space and time. Compared to the time given by a clock on the axis, the *smaller* the elapsed proper time, the *greater* the distance traveled through *space*. (Equation 19.)

is directly linked to a "*speed*ometer." In any case, that's what Epstein calls his 90° projection of the usual Minkowski diagram. (He also calls it a "space-*proper*time diagram, as distinct from the usual Minkowskian space-*coordinate*time diagram.) [60] Figure 10A shows how the two different projections of the diagram are related to each other. According to this diagram a light ray is represented not by a 45° line but by the *space*-axis (abscissa). And the ordinate represents not *coordinate* time but *proper* time. The range of the speedometer "needle" is thus a quarter circle whose height is a coordinate time unit (e.g., second) and whose width is the coordinate distance traveled in that time.

In terms of this diagram, a light ray thus travels entirely through space (zero proper time). Whereas an observer at rest (the RC's axis) travels entirely through time. For this latter extreme, coordinate time equals proper time. Whereas, for all speeds in between, the Pythagorean Theorem gives an easy decomposition in which the proper time is always less than the coordinate time when some portion of the total "speed" is diverted through space. Figure 10B shows two representative uniform speeds.

For the cyclic motion of the RC's world, the whole diagram is most intuitively depicted by rolling the individual diagrams corresponding to particular speeds into a set of nested tubes (Figure 11). As we see, the variation in "speed through time" is clearly related to the variation in "speed through space." Let's itemize a few other key points arising in this scheme (Figure 11):

> **1.** All curved paths drawn on the tubes are as "speedometer needles" whose identical lengths represent the same distance through coordinate time: one half turn around the Cylinder.
>
> **2.** Though not explicitly shown in the figure, it nevertheless follows from the RC's practical center-flash synchronization method, that all clocks on a given circle (tube) are synchronized with one another. Quite possibly the RC's would want to have a second set of clocks on each circle whose rates would all be syntonized with clocks on one particular circle – perhaps that of the cylinder wall, where they usually live, or perhaps that of the axis, or perhaps all three. In any case, they would certainly want to have a range of clock sets that reflect the natural unsyntonized rate differences from axis to wall.
>
> **3.** Finally, note that the tubes in the figure represent a passive backdrop; the blank map of empty spacetime. As far as the RC's are aware, there is a rather sharp discontinuity where empty space meets rotating (or even non-rotating) gross matter. Whether we depict only one or many clocks per circular cross-section, the corresponding helical lines represent movement of these clocks *through spacetime*. The extreme cases, $r\omega = 0$ and $r\omega = c$ correspond to the maximum or zero passage of proper *time* and zero or maximum speed through *space*, respectively. But in both cases, it is either a material object (clock) or a light ray (non-clock) that moves. It is not *spacetime* that moves. Motion is conceived as the change of position of something with respect to the passive backdrop of space, whose coordinates are typically based on one designated material body (e.g., the Rotating Cylinder) or a plurality of bodies (e.g., the "fixed" stars).

Item number 3 above might have been abbreviated by stating simply that tubular motion models are conducive to illustrating the difference between motion *through* space as opposed to motion *of* space. Motion *of* space is not part of standard physics; it is mentioned here as a hint of an idea that the RC's will make use of later. As noted above, the nested tubular "odometer" of Figure 11 is directly applicable only to the motion of different radii of their Cylinder. The spacetime paths and corresponding odometer data representing independently powered, steerable vehicles would require more complicated deviations from this base helical system.

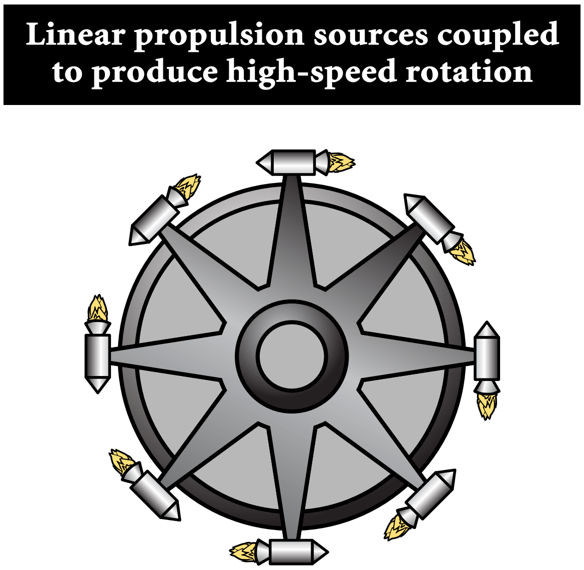

Fig. 12: The limit to linear speed applies also to the angular speed of a rotating body. If the acceleration is uniform, as the speed with respect to the axis approaches that of light, Equation 20 will be obeyed instead of the linear Newtonian relation. (This has been empirically demonstrated in particle accelerators, where the propulsion is produced by electromagnets, not rockets.)

8.4 Acceleration and Universal Speed Limit

We need to understand a few more kinematical principles that the RC's have adopted. They have arisen not only in the course of their natural technological development and their Space Exploration Program, but in their conjecture as to what got their Cylinder rotating in the first place.

In the tubular odometer of Figure 11, we see that the rotation speed increases linearly with tube diameter. The RC's have long recognized that, for a given angular velocity, ω, there is a corresponding size limit, because the product $r\omega$ cannot exceed the light speed constant c. Rather than investigate the limit by adding material to increase the radius, the RC's apply what they have learned about *linear* acceleration. It has long been known that uniform proper acceleration (as by a rocket) will increase one's speed with respect to the Cylinder's axis, not linearly by the simple formula, $v = at$, but by a modification which takes the limit into account:

$$v = \frac{at}{\sqrt{1 + a^2t^2/c^2}}. \tag{20}$$

Even for small accelerations a, as t becomes very large v approaches, but never reaches c. The origin of the RC's world and, in particular, how it acquired its rotation, is an unsettled question. The most widely accepted hypothesis is that a large fleet of powerful rockets were attached all around the circumference, each pair being "coupled" by a huge "spoke" (inner bracing) spanning the Cylinder's diameter. (See Figure 12.) Given an endless supply of fuel the Cylinder's angular velocity could ideally be increased so that the circumferential speed would approach ever closer to c. Well before it attained such a huge speed, they surmise, the rockets were turned off and removed, leaving the Cylinder to "forever" maintain its state of uniform rotation. So the story goes.

The pertinent kinematic principles can be starkly illustrated by imagining that the Cylinder's circumference has been brought up to the speed $2c/\sqrt{3} \approx 0.866c$. This is the speed at which the rates of clocks and the lengths of tangentially oriented measuring rods at the circumference will be 50% of their rest rates and lengths. On the speedometer diagram we see that this means the needle will dip to $30°$ from the space-axis.

Now imagine that a section of the Cylinder has been nested inside a huge non-rotating "ring dial" that nearly touches the Cylinder's outer wall. This portion of the outer wall and the ring dial both have numerous equally spaced clocks arrayed all along their edges. The ring dial is basically one continuous circular "ruler" having tick marks along its inner edge. Whereas, the outer wall of the Cylinder is equipped with many small measuring rods – each one being attached by a single "point" fastener to allow the rod to expand or contract. Referring to Figure 13A, we show the position of a rotating measuring rod and rotating clocks as they move with respect to the non-moving ring dial. All clocks have been set (by a center-flash synchronization signal) to read zero at this instant. Figure 13B shows the same object one "smallo-second" later by the ring dial clocks. With respect to the ring dial instruments, as expected, this reveals the rotation speed $v = r\omega = 0.866c$.

These circumstances could also be modelled with a spacetime diagram as in Figure 4. But showing how the clocks and gauges might actually be arranged in physical space lends some concreteness to the idea that could not be so easily conveyed in Figure 4. Also we have troubled to illustrate these details so as to make it abundantly clear how *un*-relativistic this reckoning system is.

Consider, for example, how the rotation speed *could* be interpreted by an observer in the rotating system if such an observer were to consider herself as being *at rest*. Suppose this observer counts the tick marks on her own measuring rod passed by the point P of the ring dial in the time elapsed between Figures 13A and 13B. She then finds $1/\sqrt{1 - v^2/c^2} =$ twice as many marks as what she would find by counting the number of *ring dial* tick marks that pass one point of *her* rod in the same time. Furthermore, this is the length that moves past the point P in $\sqrt{1 - v^2/c^2} =$ *one half* "smallo-second" according to her own clock. Using these values, she could calculate that her speed is length/time $= 2\sqrt{3} \approx 3.464c$.

Having been born and raised in the Rotating Cylinder, however, she knows that she is not really moving that fast; she knows that her speed reckoning should always be referred

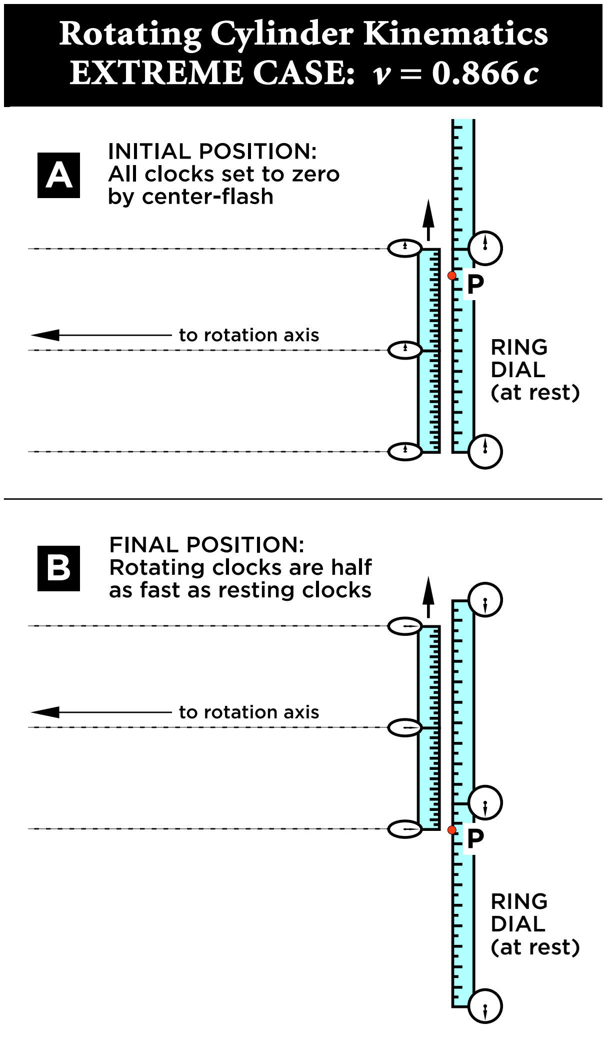

Fig. 13: Using center-flash synchronization means different velocities could be measured, depending on whether the data are referred to rotating rods and clocks or resting (Ring Dial) rods and clocks. Note that if Einstein synchronization were "locally" applied to the adjacent rotating clocks, they would appear here to be *desynchronized* exactly so as to make the velocity appear to have the same magnitude in both frames. But this results in a most extreme and impractical time gap; it implies a "relativity" to motion which contradicts the RC's physical experience.

back to the axis, which is the same thing as the ring dial. Her absolute speed is the number of distance ticks that a point on *her* rod passes on the *ring dial in ring dial time*. That is, $0.866c$.

Our observer recalls having found in the RC's obscure philosophical journals discussions about a conceivable alternative called "Minkeinstein spacetime." According to Minkeinstein, a rotating observer may begin with a series of closely spaced rotating clocks that are "locally" "synchronized" by a linear method which assumes that the speed of light is the same in both directions. She could then "locally" "measure" that the number of tick marks on her rods passed by a point on the ring dial in the time given by her (Minkeinstein synchronized) clocks yields the speed $v = 0.866c$. Although appreciating the imagination needed to conceive it, this scheme – wherein the speed of light is artificially forced to be isotropic so that all velocities will exhibit reciprocity – seems funny to her. She alternately chuckles and winces to think anyone might actually employ it because the scheme obviously breaks down globally. Light signals sent around the circumference back to herself reveal speeds $(c \pm v)$ *regardless* of how she might synchronize her clocks. So (again) the most reasonable synchronization scheme is that which is consistent with this absolute light speed anisotropy for rotating observers; i.e., which honors the axis as the true reckoning rest frame. The RC's have never entertained any abstract "relativity principle" as having any special virtue, so the idea of velocity reciprocity means little to them. What's much more important is having clocks synchronized so as to reflect the *empirical fact* of light speed anisotropy in opposite directions around their Cylinder. That this makes virtually all velocities asymmetrical, non-reciprocal and absolute does not trouble them. On the contrary, it serves to simplify their reckoning by fixing their bearings.

Our observer and her world are rotating through space; the axis and the ring dial are at rest in space. Acknowledging this axis as the "preferred" rest frame amounts to the same thing as the Reichenbach-inspired ϵ-synchrony scheme, according to which, for this frame, $\epsilon = \frac{1}{2}$, and the non-preferred $\epsilon \neq \frac{1}{2}$ frames are all determined correspondingly. Recalling Reichenbach, not only is it true that this scheme "could not be called false," given their experience, this is clearly the most reasonable kinematic scheme the RC's are likely to come up with.

8.5 First Encounter

During their entire evolution the RC's were deprived of a solar system; no orbiting planets; no orbiting moon; no large dense bodies massive enough to arouse any inkling of gravity. They understand electromagnetism well enough; they have a pretty sophisticated understanding of *motion*; but the RC's have no conception of gravitational attraction. Of course, they have often contemplated the apparent vastness of space and the mysterious points of light scattered across the otherwise black sky. After many years of practicing their excursions beyond their womb-like cylinder, gradually increasing the distance and the time away from home, they've just finished preparations for a much longer voyage.

A handful of RC's set out in two rocket ships (1 and 2) to explore the Universe. Many years pass and eventually they happen to find the end of one of our tall poles. They are delighted to find the accelerometer and the clock, etc.

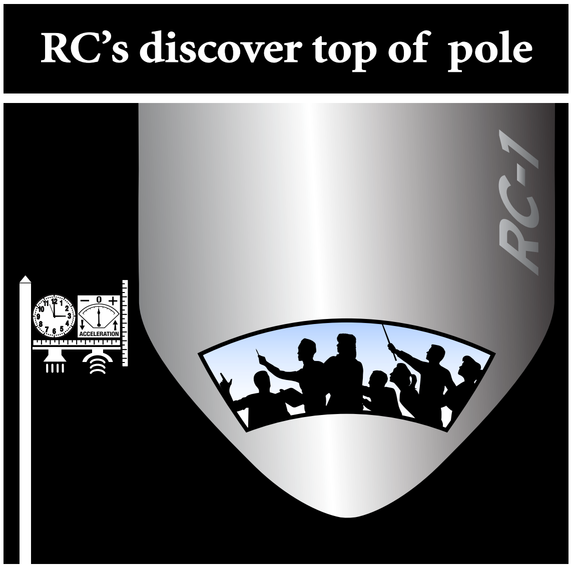

Fig. 14: After years of exploring in the void of deep space, the RC's happen upon a sign of intelligence: Instrument Station #1.

Having stopped to inspect these things, they notice that the accelerometer reading is extremely close to, but not exactly zero. Of course they realize that the rates of clocks are trickier to read and interpret. For example, they need at least a pair of clocks to get a meaningful assessment. The best they can do is to ascertain that while "co-moving," the instruments on the pole behave the same as those on the spaceship. In any case, the RC's recognize this station of instruments as a sign of intelligence – motion indicators have the same meaning anywhere in the Universe.

The RC's decide to leave their engines off and drift with respect to this pole. After a long while, they begin to visually discern a slight motion. After a very long time they find the first Instrument Station (IS-1) very slowly receding from view. Eventually, they are pleased to see another IS (IS-2) coming into view further along the pole. Since the pole is moving so very slowly, the RC's take readings and carefully inspect IS-2 without having to fire their rocket.

They are in no hurry, so the RC's continue this process for many more IS's, and a pattern begins to emerge. The accelerometer readings are slowly increasing and the speed of the pole past them is increasing correspondingly. They think, "whatever is at the other end of this pole, it is ever so gradually accelerating and ever so gradually *increasing* its acceleration." It's too early to draw further conclusions; they need more data.

As the RC's continue in this manner, after many more IS's have accelerated past them, it becomes necessary to scan the passing IS's with electronic instruments, as the pole's speed is now too great for casual visual inspection. The RC's curiosity grows as the speed increases, so a strategy is devised for unravelling the mystery and making a more thorough investigation. Since they have two ships, they decide to send Ship 2 toward the propelling end of the pole. After blasting onward for a while, Ship 2 eventually discerns not only a "spot" in the direction of the pole, but faint evidence of other poles that seem to be emanating from the same spot. Rather than continue directly toward the spot, it is decided that Ship 2 will veer out a ways beyond the pole to get a different view of the thing and take a long way around to the other side. Maybe the source of propulsion is on the other side of the spot. Or, if there's also a pole on the back side, then the plan is to see if the propulsion source is at its far end – or at least go as far out along it so as to be symmetrically positioned with respect to Ship 1, with the spot midway between them.

Ship 2 approaches the destination near the far side pole and the RC's are puzzled to discover that, in order to progress along it, they need to exceed a minimum acceleration *away* from the spot. If their acceleration is less than this minimum, the pole appears to accelerate in the direction *opposite* to that of the initial near side pole (which Ship 1 is still exploring). This is most baffling; the RC's have never before seen anything like it.

It's as though the spot is propelling both poles in opposite directions. Ship 2 manages to attain a symmetrical relationship between itself, the spot and Ship 1, which means rockets are off, yet the poles are accelerating past both ships in opposite directions. Could it be that the poles are being built up from "below," as though growing like trees planted on the spot? Not an unreasonable hypothesis, except that the spot now too appears to be getting ever larger. With their "neutrino-beam" radio, the ships remain in communication with each other even as the spot ominously begins to fill the view along the line of sight between them. Before long the RC's become totally awed by the size of the spot, which they now realize is a huge spherical chunk of matter. Another bewildering clue is that the signal propagation delays have grown shorter and shorter, as though the distance between ships were decreasing. How could that be if (as is the case) they have never registered any *acceleration* toward each other?

8.6 Puzzled Landing

There's no sign that the outward acceleration of the Sphere and its poles is going to let up; on the contrary, it continues to increase. The idea of maintaining a safe distance from the Sphere by veering off into elliptical orbits, perhaps understandably, does not occur to the RC's. So, it begins to dawn on them that the only way to avoid fatal collisions is to blast their rockets, to accelerate radially away from the Sphere, and to match its surface speed and acceleration just before contact. In the nick of time the feat is accomplished. The ships "land" without mishap.

What are they to make of this? It's still too early to say. But an important clue observed by both ships is that the acceleration of the poles – as indicated by the pole's accelerometer readings as well as their motion – increased as the Sphere got closer according to a definite pattern. *The acceleration varies as the inverse square of the distance from the Sphere's center.* Having avoided any local maneuvering (except for the landing) Ship 1, especially, was able to compute that the velocity of the pole at every point along the way was exactly as if the acceleration shown by the accelerometers were continually applied to it. Having access to the data signals from the IS's, they know that this does not mean all points of the pole shared this increase. Rather (remarkably) the acceleration shown by every accelerometer remains *constant* over time. The seemingly rigid pole somehow maintains a *range* of constant accelerations – varying as the inverse square of the distance. Surely this is a key piece of the puzzle.

Now that they have managed soft landings, they are able to confirm another piece of the puzzle, which is also provided by the IS's continual data signals. It turns out that the *frequencies* of the IS's clocks are diminished exactly as they'd expect from the measured *velocities*. For example, for three particular distances, $r = 16R$, $r = 4R$ and at the surface, $r = R$, the corresponding clock rates are in the ratio

$$\frac{f_{16R}}{f_R} = \frac{\sqrt{1 - \frac{GM}{8rc^2}}}{\sqrt{1 - \frac{2GM}{rc^2}}} \quad \text{and} \quad \frac{f_{4R}}{f_R} = \frac{\sqrt{1 - \frac{GM}{2rc^2}}}{\sqrt{1 - \frac{2GM}{rc^2}}}, \quad (21)$$

where G and M are unknown constants. As these equations imply, the RC's have already deduced that the effects have something to do with the Sphere's mass. But the mind boggling revelation is that the mass evidently combines with an entirely new constant, which they surmise they've overlooked because of their prior lack of experience with any mass so large as the Sphere. A poignant clue from these equations, of course, is that $2GM/r$ is the square of a velocity, which indeed corresponds to the velocity of the poles as they accelerated past ships (1) and (2). So some things add up, even as other things remain most puzzling and uncertain.

After a period of intense contemplation, the RC's emerge with two hypotheses. Note that, although previously ignorant of gravity, the RC's are mathematically sophisticated. They had long ago developed geometries involving non-Euclidean ideas; they are fully capable of thinking *abstractly* in terms of curved (or flat) spaces of three and more dimensions. It blows their minds to now be thinking that some of these ideas may be needed to describe the real world. Their experience, with all the data they've obtained, leaves them no choice. The conception of a static (3+1)-dimensional Euclidean space has been convincingly shattered. But it's not yet obvious which of their hypotheses, which we'll call Hypothesis 1 (H1) and Hypothesis 2 (H2), would become the appropriate replacement.

Remember, the RC's have never before had even the slightest reason to doubt the truthfulness of their motion sensing devices: accelerometers have *always* reliably told them how fast they were accelerating; and clocks could *always* be trusted to determine a velocity with respect to their rotation axis and provide a record of distance traveled (according to the elapsed time-odometer connection). So the vast majority of RC's naturally lean toward that hypothesis which honors the heretofore utter reliability of these instruments.

8.7 Accelerometers and Clocks: Betrayal?

The main reason most RC's dislike H1 is because it would involve the conclusion that their motion sensing instruments are not necessarily reliable. This would be the case if there were some kind of strange mechanism emanating from this big chunk of matter, that latched onto every nook and cranny of every other chunk of matter, producing a kind of "pulling at itself." If this were really happening and the "strength of the interaction" happened to be extremely "fine tuned," (that is, exactly the same for every kind of matter) then an accelerometer that was being so pulled, would yield a zero reading, even though it was accelerating.

The redeeming quality of this view is that it maintains the impression that the RC's had had for thousands of years, that material things did not show any obvious sign of "moving by themselves." On the scale of familiar objects in their experience, matter was seen as "inert." Either rotation or some kind of propulsion was needed to make a body move. Some kind of propulsion was needed to get a body to start rotating in the first place. And space was pretty much just a vast passive emptiness within which matter could be found to move or not. H1 involved only a relatively subtle modification of this ancient world view.

In their prior experience, however, forces always pointed in the same direction as positive accelerometer readings. Most RC's found it troubling that the mutual pulling idea would mean that this would no longer generally be true. On a large massive body, in fact, an accelerometer reading would appear to mean *the opposite* of what it says. If the massive body is really an unmoving (static) thing, in spite of the accelerometer readings, then a *positive* reading would indicate a state of *rest*. That is not only bizarre, it cuts against every deepest instinct the RC's have concerning motion.

A similar conclusion follows with regard to clock rates. Clocks attached to the Sphere have rates that would "ordinarily" indicate that they possess a substantial velocity. So now it has been proposed that these clocks may actually be in a state of "rest"? Most RC's didn't like it, but they could not yet prove otherwise. In trying to put as positive a spin on this hypothesis as they could, its proponents cast it as a "geometrical" account of space, time and matter in which this mutual pulling of static chunks of matter was a matter of course.

H1 proponents even found it fitting to "bend" the common meaning of words in their effort to explicate their geometrical model to others and themselves. For example, they invented the expression, "acceleration of a particle at rest," which refers to an object attached to the Sphere. Asking, well, is the particle accelerating or resting, one comes back to the fact that, according to H1, the Sphere is an *utterly static* thing. Therefore, the "deep down" answer is evidently that H1 proponents really think of it as being *at rest*.

They use the word "acceleration" to refer to a state of rest because of the positive accelerometer readings and because this is the state of motion with respect to *geodesics*, i.e., trajectories of "falling" objects that are thought of as "really" accelerating, but which have some properties of not accelerating. As we might expect, H1 proponents have an uphill battle trying to convince H2 proponents – who are simply inclined to a straightforward interpretation of their motion sensing instruments and aren't impressed by counter-intuitive geometrical word games.

At this stage it must be admitted, however, that the facts do not yet warrant a decision. It just might suffice to posit that space and time in the vicinity of a sufficiently massive body are perceptibly neither Euclidean nor Minkeinsteinian, but pseudo-Riemannian.

What could possibly make it so?

8.8 Space, Matter and Time in Light of a Faithful Interpretation of Accelerometer Readings and Clock Rates

8.8.1. Stationary Motion. Even as it allows them to retain the sanctity of their motion sensing devices, H2 is in some ways even more radical. Before getting to the radicalness, however, we should emphasize that the new matter-induced motions bear a profound similarity to the RC's experience with their Rotating Cylinder back home. In the Cylinder an array of accelerometers and clocks manifests readings and rates indicating a pattern of *stationary motion*. The bracing and the cylinder are practically rigid, yet they reveal a range of accelerations and velocities that do not change over time and that leave the structure intact.

These latter characteristics are exactly what they have found to exist with respect to the huge massive sphere. The RC's clearly perceive the analogy and seize it as a basis for building and comparing hypotheses. A key question, of course, is how far does the analogy extend? At what point would new ideas be needed to maintain the validity of the analogy and what exactly would those ideas be? The RC's organize their thoughts as follows.

In the Cylinder, the stationary motion pattern has a "concave-planar" character. The magnitude of the acceleration increases with distance from the axis, it is directed *inwardly* and because there is only one linear axis, a circularly symmetric cross-section can be found only on a plane that is perpendicular to this axis. Rotation, of course, means that the stationary velocity with respect to the axis, is similarly perpendicular; all velocities are on *tangents* of the circular cross-sections. Note that the symmetry of the circle thus also has a handedness. With respect to the Cylinder's circumference, clockwise motion differs from counterclockwise motion in its effects on rulers, clocks and the speed of light. The Rotating Cylinder thus manifests the combination of stationary *inward* acceleration and stationary *tangential* velocity.

The massive Spherical world also exhibits a pattern of accelerometer readings and clock rates that persists without change. This evokes the possibility that it too should be regarded as a manifestation of stationary motion. In this case the pattern has a "convex-spherical" character. The acceleration is directed outwardly; beyond the surface the acceleration decreases with distance; and a circularly symmetric cross-section can be found on planes cutting the center along *any* axis. Since the acceleration is directed *outwardly*, and given the above symmetry properties, it follows that the velocity is also directed outwardly. Of course, this is also dramatically consistent with the RC's recent experience. Thus, H2 involves the concepts of stationary *outward* acceleration and stationary *outward* velocity, by analogy with the cylindrical, rotation-induced stationary motions.

8.8.2. Unaccelerated Test Objects. One of the most obvious differences between these two circumstances has to do with the behavior of small bodies that are not firmly attached to the Cylinder or the Sphere. (For the sake of argument we presently allow, of course, that in spite of its large size, the mass of the hollow Cylinder is so small that its gravitational effects have gone unnoticed by the RC's.) In the case of the Cylinder, both inside and outside, it is possible to place objects at fixed locations with respect to the axis and their spatial relationships will not ("noticeably") change.

This is most unlike what they find even at large distances from the surface of the massive Sphere. The motional effects of the hollow Rotating Cylinder have a negligible effect on the spatial relationships in the surrounding space. Whereas the motional effect of the solid massive Sphere very noticeably alters the spatial relationships of objects similarly placed ("falling") in the surrounding space.

8.8.3. Dimensionality: Motion Through or Motion Of Space. This difference in the behavior of unaccelerated test objects is clearly related to the characterization of the rotational motion as "concave-planar" and the matter-induced motion as "convex-spherical." *Rotational motion has a lower dimensionality*. It is confined to the thing that rotates and has no noticeable effect on the surrounding volume of space. It can be conceived as motion *through* this three-dimensional volume of space [(3+1)-dimensional spacetime].

Whereas matter-induced motion evidently requires one more space dimension because the pattern is not of motion *through* space but appears to be more accurately characterized as motion *of* space. Radical may be too mild a word for the implied shift in perspective. But it makes sense because of (among other things) the inverse-square acceleration pattern.

If the accelerometers are telling the truth about their states of motion, it would *not* make sense to think of it as happening in (3+1)-dimensional space because the coherence of the sphere and its poles would not be possible if all the accelerations were naively thought of as so many different rates of "material expansion" in 3D space. But if matter is the *driving agent* of the (4+1)-dimensional *motion of space*, the inverse-square acceleration is exactly the pattern one should expect. Matter would then be seen as the *source* of space; matter is perpetually *generating space*.

The RC's regard this not only as a reasonable interpretation of their experience, but as being directly related to the physical dimensions of the new constant appearing in Equation 21. In terms of Length, Time and Matter, the dimensions of the constant, G are L^3/T^2M. In words, G may thus be expressed as "acceleration of volume per mass." Multiplying by M/r^2 gives the linear acceleration corresponding to how the generation of space spreads out over distance. At any two given distances from a massive sphere, in a given small increment of time, shells of *equal volume* are accelerated outwardly. The radial thickness of both shells is given by the inverse square law.

So the idea is that matter propels itself and its surrounding space ever outward. Fortunately, there are several ways to continue building on the rotation analogy and ways of comparing the (3+1) vs (4+1)-dimensional schemes so as to facilitate visualization and improve palatability of this, in some ways disturbingly new idea. Ultimately, the ideas need to be tested with experiments, of course. But this is all so new to the RC's that they are still mentally digesting the hypotheses – which process will naturally lead to the most effective experimental ideas in due course.

8.8.4. Embedding Space. An essential concept for understanding the relationship between a given dimensionality and the next higher or lower dimensionality is that of *embedding*. The RC's recognize that H2's appeal to a higher a dimensional space is analogous to a more familiar circumstance involving lower dimensional spaces.

Consider the surface of a sphere. Imagine that the surface is inhabited by beings having no extension perpendicular to the surface. They extend only along the east-west and north-south directions in the surface. Virtually of necessity, the description they would devise of their world would be two-dimensional; it would involve only two coordinates (e.g., longitude and latitude). This is known as the *intrinsic* description, which is *sufficient* because it's all that's needed to specify any location on the surface. It is also sufficient to reveal the essential properties of the surface, that it closes back on itself and that the sum of angles of intersecting straight lines (great circles) depends on the distances between vertices, etc. The two-dimensional geometry of the sphere is thereby shown to be quite different from the two-dimensional geometry of a plane or a torus, for example.

Beings who extend in *three* space dimensions should well appreciate the sufficiency of the intrinsic description. But an obvious alternative for them is to include the third dimension, to add a coordinate which facilitates understanding that the spherical surface is but a *cross-section* of their much larger world view. The 3D beings reside beyond, above and below the surface, so their three-dimensional description is called *extrinsic*. The 2D sphere (and its flat inhabitants) is *embedded* in the higher space of three dimensions. The third dimension is the embedding space and it is, for mapping purposes, superfluous. For other purposes, however, the third dimension is essential.

Since the extrinsic description is an obvious possibility for 3D beings, it is at least conceivable that, were the 2D beings sufficiently imaginative, they too could conceive a higher dimensional embedding space. It is even conceivable that they could devise ways of testing whether their world was *really* physically so embedded. For example, suppose the sphere were subject to various deformations due to rain, spherequakes, meteor impacts, heat, etc. Nothing on the surface itself helps to explain these things. Perhaps the inhabitants note the regularity of some of these phenomena and have figured out how to predict the time of their recurrence. If they insist that the whole world consists of their 2D surface, these phenomena could only be ascribed to magical gods or "forces of nature." Whereas, if they allowed that the world extended into another dimension, the influences impinging on their surface could be seen, or at least imagined, as originating in the higher dimension and ultimately having an explanation that did not require magic.

The point of discussing the relationship between the second and third dimensions, of course, is to establish an analogy for the relationship between the third and the fourth. Though only two dimensions are needed to *map* a sphere, for the sphere to *exist in physical reality* it requires the embedding space of the next higher, third dimension. If our analogy does in fact extend to the relationship between the third and fourth dimensions of space, it implies that, *for a seemingly three-dimensional spherical mass to exist in physical reality, it requires the embedding space of the next higher, fourth dimension*.

Before exploring this idea further, note that a similar train of thought was entertained by R. Swinburne in 1968. In his chapter on "The Dimensions of Space," after setting up the analogy much as we have, with two-dimensional beings embedded in the surface of a sphere, Swinburne presents various possible consequences and implications. His arguments led him to conclude that three space dimensions are all we get, that the idea of a fourth physical space dimension is not logical. In the chapter's last paragraph, however, Swinburne perspicaciously admits:

> But, it may be objected, how do we know that we are not in the same situation as the inhabitants of the purported two-dimensional world described earlier? It seemed to them that only two lines could be mutually perpendicular. They made this mistake because it was not physically possible for the 'objects' with which they were familiar to move outside their surface. Might not we be making a similar mistake in supposing that our space is three-dimensional because it is not physically possible for the objects with which we are familiar to move outside the three-dimensional hyperplane? It must be admitted that we *might* be making just this mistake. [61]

That Swinburne did not further explore this objection is not surprising, as his whole chapter rests on the tacit assumption that an understanding of the motion of objects on and around a large massive body should scarcely be affected by speculations about extra dimensions. Having Newton's and Einstein's theories of gravity already deeply ingrained in his mind, he was not alert to the possibility that his discussion – especially his final paragraph – points to a possible profound connection. Perhaps this connection could lead to an understanding of *why* Newton's and Einstein's theories appear to work so well. Could it be that "moving outside the three-dimensional hyperplane" is the same thing as the outward generation of space by matter?

We'll come back to this question. First, let's widen our perspective to see how the RC's might come to it without hesitation, because their minds are not ingrained with any gravity theories at all.

8.8.5. Lore of Hyperdimensionality. The RC's mathematicians have a long history of hyperdimensional and non-Euclidean research. To some extent the mathematician's results have been adopted by theoretical physicists and have spilled over into popular culture. Since the RC's are now faced with a real world need to discover which part (if any) of this research rings true, they find it worthwhile to freshly contemplate some of the key ideas.

By definition, increasing the number of space dimensions means introducing a new perpendicular direction along which motion can occur. Though aware of this definition for many years, it had not previously occurred to the RC's that one of the underlying concepts in the definition was unwittingly being slighted: motion. The extra dimension had been conceived, rather, more as an abstract device for developing theories or accounting systems for phenomena or data that were so complex that three dimensions, or coordinates, were not sufficient. So the perpendicular "motion" was generally not in physical space, but "parameter space."

On occasions when the RC's *physicists* guessed they might benefit from having more than three space dimensions to describe physical reality (which had especially lately become common) they were inclined to invoke a lot more than just one. But the purpose was always to do with simply (or complicatedly) widening the stage, so to speak, to provide more coordinates with which to describe physical phenomena. This did *not* change the primal view, mentioned above, that gross matter and space were essentially static things. So even the physicists' invocation of a fourth space dimension slighted the concept of motion – at least any motion that has a perceptible connection to the "first three" space dimensions.

The appearance of the fourth spatial dimension in RC pop culture typically shared this lack of connection to physical motion. A 4D creature supposedly would be capable of materializing and dematerializing at will; and could do magical things like remove the contents of locked boxes. But matter-induced motion played no part in such ideas. The fourth dimension also enjoyed a phase amongst certain "spiritual" RC's and continues as a recurring theme in their science fiction genre.

A key aspect of the use of the concept by some of the RC's academic physicists is the idea that a given space dimension may have a characteristic *size*, like a meter, a millimeter, the wavelength of a subatomic particle, or some unimaginably tiny length scale. In this context, the unobservability of the extra dimension would be explained as being due to its being curled up, "compactified," into these extremely small sizes.

Most of these ideas stand in sharp contrast to the present one (H2), whereby the fourth space dimension doesn't have a "size," is not compactified or invisible, and it doesn't merely add an extra static coordinate, because it represents the motional extension of every "lower" dimension. Based on their recent experience, the RC's now conceive this "motional extension" as the *perpetual generation of space by matter*.

A common graphic image presented in discussions about the fourth space dimension is very suggestive of this space generation idea. It's the so-called "hypercube" shown at the upper right of Figure 15. Also depicted in Figure 15 is the "hyperspherical" counterpart. Considering one alleged proof of the three-dimensionality of space, the historian of science, Max Jammer, described a progression much like the first three steps of Figure 15. In conclusion, he wrote that this kind of

> ...deduction comes to an end apparently because *the conception of a motion of a three-dimensional space as a whole lacks all intuition based on experience.* [62] (My emphasis.)

The implication that *motion* should play a fundamental role in the hierarchy of dimensions was hit on by the inventor-philosopher, Arthur Young, who said: "In terms of dimen-

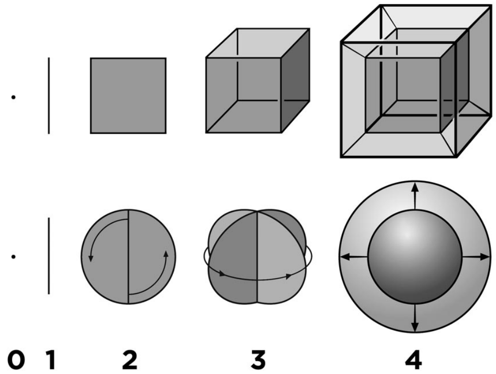

Fig. 15: Hierarchy of dimensions: Linear, rotational and omnidirectional projection.

sions, the line is extension and the birth of time." [63] Considering the profound and pronounced (motional) effects a large body of matter has on accelerometers and clocks, considering this image of the hierarchy of dimensions and the comments by philosophers, mathematicians and historians about what hyperdimensionality entails, isn't it ironic that the interconnections are not perceived as an *obvious* possibility? Just imagine a lot of little accelerometers attached to Figure 15's "hyper-objects." If the extension of a (zero-dimensional) point to create a *line* is the "birth of time," how much more may time be "birthed" by the (hyper) extension of a *massive* three-dimensional *volume*?

With the kind of intuition the RC's had about motion after thousands of years of evolution in their Rotating Cylinder, topped by their recent experience and present situation, it's easy to imagine that they would proceed, at least tentatively, beyond the third dimension. As explorers, they are rightly inspired even to *name* their hypothetical new territory. Their map now includes: height, width, depth and *gravity*.

The cubical culmination of Figure 15 had been recognized by the RC's geometers as the simplest "head on" view or "central projection" of a four-dimensional hypercube. [64] But before encountering the large massive Sphere, it had not occurred to them that the "projection" aspect might be due to a *process* that was constantly *happening in time*, or that its physical counterpart necessarily involved matter. Having now discovered that matter has a motional effect on space, that space appears to be constantly, outwardly projected by matter, in H2 the RC's conceive that space is not just a preexisting background. Rather, space would not exist, *there would be no dimensions at all were they not all being perpetually generated by matter.*

8.8.5. Curvature and Hyperdimensional Motion. The difference between H1 and H2 is crucially exemplified by the significance of curvature in the respective hypotheses. In H1 curvature is a given property of spacetime, rather like the mysteriously impinging phenomena in the case of the 2D spherical surface. In other words, the curvature is regarded as an intrinsic feature of $(3 + 1)$-dimensional spacetime, which has no extrinsic reality because an extrinsic reality would require a higher dimensional embedding space. H1 proponents explicitly deny that an embedding space is necessary or helpful. This point of view makes sense if (big if) matter and space are static things. Thus, in H1, aside from that it is somehow related to mass, *nobody knows what causes the curvature*. The RC's draw a blank to the question of what mass must be *doing* to make spacetime curve. The motional implications of the readings of accelerometers attached to large massive bodies go ignored and unrecognized like a tyrannosaurus on the couch. (H2 proponents don't understand how the H1 proponents can be so oblivious!) Yet the existence of static curvature and the way it varies from place to place are presumed to be known in fine detail, based on the equations of the hypothesis.

Now recall that the RC's had made observations in their Cylinder of the contraction of lengths due to motion. Length contraction always occurs in the same magnitude as the slowing of clocks. Although the RC's had noted that this combination of effects could be described in terms of non-Euclidean geometry, it was not suspected that this might have any particularly far-reaching implications. For all practical purposes, the length contraction and clock slowing were accounted for in the RC's kinematical scheme. Non-Euclidean geometry could well be used to describe the metric relationships on a rotating disk, as the lengths of rods placed on its surface (and the rates of clocks) varied with location. But since this had no noticeable effect on the behavior of objects not attached to this surface, for the most part, it was of no more than academic interest.

In their present circumstances, the RC's focus on the most important consequence of this prior research: it's that the curvature was entirely *caused by motion*. So one of the most questionable things about H1 is that not only does it lack an explanation for how curvature is caused, it then regards *static curvature as a cause of motion*. By this view the RC's experience in the Cylinder and now on the Sphere would most properly be regarded as *anti*-analogs, because the cause-effect relationship is reversed. This idea (H1) thus strikes most RC's as being utterly backwards. They see the "proper" analogy as being to ascribe to the same effects, as far as possible, the same causes. Rotational motion causes spacetime curvature. Uniform rotation is widely regarded as being an instance of absolute stationary motion. Therefore, *evidence of spacetime curvature implies the existence of absolute stationary motion.* (Possibly, it does not *require* the existence of absolute stationary motion; but it's not a bad guess that it does.) Concerning the evidence the RC's have found on and around the Sphere, the significance of this connection is far more than academic because the length and time differences, rather than being confined to a 2D surface, would pervade all of volumetric space. This difference again suggests the existence of a higher space dimension, as it underscores the difference between motion *through* space and the movement *of* space (concave-planar symmetry vs convex spherical symmetry).

In and around the Cylinder it made sense to regard motion as being through space. But now, if the motion attributed to the mass of the Sphere was conceived as being through space, it would require the (H1-motivated) backwards interpretation of the relationship between motion and curvature, which corresponds to the equally backwards interpretation of accelerometer readings and the rates of clocks. Another graphic construction helps to convey these features and may then be adapted to convey the possibly more "forwards" interpretation embodied by H2.

8.8.6. Odometer-Speedometer for Matter. The nested tubes of Figure 11 represent Epsteinian space-propertime diagrams that are rolled up so that time goes up the axial direction and space goes angularly around the tube. This is well-suited to represent the range of speeds corresponding to different distances from the home Cylinder's rotation axis. To represent the effect of *matter* on space and time, Epstein has rolled his diagram in the perpendicular direction, so that time goes around the tube and space is represented by the tube's axis. [65] We will first describe how Epstein's tube corresponds to H1, and then we'll describe an adaptation that is better suited to represent H2.

Before considering how a tube's shape may be dictated by matter, let's first apply this rolled up space-propertime diagram to *zero* mass inertial frames. In this case a rest frame corresponds to zero motion through space and so to just looping around the tube in endless circles along the time coordinate (or perhaps "infinitesimally thin" rolled up layers). Uniform motion with respect to the rest frame then corresponds to looping around in a uniformly pitched helix. The maximum speed of a light ray corresponds to motion parallel to the axis with no turning (zero passage of proper time). The diameter of the tube is arbitrary; it may be chosen, for example, so that one turn corresponds to one second of time, or to the distance light travels in one second of time.

Now let's consider the effect of matter. We imagine that the center of a very long tube corresponds to the center of a massive sphere. In this case, whatever diameter we may have chosen for pure inertial frames far from matter, it will change as distance to the sphere decreases so as to reflect the change in clock rate. Specifically, the diameter of the tube then corresponds inversely to the rate of a clock attached to the Sphere. This causes the tube to flare open, getting wider as distance to the sphere decreases. The *angle* of this flaring with respect to the axis then corresponds to the degree of *space curvature* (length contraction). The tube itself is static. The motion of objects or observers attached or unattached to the Sphere is

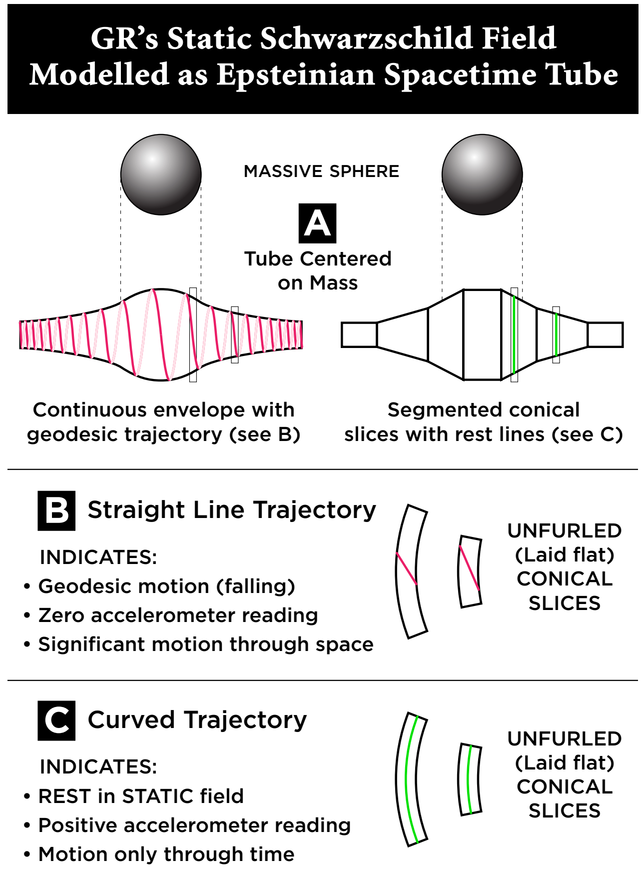

Fig. 16: Key characteristics of Epstein's tube model for rest and motion in a gravitational field. *Rest* in the field (through time only) is indicated by lack of motion along tube's axis; i.e., straight up and around the tube (green lines). *Motion* in the field (through space) is indicated by deviation from straight up the time axis (red line). Rest corresponds to *positive* accelerometer readings; motion (falling) corresponds to *zero* accelerometer readings. Our motion-sensing devices seem to be telling us that this scheme is backwards.

represented by lines traced on the tube. Objects that are firmly attached to the sphere move only through time, so their paths ("speedometer needle") are straight up and around the tube. Since the tube flares, these lines are not geodesics, which means that if the approximately conical "slice" of tube containing the time line were split and laid out flat, the line would curve. This indicates an acceleration such as would be measured by an accelerometer. On the other hand, the lines corresponding to objects that are unattached to the Sphere trace out a helix along the tube. They are geodesics, which means the corresponding cone slices, when laid out flat, would reveal the lines to be straight. And this corresponds to a zero accelerometer reading. (See Figure 16.)

The slowness of clocks at rest near the surface corresponds to the smaller angular travel of the "speedometer needle" around the tube; a needle of the same length far away from the Sphere travels a further angular distance around the tube in the same increment of coordinate time. The staticness of the tube is explicitly represented by the "straightupness" of the time lines of objects attached to it. Since the tube is static, even though lines traced on it have the correct character corresponding to geodesic vs non-geodesic or zero vs positive accelerometer reading, the question remains as to how the lines come to be drawn at all. The curvature correctly accommodates the different kinds of linear motion through spacetime; but it sheds no light on how either the curvature or the motion come to be in the first place.

The inadequacy of H1 to answer such questions is revealed by Epstein himself, who, in the course of presenting his model, at one point asserts that, "gravity slows the speed of time." Then, a few paragraphs later, he asserts that the difference in "the speed of time...is sufficient to cause gravity itself!" [66] Circular reasoning leaves you where you started.

The shape of Epstein's tube reflects the variable quantity known in H1 as the *metric coefficient*, $(1 - 2GM/rc^2)$. H2 also involves a similar quantity, which will be referred to as the *curvature* coefficient. Since this is a second order quantity (with a "velocity squared" in the argument) whose magnitude typically deviates from unity by a very small amount, Epsteinian tubes representing typical stars and planets have only a very slight flare. The extreme is reached, not at the surface but at the center. If the sphere is uniformly dense and its surface radius is called R, then the *temporal* metric coefficient corresponding to the center becomes $(1 - 3GM/Rc^2)$. (Where the corresponding *spatial* coefficient $= 1$.)

Now let's consider the H2-motivated Space Generation tube model. Its key features are: 1) The outer envelope represents not the (second order) curvature, but the first order *speed*, i.e., the *stationary outward velocity*. Its magnitude is (approximately) given by $\sqrt{2GM/r}$. This is the speed that the pole appeared to have acquired with respect to the RC's spaceships and the speed that would then cause clocks to slow and rods to contract. 2) The tube turns. Every body of matter is evidently a source of motion and a source of space. This motion may be represented so that the graph of every body turns at the same angular rate. And 3) The *interior* of the tube is a coordinate space where additional motion data are plotted. (Unlike the Epsteinian tube, whose interior contains no information.) The whole volume within the tube moves and the motion causes curvature whose magnitude is plotted in the interior space. (See Figure 17.) Both of these features are reflections of the fourth dimension of space, whereby the (moving) graph now represents motion *of* space rather than motion *through* a non-moving graphical surface. It's not just lines on the surface, nor even just the surface itself, but rather the entire tubular volume that is in a state of, and therefore cogently represents, stationary motion – motion into (or outfrom) the fourth dimension of space.

Every cross section of the tube contains the curvature data, in the form of space-propertime diagrams, where the speed is proportional to turning radius. As we recall, this correlates to the angle of the speedometer needle. The outer envelope represents the stationary outward velocity of objects attached to the Sphere and so has a maximum deviation from straight up the time coordinate. Whereas the tube's axis represents the trajectories of objects fallen from infinity whose clock rates are a maximum and so have no deviation from straight up the time axis. Proponents of H2 suspect that their experience of watching the entire length of pole accelerate and speed past them means their own ("falling") trajectory corresponds to this center-of-the-tube spacetime path. Being the extreme case which represents the trajectory of a clock having a maximum rate, the RC's call it a *maximal geodesic*. What may appear as accelerated motion of an object falling radially from infinity (maximal geodesic) is therefore, according to this scheme, more accurately regarded as the closest thing known to a state of *rest*. A maximal geodesic is the purest "tracer" of the *motion of space* past it, which is most rapid near the generating mass's surface (assuming uniform or nearly uniform density beneath the surface).

This latter parenthetical remark draws attention to one of the most obvious differences between these models: The H1 tube has a central bulge; the H2 tube converges to a point. This then also reveals where the models diverge most dramatically with regard to empirically testable predictions. It didn't take long for the RC's to realize that *conclusive proof* revealing which hypothesis was closer to the truth would have been at hand if, instead of landing on the Sphere's surface, the spaceships "fell" into a hole that spans a diameter. According to H1, the mutual pulling of matter ensures that the speed of objects pulled into the center would increase until the center was reached. Therefore, in this case the spaceships would have experienced a high speed collision. (If only one ship were swallowed by the hole, it would have gone up and out the other side to its original radial distance.) The central bulge of this model thus corresponds to the minimum clock rate. The decrease in clock rate toward the center is supposed to *cause* objects to reach their maximum speed there.

According to H2, on the other hand, matter does not pull objects inward through a pre-existing space; it moves new space outward. Recall the RC's bewilderment when their neutrino beam radio communications indicated that the distance between Ships 1 and 2 was decreasing, when they knew they had never accelerated toward each other. We now see how the RC's conceive this: As stated, they never did accelerate toward each other. Rather the space between them was continually moved past them. Near the surface the speed of space is a maximum; to keep up with it requires maintaining the constant acceleration revealed by accelerometer readings on the surface. But at the center the speed and acceleration go to zero. So the rockets would appear to asymptotically approach each other as the amount of space between them similarly diminished. The convergence to a point thus also corresponds to a *maximum* clock rate – the same rate as clocks

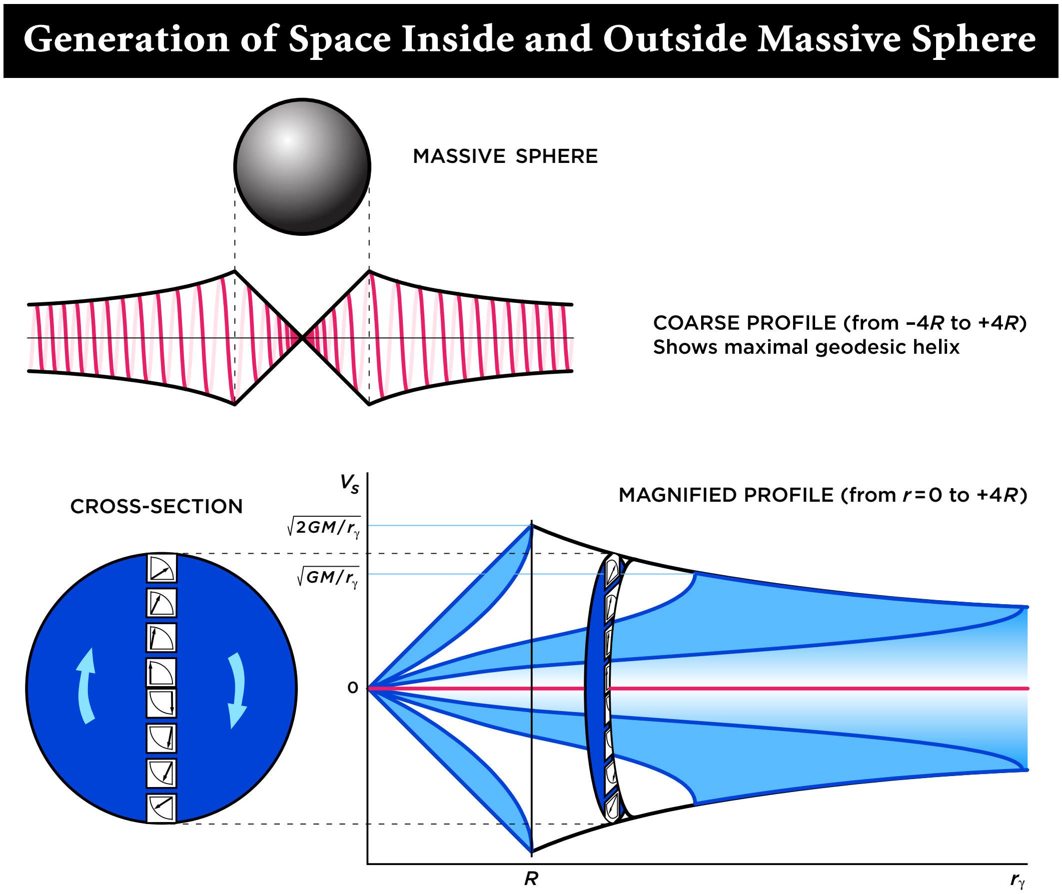

Fig. 17: COARSE PROFILE and MAGNIFIED PROFILE: Movement of space is represented by rotation of whole volumetric tube. Helical trajectory is a projection ("tracer") of unaccelerated object released at infinity (maximal geodesic). The outer envelope is undergoing stationary outward motion. Maximal geodesics, being always unaccelerated, have maximum clock rates and so are represented by the axial line of zero velocity. Trajectories branching off from outer envelope represent objects released from $r = R$, $r = 2R$ and $r = 4R$. CROSS-SECTION: Space-propertime diagrams ("speedometers") at every point within the tube indicate the degree of clock slowing and length contraction applicable to objects released from various distances. In every case both metric effects are a maximum prior to release. At the extremes are the stationary outward velocity (V_s) of outer envelope and the standard of rest (maximal geodesic) of the central axis.

at or falling from infinity. At the center all causes of motion are cancelled by symmetry, so a maximum clock rate is intuitively quite reasonable. (Contrast this with the central bulge in the mutual pulling hypothesis, where nobody has any idea *what could cause* a clock at the center to run slow.)

8.8.7. Horizons, Singularities or Well-Behaved Limits? As the RC's often found to be true, considering an extreme case proved most illuminating: What happens when the stationary outward velocity (by H2) becomes so large as to approach or exceed the speed of light? Or (by H1) when the curvature becomes so great as to freeze light and prevent its outward motion? First, H1: Consider again the metric or curvature coefficient $(1 - 2GM/rc^2)$. As noted earlier, the RC's surmise that G is a Universal constant and that its magnitude is rather small. (That's why they failed to notice its effect back home in their Cylinder.) The question then amounts to what happens when the ratio $2M/r \to c^2/G$? Or could $2M/r$ even equal or *exceed* c^2/G so that the coefficient becomes zero or negative? These questions motivated the RC's H1-sympathetic theorists to invent different ways of expressing the problem so that the answers were not so objectionable when $2M/r = c^2/G$ (where they find a *horizon*). They failed, however, to mitigate or eliminate the problem for the case that $r = 0$ (where they find a *singularity*). An industry could perhaps be built on the perplexities surrounding these consequences – if H1 were found to prevail.

But what of H2? Rather than jump immediately to this extreme, the RC's perceive the benefit of starting from the opposite extreme. They begin by considering a very small body of ordinary density – perhaps a grain of sand or a speck of dust. Having discovered quite by accident how the acceleration and velocity produced by a massive body varies with distance outside the body's surface, the RC's reason that with each additional shell of matter of the same density, these accelerations and speeds will increase in like proportion. For example, if the radius were doubled by a layer of matter of the same initial density (which would thus multiply the mass by eight) the speed and acceleration at the surface would also both be doubled.

The first benefit of this approach, as we'll see, is that it lends credence to the RC's prediction concerning falling into the center of a massive body. The inverse square law entails the consequence that spherical shells of matter should produce motional effects at or beyond their outer surfaces, but not *within* their inner surfaces where the effects are cancelled by symmetry. Therefore, the only thing that can affect the speed at a given distance within the sphere as a whole is the matter within that given distance. By adding successive layers of matter, one arrives at a new and larger maximum velocity at the surface of each new layer. But this cannot increase the velocity *within* that layer. Which means both acceleration and velocity vary directly as the distance.

Although this is certainly a reasonable (and testable) first approximation for small bodies of matter, the RC's see that the process of adding shells of matter indefinitely superficially leads to problems similar to those of H1. Consider that by H1, if all the mass of a body is contained within the distance r and $r = 2GM/c^2$, then the spherical suface at r constitutes a "light front," a surface at which light is frozen. Perhaps if one accepts the mutual pulling idea (or if it proves to be true by experiment, of course) extremes of curvature where this could happen may not seem so objectionable. But by the Space Generation idea, if the body's mass were increased so that the distance $2GM/c^2$ corresponded to its surface, that would mean the matter there was making space move at the speed of light. Also, clocks at the surface would stop ticking. The RC's surmise that both of these things are impossible. It would be quite like trying to accelerate a body of matter to the limiting speed c with some other means of propulsion. Happily, the RC's have already dealt with this circumstance (Equation 20, repeated here for convenience) in the context of uniform *linear* acceleration (*through* space):

$$v = \frac{at}{\sqrt{1 + a^2t^2/c^2}}\,. \tag{22}$$

The RC's surmise that a similar limiting relationship should exist for *volumetric* acceleration (*of* space). By analogy, the expression for the velocity caused by this acceleration would be

$$V_s = \frac{\sqrt{2GM/r}}{\sqrt{1 + 2GM/rc^2}}\,, \tag{23}$$

where V_s is the stationary outward velocity and r is the usual coordinate distance; i.e., the distance as gauged by an observer at infinity. This equation implies the need for a *new distance coordinate*. Squaring Equation 23 gives

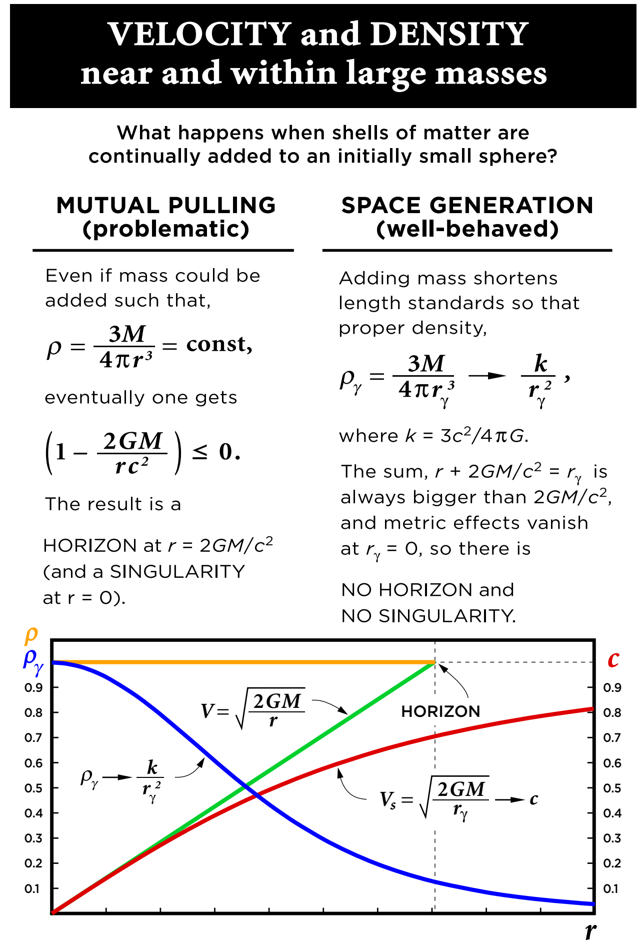

Fig. 18: *Mutual pulling (static field) approach*: As mass is added, when $2GM/c^2$ comes to equal the coordinate radius r, light is frozen and clocks stop (horizon). Since clocks are supposed to get even slower toward the center, there is an unavoidable singularity at $r = 0$. *Space generation approach*: As mass is added, it's not the coordinate distance, r, by itself that matters. Rather, it's the larger distance $r_\gamma = r(1 + 2GM/rc^2)$. No matter how much mass is added, r_γ increases so that $2GM/r_\gamma$ only *approaches* c^2. The variation in proper density ρ_γ thus approaches a $1/r_\gamma^2$ law. There is no horizon and, since all metric effects go to zero at the center (as the mass within r_γ goes to zero) there is no singularity.

$$V_s^2 = \frac{2GM}{r + 2GM/c^2}\,, \tag{24}$$

whose denominator is the new distance coordinate. The RC's call it

$$r_\gamma = r + 2GM/c^2\,. \tag{25}$$

This is the whole distance of relevance for gravitational phenomena. The curvature coefficient may then be expressed as

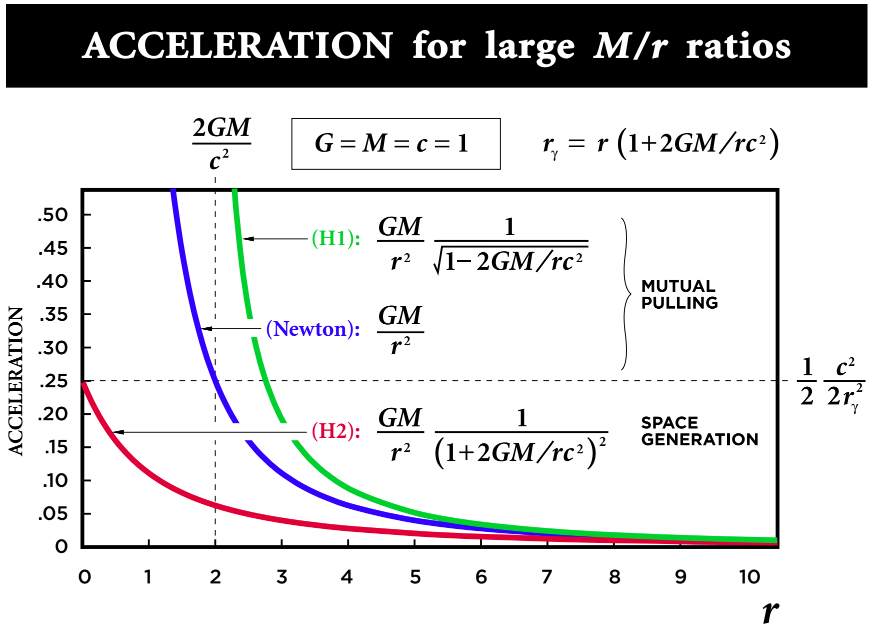

Fig. 19: Mass-induced acceleration compared for three models. The magnitude may go to infinity for Mutual Pulling models. The magnitude is prevented from reaching infinity in the Space Generation model.

$$\left\{1-\frac{2GM}{r_\gamma c^2}\right\}=\left\{1+\frac{2GM}{rc^2}\right\}^{-1}, \tag{26}$$

which is greater than zero for any finite M, r or r_γ. Inside the massive body M goes to zero as r goes to zero, so there is no singularity. Outside the body mass shells may actually be added indefinitely, but the proper density cannot remain constant. In the limit this density would vary as $1/r_\gamma^2$, as $2GM/c^2$ grows larger than r and as

$$V_s=\sqrt{\frac{2GM}{r_\gamma}}\longrightarrow c\,. \tag{27}$$

(See Figure 18.)

We should now compare how the acceleration itself differs from one hypothesis to another. For this purpose, instead of having the mass vary by adding successive shells of matter, let's keep the mass constant. In all cases, when the spacetime is close to flat, the acceleration reduces to the simple (Newtonian) approximation GM/r^2. As one would expect, the "horizon" appearing in H1 for the velocity calculation appears also in the acceleration:

$$g_{H1}=\frac{GM}{r^2}\frac{1}{\sqrt{1-2GM/rc^2}}\,. \tag{28}$$

When all the mass is within r and $r=2GM/c^2$ this "acceleration of a particle at rest" thus becomes infinite (which corresponds to the idea that $r=2GM/c^2$ represents a "light front.")

By contrast, replacing r with r_γ, as per H2 gives

$$g_{H2}=g_\gamma=\frac{GM}{r_\gamma^2}=\frac{GM}{r^2}\frac{1}{(1+2GM/rc^2)^2}\,. \tag{29}$$

The graphs for H1, H2 and the Newtonian approximation are shown in Figure 19. A couple noteworthy features should be pointed out. First, bear in mind the common Newtonian relationship between escape velocity, $V^2=2GM/r$ and g:

$$g=\frac{GM}{r^2}=\frac{1}{2}\frac{V^2}{r}. \tag{30}$$

Unlike H1, there is no horizon here. With decreasing r the acceleration increases indefinitely and becomes infinite when $r=0$. In H2, on the other hand, r_γ cannot become zero as long as there is mass. So as r_γ *approaches* zero, we get

$$g_\gamma=\frac{GM}{r_\gamma^2}=\frac{1}{2}\frac{V_s^2}{r_\gamma}\rightarrow\frac{1}{2}\frac{c^2}{r_{\gamma(\to 0)}}. \tag{31}$$

This limit is shown in the graph (Figure 19).

8.8.8. Indelible First Impression. Beyond these theoretical ideas, in the forefront of their deliberations to assess the merits of H1 and H2 is the RC's unforgettable, thrilling experience of "falling" alongside the poles. During the whole trip (minus local maneuvers and final landing) the ships' accelerometers read zero. Whereas accelerometers on the poles were positive. The RC's thus, in effect, motionlessly watched as the spherical "spot" grew ever larger. It seemed obvious the whole time that the Sphere was moving toward them, and not the other way around. Then, finally, the experience of navigating "soft landings": The RC's needed to accelerate *away* from the Sphere's center. Remember, "gravitational attraction" is not part of their training. Rather, it is in their blood and their bones to believe their accelerometers perhaps even more than it is in an Earthian astrophysicist's blood to believe Newton's equations. Their motion sensing instruments are telling the RC's that *space itself moves*. In the process of generating it, *matter moves space*. While "falling," the space between their ships and the Sphere was being constantly moved outwardly past them. That's their best guess and they'll stick to it unless it leads to an untenable conclusion.

8.9 Further Considerations and Testability

From all of the above we see that, except for their small contingent of H1 proponents, the RC's prefer space generation over mutual pulling. But they know better than to think they can just take their pick. They must settle the question with conclusive empirical evidence. Their experience so far does not provide that. So they conceive of two crucial tests. One of these tests could be performed using the instruments on the poles and one of their rocket ships. Since the RC's nearly ran out of fuel in order to make their soft landings, they would rather save what little they have left, if they can. Still it is worthwhile to explain what they had in mind, since it well illustrates a key difference between the competing hypotheses.

When the RC's first encountered the top of the pole, and upon passing later IS's, they took note of the clock readings with the intent of analyzing this data later. As it turns out, in the course of all their maneuvering and their general befuddlement, they ended up losing this clock data. If they hadn't lost it, they could perhaps already safely decide between H1 (mutual pulling) and H2 (space generation). According to H1, since the "field" around the sphere is supposed to be a static thing, the rate of a clock that "falls" in the field is supposed to depend only on its location and its *speed*. Its *direction* is not supposed to matter. Mathematically, this is expressed by adding a velocity *squared* term to the argument in Equation 18 or 21. Therefore, as a clock falls, its rate decreases, reaching a minimum just before colliding with the surface.

The RC's have reservations about this picture for the following reason. It seems to cause twice as much clock slowing as it "should." The spacetime curvature somehow causes or is a manifestation of slow clocks that are not moving at all. Then, with respect to these clocks the curvature also causes motion which has the potential to slow falling clocks' rates by another factor of that which applies to the static clocks. This is the factor that somehow causes clock slowing due to *curvature*:

$$\sqrt{1 - 2GM/rc^2}\,, \qquad (32)$$

and for the case of a clock radially falling from infinity, one is supposed to find another factor due to *motion*:

$$\sqrt{1 - 2GM/rc^2}\,. \qquad (33)$$

As a consequence, within the pulling field there is no state of motion corresponding to the rate of a clock at infinity; the only analog for a clock on their cylinder's rotation axis is a clock *at* infinity. So the picture is of a *static* thing that somehow manifests the instrument detectable characteristics of motion. Then this static thing produces motion of other things which manifest even more effects of motion with regard to their clock rates. But these latter (falling) things manifest *no* effects of motion with regard to their accelerometer readings. For most RC's the gut reaction is that the whole scheme is scrambled and nonsensical.

Fortunately, as mentioned above, the picture is different enough from H2 that it lends itself well to a confrontation by experiment. According to H2, since the "field" around the sphere is stationary, i.e., always moving with the same outward acceleration and velocity (at any given location) the rate of a clock that radially falls cannot get any slower, it can only increase. The extreme case would be a clock that falls radially all the way from "infinity." That is, of course, what the RC's just did, in essence. Such a clock would retain the maximum rate it had *at* infinity, because nothing ever changes its velocity.

During the whole trip, motion sensing devices consistently indicate that it's the pole that does the moving, not the falling clock. As noted eariler, the RC's have come to call these special case trajectories (falling radially from infinity) *maximal geodesics*. (See Figure 17.) The rate of any clock in this "space generation field," whether rigidly attached to the central mass (e.g., attached to a pole) or falling in any direction, will depend on its speed *with respect to the local maximal geodesics*. Maximal geodesics are thus the local "preferred" frames; the local standards of rest.

If the RC's had not lost track of their clock data, the idea would be to compare the elapsed time of their own clock with the elapsed time that they could calculate would have elapsed if the rate decreased as per H1. According to the latter hypothesis the expression for the rate of a "probe" clock moving in the field is

$$\frac{\Delta f_{H1}}{f_G} = \frac{f_P - f_G}{f_G} = \frac{\sqrt{1 - \frac{2GM}{r_P c^2} - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{2GM}{r_G c^2}}} - 1\,, \qquad (34)$$

where G and P denote ground and probe, and v is the probe's speed. We see that, by adding (subtracting) a squared velocity to the matter-induced clock slowing factor, $2GM/r_p c^2$, the direction of the moving clock is irrelevant. By H2, on the other hand, the expression is

$$\frac{\Delta f_{H2}}{f_G} = \frac{f_P - f_G}{f_G} = \frac{\sqrt{1 - \frac{\left[\sqrt{2GM/r_P} \pm v\right]^2}{c^2}}}{\sqrt{1 - \frac{2GM}{r_G c^2}}} - 1\,, \quad (35)$$

where we see that the matter-induced factor is treated as a speed (stationary outward velocity). This speed is then added (before squaring) to the apparent speed of a clock moving with respect to the stationary body so as to get the total clock slowing effect. Since the falling velocity is positive or negative depending on direction, the sum results in a significant difference between H1 and H2. For a maximal geodesic the sum is zero. (Note that Equation 35 has been somewhat simplified by using r instead of r_γ for the radial distance.) If the RC's had enough fuel, they'd launch a clock to a high enough vertical distance so as to make the elapsed times large enough that the difference would be measurable.

(Some readers may be aware that the prediction of H1 would seem already to have been borne out by the Vessot-Levine experiment. [8] If true, this would of course rule out H2. It is shown in §10 that the results of this experiment are actually equivocal. When account is taken of the apparatus used to make the measurements, the prediction of H2 actually agrees quite well with the experimental result. What follows here is a simplified description of a modification of the Vessot-Levine experiment whose result would *not* be equivocal.)

The idea is to take data at three key events: Soon after releasing the launched clock upwardly onto a ballistic trajectory, record the elapsed time at a well-defined height; call this the "perisphere." Another recording would be needed at the midway (apex) point; call this "aposphere." And finally, record the elapsed time at perisphere again on the descent phase. According to H1 the elapsed time for the ascent phase would be the same as that for the descent phase. By this view, the trajectory is symmetrical in all respects. The launched clock goes up and then comes back down. But the direction does not affect clock rate; only speed (and location) matters.

According to H2, by contrast, the elapsed time for the ascent phase should be significantly less than for the descent phase. The trajectory is actually asymmetrical. *The ballistically launched clock only goes up*; the only force ever applied to it was in the upward direction; nothing ever forces it back "down." Just before being released onto the ballistic trajectory the clock had the maximum upward speed, so it had a minimum rate. After that the clock's *speed with respect to a maximal geodesic* only decreases, so its rate increases on both the ascent and the descent phases.

Two comparisons should thus be made: 1) Elapsed time for ascent phase (H1 time > H2 time) and 2) Elapsed time for descent phase (H1 time < H2 time). Both H1 and H2 predict the same total elapsed times for the whole trip; so the key to a conclusive result is getting accurate determinations for the two distinct phases.

Although the results of this experiment should clearly reveal whether the field is stationary or static, an even more ironclad, and fortunately less technologically demanding experiment could be conducted in a laboratory at the sphere's surface. This will be discussed in the next section, not from the point of view of hypothetical space explorers, but from that of diligent Earthian physicists.

9 In Progress

When it's actually finished, §9 will have a different name. Sections 9-13 are still in progress. Having recently hammered §8 – the most important section in this essay – into a reasonably complete state, I deemed it appropriate to make this draft available. §9 is also important in that it discusses the most feasible experiment to test the gravity model. But most of §9's content is already available on the GravitationLab.com home page and in Paper 1: "Laboratory Test of a Class of Gravity Models."

Reader comments are encouraged:
webmaster@gravitationlab.com.
Richard Benish, June 26, 2008.

Appendix

A Modern Commentators on the Sagnac Effect and Relativistic Rotation

The practical purposes of positioning and navigating are extremely well served by the GPS. The system's clocks are synchronized, in effect, by the central-flash method. Accordingly, when used to "measure" the one-way speed of light, they indicate that the orange beam (of our previous example, §6.8) returns to the observer before the green beam because the orange one travels with respect to the observer faster than c, and the green one travels slower than c. The difference is plus or minus the speed of rotation. This concurs with the conclusion of our "child" observer in our "amplified" example in which the time-gap is so large as to be discernible using only human eyes and mind. It is a simple physical property of rotating reference frames. Most children, one can be sure, would have a difficult time understanding why so many grownups would make such a big deal of it.

Happily, since the GPS has been up and running, one no longer (to my knowledge) finds blanket "impossibility" statements. The spirit of Einstein, however, still resides in many theoretical accounts of the System. In spite of how simply the situation can be described, there is still some debate about the "proper" way to interpret the Sagnac effect. A key focus in these discussions, as indicated above, is the time gap. It is useful to see exactly what's been said. Three examples thus follow. First, Rizzi and Serafini:

> The deep physical, non-conventional, nature of the time-lag...the dark physical root of the Sagnac effect [is] desynchronization of slowly travelling clocks. ...such a desynchronization is just a variance of the well known twin paradox. [67]

Next, D. Dieks:

> Because of the difference in arrival times of the two light signals, the velocity of light obviously cannot be everywhere the same in the rotating coordinates. This is a consequence of the fact that in the rotating frame events with equal time coordinate t are not standard simultaneous. [68]

N. Ashby:

> The Sagnac effect can... be regarded as arising from the relativity of simultaneity in a Lorentz transformation to a sequence of local inertial frames co-moving with points on the rotating earth, or as the difference between proper times of a slowly moving portable clock and a Master reference clock fixed on earth's surface. [69]

Before considering the general import of these statements, I'll begin with some remarks about each one. Rizzi and Serafini refer to another physical operation besides counter-rotating light beams that would result in an identical time difference. Two very slowly transported clocks, upon returning to their starting point, will have become "desynchronized" by the same amount. Notice how far removed this is from the original empirical fact. The authors may have pointed out some kind of corollary or *analogy*, but is it true to say that this is the "dark physical root" of the Sagnac effect? No clocks are needed to demonstrate the effect. So how can clocks be the "physical root"?

The t Dieks refers to corresponds to the time of the rotation axis, or the time clocks on the rim would have (to first order) if they were synchronized by a flash from the center. He points out that the velocity of light cannot be isotropic with respect to the rotating disk when using this time coordinate. However, one is still struck by Dieks' characterization of light speed anisotropy (or time lag) as being a *consequence of the synchronization scheme*. Remember, the lag (anisotropy) occurs with respect to any *one* clock on the rim. The (physical) effect is patently *independent* of (abstract) synchronization scheme.

Ashby is recognized as one of the foremost authorities on the GPS. He attributes the Sagnac effect to the relativity of simultaneity or a clock transport operation like the one mentioned above by Rizzi and Serafini. He reiterates these and other causes in at least three papers. [70] [71] [72] In describing the details of the System, Ashby often refers to the "constancy of c" in the underlying (Earth centered) inertial coordinate system. With regard to rotating observers and the signals they send with or against the rotation around the circumference, he refers to "path-dependent inconsistencies" or "discrepancies"; he refers to differences in the travel times or differences in the path lengths of such signals. But, to my knowledge, Ashby never explicitly describes what happens as being due to non-c light speeds with respect to rotating observers. It is amazing to me how consistent he is, at every step avoiding the simplest explanation. Instead, he concludes: "In fact observers in the rotating frame cannot even globally synchronize their own clocks, due to the rotation." [73]

Navigators and time keepers around the world are surely satisfied with how well synchronized the clocks of the GPS appear to be. Yet one of the physicists who helped to build and maintain it regards the clocks as being *de*synchronized. In the discussion section of the book from which the above quotations were taken, we see that Ashby graciously grants as "admissible" the viewpoint that the rotating clocks are indeed synchronized (with the corresponding anisotropic light propagation which follows). But he evidently prefers not to think of it this way himself. Why not? Evidently, it's because it would mean that the speed of light with respect to rotating observers would then even locally not be equal to c. While allowing that others may look at it that way, Ashby appears to maintain that, for himself, it is against his principles.

B Classical, Relativistic or Generic Fact?

Although Ashby and others have explicitly referred to the Sagnac effect as a "Special Relativity effect" [70] [74], K. Brown, who has provided an in-depth analysis of the problem, points out that, actually, it is a classical effect:

> It's worth emphasizing that the Sagnac effect is purely a classical, not a relativistic phenomenon, because it's a "differential device," i.e., by running the light rays around the loop in opposite directions and measuring the time difference, it effectively cancels out the "transverse" effects characteristic of truly relativistic phenomena. [**?**]

We should add that there's little doubt that Sagnac himself would have emphatically denied that the effect that he demonstrated is "relativistic."

Be that as it may, Brown derives the Sagnac "time-gap" four different ways and comments: "We're simply decomposing those absolute intervals into space and time components in different ways." [75] Though Brown regards it as a classical effect, his analysis is clearly couched in Relativistic terminology. While a "classical" physicist (e.g., Sagnac) would not hesitate to describe light propagation with respect to the rotating body as manifesting a speed anisotropy with respect to the body, for the most part, Brown avoids this point of view. He points out that "Special relativity does not entail invariant or isotropic light speed with respect to non-inertial coordinates." Yet his "space and time decompositions" are all from the point of view of inertial observers, relative to whom, he repeatedly asserts, the speed of light equals c. Indeed, in the course of defending SR against those who would suggest that the Sagnac effect is somehow inconsistent with SR, Brown emphasizes the special status of inertial frames:

> It's self-evident that since the speed of light is isotropic with respect to at least one particular frame of reference, and since every other frame is related to that frame by a transformation that explicitly preserves light speed, no inconsistency with the invariance of the speed of light can arise.

As we recall from §4, however, we see that Brown is tacitly denying the possibility of adopting a value other than $\frac{1}{2}$ for the synchrony parameter, ϵ. What Brown says is true "in theory" if we insist that all uniformly moving observers adopt $\epsilon = \frac{1}{2}$. One can "uphold the law" (second postulate) by this approach. As Brown writes: "The second principle states that light always propagates at the speed c, assuming we define the time intervals...as whatever they must be in order for the speed of light to be c." [76] Sounds like stacking the deck, doesn't it? This is what we get if everyone agrees that ϵ must equal $\frac{1}{2}$. Physics by consensus? Judges and lawyers would perhaps be happy with it. But detectives would not.

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