Preliminary Experimental Result of the Grand Unification Theory of Physics on Gravity

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Abstract: This paper reports the results of a ground-based experiment on the gravitational property of a simple pendulum using two selected materials. The experiment is motivated by the grand unification of physical theories by way of the extended energy-momentum relation that was established in late 2019 by the author. The preliminary results seem to align well with the theoretical declaration that inertial mass, which is affected by mass deficit, differs from gravitational mass. The results offer feasible theoretical explanations to discrepancies found in existing gravitational *G* factor measurements. This paper calls for refined observations and analyses including ground and space experiments from a unified perspective to reaffirm or refute existing and future results, thereby lay a dependable experimental foundation for further developments of physical theories in the framework of grand unification. The results of relevant experiments, whether positive or negative, have fundamental significances in that they can unambiguously prove or disprove either the grand unification theory or Einstein's general relativity theories, and will definitively point to the right direction for robust developments of physics theories and experiments.

1. Background

Since the establishment of the grand unification of physical theories by the single equation, termed as the **extended energy-momentum relation** of $\boldsymbol{E^2 = (mc^2 + V)^2 + (pc)^2}$ by the author in late 2019, significant theoretical developments have been accomplished utilizing this new equation to unify classical mechanical physics, quantum physics, general relativity and special relativity theories. Specifically, this equation has been successfully used to calculate the atomic hydrogen energy levels with relativistic accuracy that exceeds those of the Schrödinger equation. The solutions to this quadratic equation naturally offer the ½ quantum number, corresponding to the concept of ½ spin of electrons, and the solutions provide two distinctive quantum states for each primary quantum number, corresponding to the Pauli exclusion principle thus explaining the origin of fermions. The extended energy-momentum relation simultaneously represents both the wave-nature and relativistic particle-nature of any particle by one single equation, microscopically through the squared spatial operator $-(c\hbar\nabla)^2$ and macroscopically through the squared relativistic total energy which can be conveniently decomposed as the dot product of a four-energy vector, representing the four orthogonal components of the relativistic total energy: one-dimensional rest energy and three-dimensional kinetic energy. Conservation of momentum and conservation of energy are unified as the single conservation of the four-energy vector and its amplitude. It has been elucidated that the bewildering time and space effects described by Einstein's general relativity are simply the coherent combination of quantum physics and mass deficit effects due to the gravitational binding energy, which is one of the core concepts of the extended energy-momentum relation. Beyond the microscopic domains, the extended energy-momentum relation has wide applications in astrophysics where gravity is the primary driving force for cosmic dynamics. It can naturally give rise to comprehensive astronomical phenomena including astronomical jets, cosmic outflows, fast radio and gamma ray bursts, supernova explosions, neutron star and black hole physics, as well as cosmological evolutions. The same physical principles of extended energy-momentum relation applied to both macroscopic and microscopic domains reveal that black hole radiation and gamma-ray pair production are essentially originated from very similar physical conditions and mechanisms. The extended energy-momentum relation unambiguously treats any natural forces equally through potential energy, and the quadratic nature of the relation offers superb mathematical and physical symmetries including matter-antimatter mass symmetry, charge-symmetry, mass-potential symmetry, time symmetry and parity symmetry. The symmetries naturally provide repulsive gravity between matter and certain antimatter − a much-needed theoretical mechanism to explain myriad of puzzling astronomical observations that are presently modeled with unconvincing hypotheses (such as dark matter and the magnetic field acceleration), e.g., the accelerating expansion of the universe, the powerful ejection of jets and outflows at near speed of light without been pulled back by supermassive black holes or galactic center, or the ejection of mass at the end of a star's life.

Although the innovative extended-energy momentum relation has demonstrated great achievements both microscopically and macroscopically and is at the same time compatible with many theoretical and experimental observations, its acceptance is non-existent in the mainstream theoretical community. Multiple submissions to recognized journals for publication of novel theories based on the extended energy-momentum relation proved unsuccessful. There are obviously no peers so far to recognize these creative theories. There are either no knowledgeable staffs to appreciate such important theoretical discoveries and start the peer-review process, or even when a passionate staff member did start the peer-review process with one reviewer showed lukewarm support, the submission still ended up rejected. Other reviewers either fell short to read through the section titles (not mentioning the content), or proposed the so-called "correct" theory of his own and rejected the extended energy-momentum relation just because his version is clearly wrong. It is safe to conclude that publication of such innovative theories in peer-reviewed journals without conspicuous experimental results is impossible, efforts should be instead directed to further theoretical developments and identifications of experimental opportunities to differentiate the new theories from present theories by irrefutable and verifiable results.

An alternative route to make the new theories public without going through the impassable peer-review process is the various preprint online services. The most well-known example is arxiv.org where preprint papers are freely accessible. Arxiv has significant influences in both academic communities and general public. Unfortunately, due to its high popularity, publishing preprint papers at arxiv is moderated, meaning that it shares the same issues with peer-reviewed journals and will not allow innovative theories drastically different from dominant theories to be published, even though it hosts more questionable papers. Trying to convince arxiv is also impractical. So far, the best preprint website is the ResearchGate.net where both academic scientists and independent researchers posted their papers freely. Most preprints are free to access even including some peer-reviewed papers, in addition, ResearchGate is a valuable social media for interactions with other researchers by means of user profiles, posted projects and updates, questions and answers, messages, comments, followships and recommendations. It is the primary media where the author publicizes the theoretical developments and updates and interacts with researchers who are interested in related topics. The participants include institutional researchers who can gain audience by posting their abstracts and peer-reviewed papers (full-text papers are supposed to be available upon request although the author never got one). Most independent researchers are very open and actively promoting new ideas from their perspective; however, due to limited resources and varied backgrounds and experiences, many of the new ideas are not well-considered and subject to criticism. Similar to some exotic theories, these new ideas can hardly develop into viable theories due to insufficient merits and inadequate depths of reasoning or contradictory to experimental results. Nevertheless, the ResearchGate is a thoroughly open system that doesn't discourage independent research efforts. It does assemble serious contents and intelligent researchers, only more efforts are needed to sift out irrelevant information.

On the other hand, institutional academic research activities and peer-reviewed papers have been the sources of the most valuable scientific indepth knowledge and practices. Particularly for physics, tremendous amounts of highly reliable, accurate, traceable, detailed, systematic and trustworthy contents have been accumulated over centuries of intensive research activities supported by vast amount of investments. Numerous brilliant scholars have contributed to shape the present theories that have reached a superb level of precision so that difference of one part in 10^{10} or more can be reliably identified. Nevertheless, recent theoretical developments in the formal academic communities lack meaningful innovations and are mostly preoccupied by the dominant theories, which are several decades if not centuries old. Physics theories are distinctively separated into isolated fields such as quantum physics, electromagnetics, particle physics and relativity physics. Noticeably, some late efforts to advance these theories are becoming more and more complicated and hypothetical with poor connection to physics, stretching toward sophisticated mathematics and abstract concepts that cannot be experimentally verified piecewise nor theoretically viable for further advancements, and yet offer misleading hopes and unconstrained claims that require unfounded subjective interpretations. Such mythical mentality combined with prejudice and complacence owing to past achievements effectively deprived the momentum of theoretical explorations, as evidenced by the long absence of major breakthroughs even diverse technologies have advanced undeterred all the way.

On the contrary, the new theories based on the extended energy-momentum relation have shown great potentials in explaining wide range of microscopic and macroscopic phenomena both quantitatively and qualitatively utilizing straightforward simple physical concepts and have helped identify many obvious faults in Einstein's interpretation of general relativity effects especially the concept of warped space-time and the related equivalent principle where gravity is interpreted as the equivalence of acceleration and gravitational mass is equivalent to inertial mass.

2. The very simplified explanation of the framework of grand unification theory

Physics theories are about deriving principles and rules from observations of the physical world. Physics theories must be the highly concise and consistent. All physics phenomena should follow as few principles as possible rather than multiple principles under different situations. The grand unification of physics should treat any natural forces equally in this spirit. This should be true in particular for gravitational, nuclear and electric forces. However, Einstein's general relativity singles out gravity as a very special kind of force – it makes gravity uniquely related to warped space and time near a mass. Despite of its success, it can be decisively declared that such theory doesn't fit in the unified physics theory because it separates gravity from other forces thereby consciously creates additional principles that cannot be shared with other parts of physics. It is also known that general relativity is incompatible with quantum physics, thereby further distances itself from the grand unification of physics.

Needless to say, the first step toward grand unification must at least unify gravity, electric and nuclear forces and treat them equally. The apparent connection among these different forces can be very naturally achieved by way of energy. Energy can be and should be exchanged equally among these forces without the necessity to identify the specific type of the force contributing to the energy or how strong or weak the contributing force is.

After identifying energy as the keyword for grand unification, the next step should be the unification of the distinct properties of the energy caused by the abovementioned forces. This is straightforward too, because all of these forces are considered conservative forces, and they all contribute to the same category of energy, the so-called potential energy. This seemingly simple concept actually carries very profound implications, because (a) potential energy exists even when all involved participants are at rest; (b) potential energy doesn't depend on the variation of the masses of involved participants; (c) potential energy can cause mass deficit.

It is very easy to understand property (a) because potential energy is different from another category of energy – kinetic energy, which must rely on motion. In other words, potential energy is orthogonal to kinetic energy in the sense that motion and at-rest are two orthogonal conditions – any object can either be at rest or in motion, but not both. This is also evident in the extended energy-momentum relation $E^2 = (mc^2 + V)^2 + (pc)^2$ where mc^2 is the rest mass energy and V is the potential energy. The kinetic energy represented by *pc* can be zero or non-zero regardless of the value of $mc^2 + V$, and vice versa, $mc^2 + V$ can be zero or non-zero regardless of the value of *pc*. Potential energy V is listed alongside with rest mass energy mc^2 because they can both exist even when all involved participants are at rest. A simple mathematical summation is adopted to reflect the fact of nuclear mass deficit where V represents the nuclear binding energy. Since binding energy is one type of negative potential energy, it will linearly reduce the total rest mass energy, so the final rest mass energy after mass deficit is written as $mc^2 + V$. An additional reason why potential energy is considered orthogonal (together with mc^2) to kinetic energy is that *pc* and $mc^2 + V$ contributes to the total energy E in the squared form $E^2 = (mc^2 + V)^2 + (pc)^2$ where $mc^2 + V$ and *pc* behave similarly to the two orthogonal legs of a right triangle contributing to the length of its hypotenuse.

Understanding of property (b) can be tricky. It is well known that electric potential energy is proportional to the product of charges that are quantized and fixed to the multiples of unit charge (the charge of an electron). Even for fractional charges inside subatomic nucleons, the charges are fixed fractions (multiples of $1/3$) of the unit charge. The charges are not affected by masses of the charge carriers, whether it comes from an electron, or heavier proton or quarks. More importantly, electric charges are not affected by the mass deficit of neurons, protons or quarks in the nuclei. For example, the electric charge potential caused by a proton in a hydrogen atom will be the same as the electric charge potential caused by a proton in an iron atom even protons and neutrons in iron atom suffer significant mass deficit (also called mass defect, or binding energy, refer to https://periodictable.com/Isotopes/026.56/index.html for values of mass deficits) of approximately −8.8 MeV per nucleon on average. For iron atoms, each proton loses about 0.94% of its rest mass due to binding energy, but its charge remains to be precisely 1 unit charge, not at all affected by its loss of mass. Given that modern atomic energy level experiments (including atomic clocks) has achieved precision better than 1 part in 10^{10}, any possible minuscule variation in electric charge many orders smaller than 0.94% would have been noticed long time ago. The reality is no charge variation has ever been found, meaning that electric potential energy will not be affected by mass deficit of involved particles, the amount of charges is conserved.

Understanding of property (c) is straightforward for nuclear force (one type of force giving rise to the binding energy) because this is where the concept of mass deficit was discovered. If grand unification of physics ever holds true, then the mass deficit concept should be equally applicable to any forces including electric force and gravity. This unification may surprise many physicists, or even make them skeptical or uncomfortable. Probably they never thought about it and are complacent with nuclear mass deficit and stop thinking further. For whatever reason, this complacence may subconsciously turn into a denial against the posibility that all potential forces can cause mass deficit equally as the nuclear binding energy does. Contrary to what one might expect, theoretically denying electric and gravitational mass deficit is much more difficult than accepting it. The denial would in essence discriminate energy among different forces, and potential energies would have been isolated into different types and would not exchange freely through mass deficit, and energy conservation would have been artificially separated into different force-specific types. This would necessitate extra theories to prevent electric force and gravity from causing mass deficit. No serious physicists would be motivated to divide potential energies into different types and pursue such theories to deny the mass deficit by all potential energies. Accepting (c) will be the proper and easy choice in the spirit of grand unification, and thereby uniformly measure the strength of all forces in terms of the amount of mass deficit they can induce. Mathematically, property (c) is represented in the extended energy-momentum relation $E^2 = (mc^2 + V)^2 + (pc)^2$. In this equation, the total rest energy squared E^2 is equal to $(mc^2 + V)^2$ when the system is at rest ($pc = 0$) where V can be the sum of any potential energies including nuclear binding energy, electric potential energy and gravitational potential energy, namely $V = \sum V_i = V_{nuclear} + V_{electric} + V_{gravity} + \cdots$. Any change in potential energy V will be linearly reflected in the system's total energy E at rest. Through the well-known mass-energy equivalence, the system's rest mass after mass deficit deduction should be $\boldsymbol{m' = (mc^2 + V)/c^2 = m + V/c^2 = m + \sum V_i/c^2}$, meaning that the rest mass experience a deficit of V/c^2 where V is the total binding energy which is a negative quantity (positive potential energy typically cannot stably bind). There are additional advantages taking this energetic point of view in that the mythical instantaneous gravitational force (including any other forces) is not a conceptual hurdle because no matter how long it takes to transmit the force even over astronomical distances, the energy conservation principle can still rule the system at any instant without the need for the force to exceed the speed of light.

Reviewing properties (b) and (c), experimental facts clearly indicate that potential energy owing to electric force only depends on the amount of charges (and depends on distance of course), the charges and electric force are not affected by the variation of mass due to mass deficit caused by binding energy, unified as potential energy which includes nuclear binding energy and electric potential energy. Now the question is: what specific physical property is affected by the mass deficit? The effect of mass deficit will obviously change the rest mass with respect to rest mass energy, which is well understood, because there is a mass-energy equivalence principle – the negative potential energy (binding energy) leads to the reduced rest mass (of a nucleus). But more importantly, the key point is that the mass to be varied by the mass deficit must be the inertial mass, the amount of mass experimentally determined by measuring the motion of the mass related to kinetic energy or momentum in magnetic fields (again, momentum has been unified as part of the four-energy vector in the framework of grand unification, not detailed in this paper). Experimental facts clearly indicate that inertial mass is directly related to the rest mass after the reduction of mass deficit, because accurate values of rest masses are determined by the motion, namely by means of the inertial mass, of charged nuclei in magnetic fields. The accurate measurement of inertial masses by the mass spectrometry technique effectively measure the (inertial) mass-to-charge ratio, and the charge is assumed to be constant, only the inertial mass is the variable to be determined. So far there is no other accurate method of directly measuring rest mass other than by way of inertial mass. Up to this point, the very important conclusion can be made: **inertial mass related to momentum is equal to rest mass related to rest mass energy, namely there is an inertial-rest mass equivalence**, which corresponds to the mass-energy equivalence. Notice that this **inertial-rest mass equivalence** is experimental facts, not a hypothesis.

Reviewing the above discussions about properties of (a), (b) and (c) all together, it is now not hard to realize that the present understanding of mass in energy-momentum relation $E^2 = (mc^2)^2 + (pc)^2$ where rest mass *m* is used for both rest mass energy $E = mc^2$ and momentum $p = \gamma mv$ is erroneous, incomplete or at least subconsciously inconsiderate without mentioning mass deficit. When at rest, that is $v = 0$ and $p = 0$, rest mass *m* in mc^2 should be $m' = m + V/c^2$ that includes mass deficit effect considering the presence of binding energy *V*, failing to do so will violate the energy conservation and mass-energy equivalence principles; when in motion, p should use the inertial mass at rest $m' = m + V/c^2$ that includes mass deficit effect, failing to do so is ignorant about the fact that mass deficits of nuclei are experimentally determined by the variation of mass-to-charge ratio where the mass is the inertial mass at rest giving rise to the momentum of nuclei in magnetic fields. In other words, the correct momentum should be $p = \gamma m'v = \gamma(m + V/c^2)v$ where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor reflecting the special relativity effect. Therefore, the correct energy-momentum relation should be $E^2 = (mc^2 + V) + (pc)^2 = (m'c^2)^2 + (\gamma m'vc)^2 = (\gamma m'c^2)^2 = [\gamma(m + V/c^2)c^2]^2$, where the first part of the equation is exactly the extended energy-momentum relation. Similar to the Lorentz factor γ, another factor $\boldsymbol{\zeta = 1 + V/(mc^2)}$ can be introduced to simplify the correct energy-moment relation as $\boldsymbol{E^2 = (\gamma\zeta mc^2)^2}$, and this can be termed as the **extended mass-energy equivalence principle**. This equation is effectively the most concise form of the extended energy-momentum relation. It is the unification of both relativistic effect and mass deficit effect, and it can be shown to explain both special relativity effects (represented by γ which is a function of speed v, and is always no less than 1) and general relativity effects (represented by ζ which is a function of potential energy V, and is always no greater than 1) without manipulating space and time as proposed by Einstein. For one example, when a planet is bound to the Sun in an elliptical orbit, ζ is smallest and largest at perihelion and aphelion respectively because the gravitational potential energy V is most negative and least negative respectively. To conserve the total energy E, γ must be largest and smallest respectively, meaning that the planet must be moving fastest and slowest at perihelion and aphelion, respectively. Superficially, it may appear nothing new as this has been known since Newton's time. But thinking more deeply, there is a brand-new concept of mass deficit represented by ζ on top of the conventional Newtonian mechanical physics – the planet's inertial mass at rest is no longer constant as Newton or Einstein thought. In a simplified qualitatively view, it means that the inertial mass at rest ζm of the planet becomes smallest and largest at perihelion and aphelian respectively, so it moves more faster and more slower than the Newtonian orbital speeds to compensate the change in inertial mass, respectively, promoting the planet to deviate from the fixed Newtonian orbit assuming invariant mass thereby take on an orbital precession around the Sun. Mathematically, taking the approximation of the positive square root of the extended mass-energy equivalence by omitting v^4/c^4 and higher order smallness in γ, $E = \gamma\zeta mc^2 \approx (1 + ½v^2/c^2)(1 + V/m/c^2)mc^2 = (mc^2 + mv^2/2)(1 + V/m/c^2) = mc^2 + mv^2/2 + V + ½Vv^2/c^2$. It can be found that the first term is the rest mass energy without mass deficit, the combination of second and third terms is the conventional Newtonian mechanical energy, but the fourth term of $½Vv^2/c^2$ although a small correction is new and missing in Newtonian physics. Therefore, the conservation of energy in Newtonian physics is inaccurate at higher orders because it fails to take $½Vv^2/c^2$, a negative term, into account. Mathematically, conservation of Newtonian mechanical energy of $mv^2/2 + V$ will lead to an orbit fixed in space as is well known. But the conservation of $mv^2/2 + V + ½Vv^2/c^2$ will deviate from any fixed orbit unless V and v^2 remain constant (namely circular orbit). For an elliptical orbit, at perihelion because V is most negative and v^2 is highest, $½Vv^2/c^2$ becomes most negative. In order to conserve energy of $mv^2/2 + V + ½Vv^2/c^2$, v^2 must be raised by the most amount relative to Newtonian orbital speed to compensate through term $mv^2/2$ (V cannot be changed because it is a function of geometry), therefore the actual speed is faster than Newtonian orbital speed by the most amount at perihelion (while at aphelion it is still faster than the Newtonian orbital speed but by the least amount) giving rise to the orbital procession. Of course, even for circular orbits, Newtonian orbital speed is lower than that predicted by the extended mass-energy equivalence as shown above. The new conservation of extended "mechanical" energy $mv^2/2 + V + ½Vv^2/c^2$ derived from the extended mass-energy equivalence can be mathematically viewed as the result of a perturbation to the Newtonian potential energy from V to $V_{grand} = V + ½Vv^2/c^2$. Taking the Newtonian approximation that gravitational potential energy is $V = -GMm/r$ and $mv^2/2 \approx -V/2$, it can be found that $V_{grand} \approx -\frac{GMm}{r} - \frac{G^2M^2m}{2c^2r^2}$. The presence of the perturbational potential energy of $-\frac{G^2M^2m}{2c^2r^2}$ different from $1/r$ will cause the planet to move more faster at perihelion and less faster at aphelion on top of the Newtonian obit which takes $-GMm/r$ as the gravitational energy, leading to the orbital precession. For another example, when an atomic clock is placed in a lower altitude in gravity, gravitational potential V becomes more negative, making ζ smaller, which in turn makes inertial masses at rest (ζm) of electrons and nuclei smaller in the clock through the gravitational mass deficit. Quantum mechanically, the frequency of the atomic clock is proportional to the masses of electrons (and nuclei to a much lesser degree), therefore the atomic clock's frequency is slower compared to that of another atomic clock at higher altitude – an effect known as gravitational time-dilation or redshift. These two examples demonstrate the much cleaner and more unified physical interpretations of general relativity effects all within the framework of extended energy-momentum principle (and the concise energy-mass equivalence principle) without the addition of mind-bending space-time manipulations, and the explanation is compatible with quantum theory. Notice that in the grand unification framework, quantum physics is the manifestation of the wave-nature of the same extended energy-momentum relation in microscopic domain by equating relativistic kinetic energy $(pc)^2$ to operator $-(c\hbar\nabla)^2$ based on the de Broglie matter wave observation. Therefore, quantum physics effects, special relativity effects and general relativity effects are now all unified by the same extended energy-momentum relation and the extended energy-mass equivalence principles.

3. Gravity in the framework of grand unification theory and experimental designs to test the unification

The concept of variation of rest mass (inertial mass at rest) in the grand unification theory is found to be fruitful in unifying both special relativity and general relativity effects in the extended energy-momentum relation and the extended mass-energy equivalence principles in a concise equation of $E^2 = (\gamma\zeta mc^2)^2$. In short, special relativity changes the relativistic mass (related to total relativistic energy and relativistic inertial mass) through factor γ owing to motion represented by speed v, and general relativity changes the rest mass (related to total rest mass energy and inertial mass at rest) through factor ζ owing to potential energy V. But variations of γ and ζ do not change potential energy V because it is only a function of geometry and the interacting force providers (or charges). As observed experimentally, electric potential energy is independent on motion (through γ) and mass deficit (through ζ). The next very essential question to be answered is: will gravitational force behave the same as electric force to be unaffected by γ or ζ? This fundamental question has never been meaningfully addressed by any existing theories and the answer to this question will have profound influence on all aspects of physics, especially on the grand unification theory itself. For the spirit of unification and simplicity of physics, gravitational force must not depend on the variation of mass by mass deficit (ζ) nor motion through special relativity (γ), just like the electric force. This proposition will immediately raise questions to two major physicists in history – Newton and Einstein. Newtonian gravitational potential energy is written as $V = -GMm/r$ where *G* is the gravitational constant, *M* and *m* are the interacting masses, and *r* is the geometric distance between them. The essential question for Newton is: what should *M* and *m* be? should they be the masses before mass deficit (ζ) and/or relativistic (γ) corrections, or should they be one of the possible masses such as $\zeta\gamma m$, ζm,or γm? Newton wouldn't foresee this question because no one knew these masses at his time, so he would probably be ok with either of the answers because he only dealt with non-relativistic physics, and the maximum error for choosing the wrong answer would not exceed the difference caused by nuclear mass deficits, which is typically less than twice of iron atoms by approximately $2 \times 0.94\%$ (doubled because gravitational energy involves the product of two masses). However, the essential question for Einstein is quite challenging as he was aware of special relativity and general relativity effects: are gravitational mass (which is likely *m* before ζ and γ correction) and inertial mass at rest (which is ζm) and total relativistic inertial mass (which is $\zeta\gamma m$) equivalent? or is his equivalence principle correct? This is a quite aggressive yes-or-no question, because it directly challenges the foundation of his general relativity theory in which gravitational mass and inertial mass are considered equivalent. If Einstein defends his relativity theory, then his gravitational potential energy becomes ζm and $V = -G\zeta_M\gamma_M M\zeta_m\gamma_m m/r$, making his general relativity theory deviate further away from grand unification on top of the incompatibility with quantum physics and the extra space-time manipulation, because his theory doesn't use the same physics principles as electric and nuclear force whose potential energies are independent of ζ and γ. Not only the theory doesn't belong to the grand unification, but also no direct experiments have ever been designed to purposely verify whether gravity is affected by ζ or γ. His equivalence principle appears more like an abrupt opinion or speculation rather than a physics principle because it is specifically hypothesized for the development of general relativity theory. It takes no physical implications nor physical principles from experimentally determined inertial mass or gravitational mass, nor properties of other natural forces; instead it deviates from them and started a new principle. On the contrary, the extended energy-momentum relation $E^2 = (mc^2 + V)^2 + (pc)^2$ and extended mass-energy equivalence $E^2 = (\gamma\zeta mc^2)^2$ are all physical principles that are individually verifiable by experiments, and are extended from existing known physics principles: all the principles are connected inherently, all natural forces consistently behave the same through potential energy. Most importantly, this paper will provide experimental evidence to indicate that gravity is probably not affected by ζ, and the gravity is better described by $V = -GMm/r$ where *M* and *m* are all the rest masses before any mass deficit (such as nuclear binding energy in particular), suggesting that in essence, *M* and *m* are more like the amounts of gravity charges, not affected by nuclear mass deficit.

Regardless of the eventual experimental results, Einstein's general relativity theory is incomplete as it does not take into account the two mass modifiers (ζ and γ) and it denies the possibility of gravity being a true natural force without giving adequate theoretical reasoning, and at least fails to either approve or disapprove such possibility by thorough thought experiments. It is insufficient in consciously distinguishing the physical differences between mass with deficit correction (inertial mass) and mass without mass deficit correction (possibly gravitational mass), even though Einstein knew the variation of rest mass caused by nuclear binding energy, described by the famous mass-energy equivalence equation $E = mc^2$. The following thought experiments are realistic enough to theoretically uncover its incompleteness and challenge the equivalence principle. Considering a deuterium atom ${}_1^2\mathrm{D}$ with mass $m_\mathrm{D} = 2.0141u$ and a hydrogen atom ${}_1^1\mathrm{H}$ with mass $m_\mathrm{H} = 1.0078u$ both falling in the gravity of the Earth, where u is the unified atomic mass unit, approximately 931.49 MeV/c^2. Assuming they fall together for a distance of h in downward direction and gain the same speed per equivalence principle regardless of the atomic composition. In nonrelativistic approximation, the sum of their kinetic energies will be $(m_\mathrm{D} + m_\mathrm{H})v^2/2$. In another scenario, the hydrogen and deuterium atoms go through a nuclear fusion ${}_1^2\mathrm{D} + {}_1^1\mathrm{H} \rightarrow {}_2^3\mathrm{He} + \gamma$ first to become a helium atom ${}_2^3\mathrm{He}$ with mass $m_{3\mathrm{He}} = 3.0160u$ before ${}_2^3\mathrm{He}$ falls the same distance of h and reach the same speed of v per the equivalence principle and gain a kinetic energy of $m_{3\mathrm{He}}v^2/2$. At the time of the fusion, ${}_2^3\mathrm{He}$ suffers a mass deficit of $\delta m = m_\mathrm{D} + m_\mathrm{H} - m_{3\mathrm{He}} \approx 5.469\ \mathrm{MeV}/c^2$, and the deficit is turned into a γ ray with energy of 5.469 MeV so that energy is conserved. This energy conservation for the fusion is expressed as $E_{D,H}^0 = (m_\mathrm{D} + m_\mathrm{H})c^2 = E_{3He,\gamma}^0 = (m_{3\mathrm{He}} + \delta m)c^2$ where δmc^2 is the 5.469 MeV energy of the γ ray. The difference of energies between the two scenarios is then $\Delta E^0 = E_{3He,\gamma}^0 - E_{D,H}^0 = 0$. During the falling process, summing the rest mass energy, gravitational potential energy, kinetic energy (in the nonrelativistic domain) and the blueshift/redshift energy of the γ ray when they all travel in gravity (as a side note, Einstein's blueshift/redshift are not supported by the grand unification theory which considers redshift/blueshift as mass related events rather than gravitational effects), the total energy for the two scenarios per equivalence principle would be $E_{D,H}^1 = (m_\mathrm{D} + m_\mathrm{H})c^2 - (m_\mathrm{D} + m_\mathrm{H})gh + (m_\mathrm{D} + m_\mathrm{H})v^2/2$ and $E_{3He,\gamma}^1 = m_{3\mathrm{He}}c^2 + \left(\delta mc^2 + \delta E_{\gamma shi}\right) - m_{3\mathrm{He}}gh + m_{3\mathrm{He}}v^2/2$ respectively because the equivalence principle specifies that ${}_1^2\mathrm{D}$, ${}_1^1\mathrm{H}$ and ${}_2^3\mathrm{He}$ will all fall at the same acceleration, so that the same h and v are used to describe all the atoms. In this sense for ${}_2^3\mathrm{He}$, equivalence principle indicates that gravity is proportional to ζm (which is corrected by nuclear mass deficit) because $m_{3\mathrm{He}} = \zeta(m_\mathrm{D} + m_\mathrm{H})$ with $\zeta = 1 - \delta m/(m_\mathrm{D} + m_\mathrm{H})$, but not for ${}_1^2\mathrm{D}$ and ${}_1^1\mathrm{H}$ who are considered "elementary". The equivalence principle doesn't seem to treat all particles equivalently and not as "equivalent" as it seems to be. Moreover, the equivalence principle will violate the energy conservation by comparing the two scenarios except at the beginning when both h and v are zero. Take the difference of energies between the two scenarios, $\Delta E^1 =$

$E^1_{3He,\gamma} - E^1_{D,H} = \delta E_{\gamma shi} \quad + (m_{\mathrm{D}} + m_{\mathrm{H}} - m_{\mathrm{3He}})gh - (m_{\mathrm{D}} + m_{\mathrm{H}} - m_{\mathrm{3He}})v^2/2 = \delta E_{\gamma shift} + \delta m(gh - v^2/2)$. Because for freefall, $gh - v^2/2$ is always zero, then to ensure conservation of energy for both scenarios, $\Delta E^1 = \delta E_{\gamma shift}$ must also remain zero. Unfortunately, this is an apparent paradox because Einstein's general relativity specifies that when the γ ray travels up or down the gravity, it loses (redshift) or gains (blueshift) energy accordingly. In other words, the γ ray cannot always travel perpendicularly to gravity because it will be bent by gravity and $\delta E_{\gamma shif}$ cannot maintain zero, so it is impossible to keep ΔE^1 zero and impossible to conserve energy. On the other hand, grand unification theory specifies that massless particles such as protons and gravitons do not interact with electric or gravitational fields because they have zero mass, they travel in straight lines in vacuum even in presence of fields, they can only interact with massive particles. Therefore, grand unification theory does not violate energy conservation. The above thought experiment clearly indicates that Einstein's theories are incomplete because they do not thoroughly consider the energy conservation because such theories are not derived from the perspective of energy. Another thought experiment points out that acceleration used in the equivalence principle is not a good physical concept to describe gravity, because it strongly depends on the speeds of the objects in relativistic domain. Consider a hypothetical compound particle consisted of two charged masses with everything identical except the opposite signs of their electric charges. The two masses are circling each other with near speed of light but the compound particle is at rest. When this compound particle is dropped in Earth's gravity, accelerating it will be very difficult because the individual masses inside are all near speed of light, meaning any minor additional velocity will increase the γ factor dramatically, therefore it takes more gravitational energy to make the compound particle fall at the equivalent speed compared to other simple masses. To avoid breaking special relativity, the equivalence principle implicitly requires the gravitational potential energy and the field equations to take into account the internal speeds of underlying subparticles as well as the overall speed, which means gravitational mass is proportional to γm. This thought experiment should be applicable to the previous thought experiment too because ${}^2_1\mathrm{D}$, ${}^1_1\mathrm{H}$ and ${}^3_2\mathrm{He}$ are compound particles and their electrons and nuclei have different orbiting speeds. Combining the two thought experiments, equivalence principle practically implies that gravitational mass is modified by mass deficit and relativistic effects, namely $\zeta\gamma m$. But all these thought experiments are not yet explicitly nor clearly addressed by present theories and the above implicates seem to contradict general relativity's notion of curved spacetime where gravity and acceleration are only the consequence of geodesic properties, irrelevant to the detailed properties of objects involved in gravitation interaction. The field equations seem to have no mechanism to derive such information in Einstein's general relativity theory but instead rely on the additional equivalence principle to fill in the theoretical incompleteness plus the coefficients of the field equations rely on a mathematical fitting to Newton's universal gravity equation which was developed more than three hundred years ago, not to mention that the field equations are difficult to reach general analytical solutions involving multiple masses without further significant simplifications and assumptions. On the contrary, the grand unification theory is much simpler and cleaner: gravity should simply be a force related to potential energy, gravity should be independent of ζ and γ, acceleration is the result of the conservation of energy just like electric force, no muss no fuss.

It would be truly incredible and embarrassing if experimental results show that gravitational mass is different from inertial mass, directly overthrowing the equivalence principle which has long been regarded as a major contribution and the foundation of general relativity theories with decades of supporting peer-reviewed papers.

4. <u>Experiment designed to examine the difference or equivalence between gravitational and inertial masses</u>

This paper describes a very simple experiment that will offer clear insights to answer the essential questions for Newton and Einstein: whether gravitational mass and inertial mass are different or equivalent. It aims to establish a solid theoretical foundation for future theoretical developments not only about gravity but also about the grand unification of physics. The theoretical grand unification framework has been briefly explained in previous sections, but it needs unambiguous experiments to either support or overhaul the framework. Cautions must be exercised to physically interpret the experimental results to avoid taking sides before thorough contemplations with clear and open-minded thought processes. To avoid unnecessary theoretical burdens and confusions, the experiment is described with minimal pertinent information leaving out unnecessary details to be filled in later papers.

The experiment is very simple, it precisely measures the oscillation periods of a simple pendulum using different types of materials and observe the periods as a function of the material's nuclear mass deficit. The pendulum oscillates to conserve its energy between gravitational potential energy and kinetic energy with the former tied to gravitational mass m, the latter tied to inertial mass m' at rest. This offers an excellent opportunity to experimentally investigate the difference between m and m'. Under low speed nonrelativistic approximation, the mechanical energy of the pendulum given by grand unification theory should be $E_{mech,grand} \approx -mgLcos\theta + m'\left(L\dot{\theta}\right)^2/2$ where L is the length of the pendulum, g is the gravitational acceleration constant, and θ is the swing angle of the pendulum. The time derivative of this equation gives $0 \approx mgLsin\theta\dot{\theta} + m'L^2\dot{\theta}\ddot{\theta}$. With small angle approximation of $sin\theta \approx \theta$, the equation becomes $0 \approx mg\theta + m'L\ddot{\theta}$ and the solution to the swing angle can be chosen to be $\theta \approx \theta_0 sin\left(t\sqrt{m/m'}\sqrt{g/L}\right)$ and the period of the pendulum is $T_{grand} \approx 2\pi\sqrt{m'/m}\sqrt{L/g} = 2\pi\sqrt{\zeta L/g}$. Notice that this result is only different from conventional pendulum period of $T_{conv} = 2\pi\sqrt{L/g}$ by a factor of $\sqrt{\zeta} = \sqrt{m'/m}$. This is the fundamental conceptual difference between the grand unification theory and conventional theory. In conventional theory (both Newton's gravity and Einstein's equivalence principle), $\sqrt{m'/m} = 1$ because there is no difference between the gravitational mass and inertial mass. But under grand unification, m' is affected by mass deficit while the gravitational mass m is not. The pendulum bob is made of many atoms of different kinds, each atom consists of a nucleus and typically multiple electrons. The largest mass deficit suffered by the nucleus is the nuclear binding energy, mostly on the order of a few MeV per nucleon (except for hydrogen atoms whose nuclear mass deficit is assumed to be zero) depending on the atomic isotope while the largest mass deficit suffered by electrons is the electric potential energy typically much less than a few hundred keV. The total mass deficit caused by nuclear mass deficit obviously dwarfs that by electric mass deficit, that is $m' = \zeta m \approx \left[1 - V_{nuclear\ binding\ energy}/(mc^2)\right]m$. It is therefore adequate to qualitatively evaluate the dependence of T_{grand} on atomic composition of the pendulum by solely considering the nuclear mass deficit for first-order approximations. Given that each nucleon has a rest mass of roughly 940 MeV and its nuclear binding energy is approximately a few MeV, $\zeta = m'/m$ will differ from 1 by a few thousandths. This difference is good enough to experimentally test the grand unification theory: conventional theory foresees no observable effect of $\zeta = m'/m$ and assumes the pendulum period $T_{conv} = 2\pi\sqrt{L/g}$ doesn't depend on atomic compositions, while the grand unification theory predicts that the pendulum period $T_{grand} = 2\pi\sqrt{\zeta L/g}$ depends on the atomic compositions because different atoms have different nuclear binding energies leading to different $\zeta = m'/m$.

In summary, the grand unification theory predicts that a pendulum's period depends on the atomic compositions of the bob because it unifies the behavior of gravity with those of other natural forces such as electric force and nuclear force. The unification theory predicts that gravitational mass is a charge-like physical quantity like electric charge and nuclear force charge, therefore not affected by nuclear mass deficit; meanwhile it is well known that inertial mass is affected by nuclear mass deficit on the order of a few thousands. So the grand unification theory directly disagrees with Einstein's equivalence principle where gravitational mass and inertial mass are presumed to be equivalent. Newtonian gravitational theory is effectively no different from the equivalent principle because it doesn't differentiate gravitational mass from inertial mass. The simple pendulum period is a perfect physical property to experimentally examine the grand unification theory because it is predicted to be proportional to the square root of the inertial-to-gravitational mass ratio, namely $\sqrt{\zeta} = \sqrt{m'/m}$.

5. <u>The theoretical prediction of periods of an actual pendulum based on the grand unification theory</u>

The actual pendulum uses a fixed length of $L \approx 5.622$ meters achieved by a thin cotton thread, fine adjustable by wrapping one end of it around a screw. The length is not exactly measured but inferred from the measured periods, and the focus of the measurements is not the absolute periods, but the difference in periods caused by the atomic compositions of the pendulum bob. The pendulum bob is made of a spherical metal shell with an external diameter of 32 mm. It consists of two identical 304 stainless-steel half shells, with a total weight of 26 grams. The atomic composition of the shells is approximately 18.5% Cr, 9% Ni and 72.5% Fe by weight. The mass deficits of these atoms are approximately 8.771, 8.748 and 8.786 MeV per nucleon, respectively. Ignoring higher order accuracies such as the electric mass deficits of electrons and assuming number of nucleons is proportional to weight, the average mass deficit is approximately $8.771 \times 18.5\% + 8.748 \times 9\% + 8.786 \times 72.5\% \approx 8.780$ MeV per nucleon for the steel shells ignoring the enclosed air which contributes negligible mass.

To demonstrate the effect of mass deficit on the period of this pendulum with a different atomic composition, another pendulum bob is constructed with the same type of steel shells but with their insides entirely filled with paraffin wax. The paraffin wax consists of C_nH_{2n+2} molecules with n ranging roughly from 17 to 35, roughly 33.3% C and 66.7% H by number of atoms. C has a mass deficit of approximately 7.678 MeV per nucleon while H has no nuclear mass deficit and electric binding energy is negligible. The average mass deficit of wax can be roughly estimated as $(7.678 \times 33.3\% \times 12 + 0 \times 66.7\% \times 1)/(33.3\% \times 12 + 66.7\% \times 1) \approx 6.580$ MeV per nucleon. After filling the bob with wax, its weight was found to increase from 26 grams to 39 grams, meaning that the bob is composed of 66.7% of 304 stainless steel and 33.3% of wax by weight. The total average mass deficit is then approximately $8.780 \times 66.7\% + 6.580 \times 33.3\% \approx 8.047$ MeV per nucleon.

The inertial-to-gravitational mass ratio $\zeta = m'/m = 1 + V/(mc^2)$ can be evaluated based on mass deficit values where V is the negative of mass deficit. Given that average mc^2 of nucleon is approximately 940 MeV, $\zeta_{empty} \approx 1 - 8.780/940 \approx 0.99066$ while $\zeta_{waxfilled} \approx 1 - 8.047/940 \approx 0.99144$ (the number of digits in decimals is extended to preserve the small variation from 1). Using the period derived with the grand unification theory $T_{grand} = 2\pi\sqrt{\zeta L/g}$, the pendulum period is $T_{empty} = 4.7367$ and $T_{waxfilled} = 4.7386$ seconds for empty bob and bob filled with wax, respectively with $L \approx 5.622$ meters and $g = 9.8$ m/s^2.

The grand unification theory predicts that the empty bob should have slightly shorter period than that filled with wax, that is $T_{empty}/T_{waxfilled} = \sqrt{\zeta_{empty}/\zeta_{waxfilled}} \approx \sqrt{0.99066/0.99144} \approx 0.99961$. The reason is that the empty bob consists of metal atoms that suffer a relatively larger mass deficit on average leading to relatively less inertial mass at rest, compared to that of the bob filled with wax where hydrogen and carbon atoms experience less mass deficits leading to relatively more inertial mass at rest. For unit gravitational mass in the pendulum bob, namely the same amount of gravity, empty bob has relatively smaller inertial mass, therefore the empty bob swings slightly faster than the bob filled with wax for the same pendulum path. This prediction is fundamentally different from Newtonian gravity and Einstein's equivalent principle assuming that the pendulum should swing at the same period regardless of the atomic compositions or nuclear mass deficits.

As a side note, the 304 stainless steel is non-magnetic, the magnetic field effect is not a significant concern. Even if there is some residual magnetism, the two bobs use identical steel shells, the effect should be approximately equal to both bobs and will not significantly affect the predicted difference.

6. Experimental considerations for the precise measurement of the periods

Because the predicted periods of the pendulum with empty and wax-filled bobs are very close (4.7367 and 4.7386 seconds respectively), with a difference of only 1.9 milliseconds or 0.039%, tremendous considerations must be exercised to obtain credible results. Notice that it is not necessary to accurately measure the absolute pendulum periods, but the periods must be precise better than 0.01% to reliably resolve the 0.039% difference. A microcontroller unit (MCU) running at 80 MHz on a quartz crystal oscillator is used for the time keeping, it can easily achieve short term stability (better than 20 ppm) and resolution way better than needed. When the bob swings, it blocks and unblocks a thin laser beam. A photodiode picks up the status of the laser beam caused by the passing bob with millisecond precision, and the MCU records the timing of the change in status and sends out the information for analysis. By averaging over a few periods, the measured period can reach the required precision. Equation $T_{grand} = 2\pi\sqrt{\zeta L/g}$ indicates that L must remain consistent better than 0.02% to avoid introducing uncertainties greater than 0.01%. With $L \approx 5.622$ meters, it requires an uncertainty of less than 1 mm (millimeter). The uncertainty of L can come from the variation of the center of mass due to uneven mass distribution in the shells and the wax, and also from the length of the thread attached to the bob. The shells are mass produced, their uncertainty is likely no greater than 0.1 mm, but the assembly of the shells can cause another 0.1 mm. The wax distribution is unlikely to cause more than 0.2 mm. To fill the wax, two shells were submerged in melted wax and cooled together for solidification, excess wax was then shaved off with a sharp straight blade. The geometric length of the pendulum is monitored by the laser beam reflected off the bob. Because the bob has a radius of 16 mm, when the length of the pendulum varies, the laser beam reflected off the bob will change direction and can be easily detected by observing the laser dot projected over a moderate distance of a few centimeters. The length of the cotton thread can be adjusted to eliminate the variation. By purposely adjusting the thread length back and forth, a precision of less than 1 mm is found to be easily achievable by checking the pendulum period. Another uncertainty may come from g, which can vary over time considering the influence of the moon and the Sun which overtime can also cause a variation of a few thousandths. It is therefore important to finish necessary measurements as quickly as possible. Practicing the measurements many times can benefit setup efficiency and help promptly identify and resolve issues that may potentially compromise the quality of collected data.

7. Actual measurement data favoring the prediction of grand unified theory

After the pendulum was started, the time of the bob entering and exiting the laser beam was recorded in text files in 0.1 ms resolution. The period can be calculated by computing the intervals between the repetitive entering time or exiting time detected by the laser beam. Notice that the periods based on the entering time is a few milliseconds shorter than that based on exiting time, the reported periods are the average of periods calculated from neighboring entering and exiting time. In addition, to avoid the dependence of period on pendulum swing angles, the periods are only calculated right after the bob crossed the laser beam in approximately 20 ms (a speed of approximately 32 mm/20 ms at the bottom of the swing). For the empty bob, the pendulum was started six times, the measured periods were 4.7381, 4.7364, 4.7398, 4.7374, 4.7365 and 4.7350 seconds averaged over two full swings, and 4.7370, 4.7374, 4.7377, 4.7393, 4.7374 and 4.7375 seconds averaged over four periods. The measurements started around 11:37 AM and finished around 11:56 AM. For the bob filled with wax, the pendulum was started five times, the measured periods were 4.7381, 4.7388, 4.7389, 4.7388 and 4.7396 seconds averaged over two full swings, and 4.7383, 4.7388, 4.7390, 4.7387 and 4.7390 seconds averaged over four full swings. The measurements started around 12:18 AM and finished around 12:35 AM on the same day.

To summarize, the results are $T_{empty,2} = 4.7372 \pm 0.0016$ seconds averaged over two full swings and $T_{empty,4} = 4.7377 \pm 0.0008$ seconds averaged over four full swings; $T_{waxfilled,2} = 4.7388 \pm 0.0005$ seconds averaged over two full swings and $T_{waxfilled,4} = 4.7388 \pm 0.0003$ seconds averaged over four full swings. The results show that measured pendulum period for empty bob is shorter than that for bob filled with wax by approximately 1.6 ms if measured over two full swings and 1.1 ms if measured over four full swings. The results qualitatively agree with the prediction of the grand unification theory in which the period for the empty bob should be 1.9 ms shorter than that of the bob filled with wax. Meanwhile, $T_{empty,2}/T_{waxfilled,2} \approx 0.99966 \pm 0.00035$ and $T_{empty,4}/T_{waxfilled,4} \approx 0.99977 \pm 0.00018$ are close to the theoretical value of $T_{empty}/T_{waxfilled} \approx 0.99961$.

Although the experimental results are very encouraging in qualitatively proving the prediction of the grand unification theory, cautions must be given to the reliability of the results. It can be found that the standard deviation of the period measured over two full swings is 1.6 ms (or 340 ppm) for the empty bob, comparable to the targeted theoretical difference between the grand unification theory and the conventional theories.

Although it drops to 0.8 ms (or 170 ppm) when measured over 4 full swings, it still indicates that this measurement is not as precise as desired, making the credibility of the conclusion marginal. The bob filled with wax on the other hand showed much lower standard deviation down to 0.3 ms (or 60 ppm). The light weight empty bob is arguably more suspectable to uncertainties such as air drag and imperfect coplanar swing, and the consequent faster oscillation decay may also contribute to the uncertainties in timing. But in general, more air drag tends to increase the pendulum period, which might also explain the slightly higher $T_{empty,2}/T_{waxfilled,2}$ and $T_{empty,4}/T_{waxfilled,4}$ than the theoretical $T_{empty}/T_{waxfilled}$. The air drag effect gives more favor to the grand unification theory because even with more slowdown by air drag, the empty pendulum still exhibits shorter period, meaning that without air drag, the empty pendulum will have even shorter period than that filled with wax, further approaching the theoretical expectation.

In conclusion, the experimental results favor the prediction of the grand unification theory and show that there does exist appreciable difference between gravitational mass and inertial mass caused by nuclear mass deficit. The results unmistakably necessitate further examinations of Newtonian gravitational theory where gravitational mass and inertial mass are indistinguishable and Einstein's equivalence principle that gravitational mass and inertial mass are equivalent.

8. <u>Theoretical contemplations of further experimental examinations of the grand unification theory</u>

It is likely that existing gravitational theories including Newtonian gravity and Einstein's general relativity all miss the physical perceptions of gravitational mass and inertial mass, therefore these theories will be unable to appreciate the differences caused by these two distinctively disparate types of masses from the perspective of the grand unification theory. Because of this, there has been no theoretical guidance nor awareness to correct for the effects of atomic composition on gravitational interactions. For example, modern attempts to measure the accurate gravitational constant *G* have always shown variations among them by a fraction of a thousandth no matter how precise they are. In 2018, a Chinese institute published in Nature their new gravitational constant measurements using two different methods (TOS and AAF, both with fused silica torque pendulums), all with the highest precisions ever achieved on the order of 10 ppm. Yet still they found a definite difference of 45 ppm between the two methods. Furthermore, their *G* results are different from recent results given by credible institutes such as NIST-82 (TOS with Pyrex glass plate), BIPM-14 and HUST-05 by much larger magnitude (more than 400 ppm) and all beyond their respective standard deviations. They ultimately suggested that sources of error unaccounted for must have caused such variations. If grand unification theory is true about gravity, it can help identify some sources of variations. For example, Pyrex glass is composed of 4% boron, 54% oxygen, 2.8% sodium, 1.1% aluminum, 37% silicon and 0.3% potassium by weight, the average mass deficit is approximately 8.116 MeV per nucleon, giving $\zeta_{pyrex} \approx 0.99137$. The fused silica is mostly SiO_2, the average mass deficit is approximately 8.196

MeV per nucleon, giving $\zeta_{silica} \approx 0.99128$. If these *G* measurements involve moment of inertia, which is proportional to inertial mass at rest ζm, the difference between ζ_{pyrex} and ζ_{silica} alone can give an error on the order of 90 ppm, with the total difference depending other factors. In addition, different institutes used different source masses such as iron, tungsten and copper to generate gravitational effects for measurements. Their respective atomic mass deficits are 8.786, 8.004 and 8.754 MeV per nucleon, corresponding to ζ of approximately 0.99065, 0.99149 and 0.99069. The different ζ values can lead to a variation of *G* measured with these materials on the order of 840 ppm depending on how the differences propagate to the final results. The actual spread of published *G* values by different institutes on the order of 500 ppm does qualitatively agree with the variation of ζ. As of the 45 ppm difference reported by the Chinese institute, the TOS method used Al-coated silica block as the pendulum body while the AAF method used gold-coated fused silica block for the pendulum body. It is unclear whether the silica blocks are using exactly the same atomic composition, but the different metal coatings might inadvertently introduce a very small amount of systematic difference because they have a mass deficit of 8.332 MeV for aluminum ($\zeta_{Al} \approx 0.99114$) and 7.916 MeV for gold ($\zeta_{Au} \approx 0.99158$) per nucleon, a difference of 440 pm. The tiny amount of coating may contribute part of the difference to the very precise *G* results.

It must be emphasized that there were past experiments to examine the proportionality of mass and weight or to detect the differences between gravitational mass (related to weight or conventionally measured mass) and inertial mass (related to motion). In essence, they were also testing the minute difference between inertial mass and gravitational mass or examining the concept of equivalence principles. However, the very miniature effects on the order of a thousandth could easily be overwhelmed by various uncertainties or overlooked as other uncertainties or eliminated by calibration processes because there existed no quantitative expectations nor theoretical guidance to focus on the essential physics and minimize irrelevant factors. For example, C F Brush in the 1920s observed a difference in periods between pendulums made of bismuth and zinc, and for a given mass, bismuth (with 7.848 MeV mass deficit) appears to weigh more than zinc (with 8.746 MeV mass deficit). However, his observations were ultimately explained by a lack of inclusion of the buoyancy of air in his calculations. Without going through the complicated experimental details and his definitions of mass and weight, his observations of differences are interesting but are qualitatively the opposite of the predictions made by grand unification.
Much earlier, Newton took experiments and claimed that all materials he found had the same gravitational acceleration within in one part in a thousand. This is understandable because inertial mass at rest and gravitational mass typically differ by a fraction of a thousandth for common objects, and measurements at Newton's time were probably not precise enough to resolve the difference. Later in 1827, comparing with a same standard clock, F W Bessel measured the periods of two pendulums with two different but fixed

lengths using various bob materials and meticulously calculated the theoretical lengths needed for a simple pendulum to generate 1 second period. He concluded that except for water, the gravitational acceleration was the same for all substances he tried, precise to one part in 60,000 (~20 ppm). From the point of grand unification, water has significantly smaller average mass deficit, which is approximately $(0 \times 2 + 7.976 \times 16)/(2 + 16) \approx 7.090$ MeV per nucleon corresponding to $\zeta_{water} \approx 0.99246$. Water has uniquely larger inertial-to-gravitational mass ratio compared to ordinary materials (for example with mass deficit of 8.5 MeV per nucleon and $\zeta \approx 0.99096$). To make a 1 second simple pendulum using water filled bob, its length must be noticeably shorter than other ordinary materials. Bessel's observation sure qualitatively agrees with the result shown in this paper using wax, but he tried hard to explain the "anomaly" attributing it to the speed of water acquired relative to the pendulum container during the oscillation. He wouldn't have an answer if the grand unification theory is truly the reason behind it, but he might have tried ice. Not sure why Bessel's observation doesn't include wax because wax would have exhibited more pronounced "anomaly" because its average mass deficit is 6.580 MeV per nucleon, even smaller than that of water. H H Potter published in 1923 that he found no difference of the pendulum periods using different bob materials including brass, lead, wax, steel and bismuth. The reported precision of better than 10 ppm (while admitting the accurate period measurements were the most difficult part of his experiments) seems doubtful and requires further error analyses. Most importantly, the equivalence principle appears to be strongly supported by the Eötvös' torsion balance method, with accuracy up to 1 part in 10^{15} as reported by CNES using a satellite-based setup (MICROSCOPE satellite mission). This is probably considered as the most definitive experimental support that gravitational mass and inertial mass are equivalent. However, it must be emphasized here that contrary to the common belief, Eötvös' torsion balance experiments carried on the Earth are not supposed to sensitively distinguish the difference between gravitational mass and inertial mass if it is not carefully designed. The reason is that in order to set up the balance, its mechanical parameters must be configured or calibrated to start untwisted by neutralizing the differential torque caused by the possible inequivalence effects due to different centrifugal forces as a result of its spatial movement along with the Earth. After that, there is no additional differential centrifugal force to break the balance unless either the gravity changes in strength or the Earth's rotation or orbital motion is changed, or the centrifugal force is changed by varying the speed or location of the balance in space. When the balance is moved to a different location, the grand unification theory predicts that because different atomic compositions with different mass deficits produce different inertial masses, the change in centrifugal forces will be different accordingly and can potentially break the balance, but only if the differential forces create a net torque which is not always true depending on how the toque balance is calibrated. Such effect was indeed observed when the experiments were carried out on moving ships but was instead explained as the motion of the ships in relation to the Earth. The grand unification theory indicates that the change of the centrifugal force should be on the order of a thousandth of the change in centrifugal force, making it an observable but weak effect. As of the MICROSCOPE satellite measurement, there does exist obvious observable differential centrifugal acceleration. But such acceleration is presently explained as the result of gravity gradient effect and was modeled as the Earth's gravitational variation over an off-center distance of 20 μm between the paired test masses at an orbital altitude of approximately 710 kilometers, or as the effect of the mass oscillation when the mass spins relative to the Earth. This explanation has the potential to overlook the possible inequivalence signal embedded in the supposed "gravity gradient effect" or "mass oscillation", while the expected Eötvös effect is truly absent due to the absence of additional change in centrifugal force in the controlled orbit. Moreover, the published results did not address the much larger gravity gradient effect and centrifugal effect: off-center distance between the satellite's spinning center and the test masses' centers (probably in the millimeter range if not centimeter range) are much greater than the estimated 20 μm off-center distance between the paired test masses. When the satellite spins, it will cause much stronger signal compared to the published signal levels. Beyond all the above, there is another potentially more deleterious issue about the satellite's acceleration and attitude control system (AACS) intended for achieving a drag-free orbit around the Earth. The AACS allegedly used the signal from the test masses and adjusted eight micro-thrusters to compensate for the "external perturbations". This action could inadvertently cancel out all the useful signals corresponding to the inequivalence effects between inertial mass and gravitational mass and falsely report an equivalence result which was the consequence of an orbital compensation. The following analysis gives the expected signal based on the grand unified theory. The two paired test masses in SUREF are both Pt/Rh (90/10) alloys with an average mass deficit of 7.994 MeV per nucleon and $\zeta_{\mathrm{Pt/Rh}} \approx 0.99150$, the two paired test masses in SUEP are respectively Pt/Rh (90/10) alloy and Ti/Al/V (90/6/4) alloy with the latter having an average mass deficit of 8.692 MeV per nucleon and $\zeta_{\mathrm{Ti/Al/V}} \approx 0.99075$. These different ζ values can result in a differential acceleration on the order of 0.00075 of the local gravy between the two SUEP test masses while the SUREF test masses should experience no differential acceleration due to identical atomic composition. It must be pointed out that both SUREF and SUEP should have differential accelerations against the overall satellite due to mismatching ζ values as predicted by the grand unification theory, and the tethering 7 μm gold wires can also introduce reactive forces to reduce the measurable differential accelerations. It is unfortunate that such complicatedly designed MICROSCOPE satellite didn't take advantage of the much easier approach to investigate the equivalence principle by observing the possible separate orbits (or world lines) of test masses with different nuclear mass deficits.

The review of past experiments has found many inconsistent and questionable results. Given the theoretical importance of these experiments to the ultimate unification of gravitational interaction, it is crucial to design better experiments aiming at minimization of uncertainties to reveal the possible tiny difference between gravitational mass and inertial mass. Test materials must include those with maximum versus minimum mass deficits such as iron/nickel/copper versus hydrogen/lithium/beryllium/boron in order to increase observability. The following experiments can be considered. (a) Observation of the orbital drifting behaviors of different materials in a space station. Without considering residual atmospheric air drag and active orbital maintenance, the space station's orbit around the Earth is affected by its inertial-to-gravitational mass ratio due to its overall mass deficit, dominated by the metal structures made of aluminum, steel and titanium, which all have relatively high mass deficits greater than 8.3 MeV per nucleon. This means that any objects with smaller mass deficits will experience minute extra centrifugal forces away from Earth if they move along with the space station. Lithium, wax, boron, water and lead with mass deficits of 5.585, 6.580, 6.838, 7.090 and 7.870 MeV per nucleon, respectively all have higher inertial-to-gravitational mass ratios than that of the space station. If they are forced to fly along with the space station, the motion will generate weak but appreciable extra centrifugal forces due to higher proportions of inertial mass, making these objects tend to float away from Earth relative to the space station. Interestingly, 96% of human body is made up of oxygen, carbon, hydrogen and nitrogen, all with mass deficits less than 8.0 MeV per nucleon, therefore astronauts inside the space station also tend to float away from the Earth. But as calculated previously, such tendency is on the order of a thousandth of local gravity, therefore the effect can be easily overwhelmed by the initial motion, air draft or even magnetic and electrostatic forces. In order to appreciate such effects, such observations should preferably be performed in a vacuum chamber while the space station is following a true freefall obit. All observed objects should be non-magnetic and coated with metal and discharged to avoid electromagnetic interference. Objects such as lithium, lead and copper balls should be adequate to represent small, medium and large mass deficits and should be released as still as possible relative to the space station. It is expected that lithium and lead balls will float away from Earth while copper ball will float toward Earth with lithium experiencing the most obvious tendency due to its exceptionally small mass deficit. This type of observation has the highest qualitative resolution in that the orbiting environment cancels out majority of the gravitational effects and reveals the most minuscule difference with long observable time. A more ideal setting would be a satellite orbiting in high altitude to reduce interferences from Earth's magnetic field and residual atmosphere. (b) Quantitative measurements of free fall accelerations preferably in a vacuum tube using different materials. In this setting, the Earth acts as a fixed attracting mass, the falling objects' free fall accelerations will be affected by their inertial-to-gravitational mass ratio as explained in previous discussions. For light atoms such as lithium and hydrogen atoms, the low mass deficits leave behind relatively larger portions of inertial masses for the same amount of gravitational mass, making the accelerations slightly slower. From the point of the grand unification theory, the energy conservation based on the extended mass-energy equivalence is quantitatively $E^2 =$

$(\gamma\zeta mc^2)^2 = \left(mc^2 + V_{nuclear\ binding} - GMm/r\right)^2/(1 - \dot{r}^2/c^2) \approx (m'c^2 - GMm/r)^2/(1 + \dot{r}^2/c^2)$ where M is the mass of the Earth, m is the gravitational mass of the falling object and $m' = \zeta m = m\left[1 + V_{nuclear\ binding}/(mc^2)\right]$ is its rest mass after deficit correction by nuclear binding energy – the inertial mass in the conventional perception. The distance of the object from the Earth center is *r*, and can be expressed as $r = R + h$ where R is the experimental location from the Earth center and *h* is the altitude from R. Plugging in *r* and assuming $R \gg h$ and $\dot{r}^2/c^2 \ll 1$, the positive root of E^2 is approximately $E \approx (m'c^2 - GMm/R + GMmh/R^2)\left(1 + \dot{h}^2/c^2/2\right)$, where the falling speed $\dot{h}$ varies with *h* as constrained by energy conservation. The time derivative of E then gives $0 = \dot{E} \approx GMm\dot{h}\left(1 + \dot{h}^2/c^2/2\right)/R^2 + (m'c^2 - GMm/R + GMmh/R^2)\dot{h}\ddot{h}/c^2$. Because $\dot{h}^2/c^2 \ll 1$, $GMm/R \ll m'c^2$ and $GMmh/R^2 \ll m'c^2$, they can be dropped out of the approximation to get $\ddot{h} \approx -(m/m')GM/R^2 = -(m/m')g$ where $g = GM/R^2$ is the gravitational acceleration constant and $\ddot{h}$ is the acceleration of the object. The purpose of the approximations is to simplify the freefall analysis in nonrelativistic (low speed) domain to emphasize the primary physical implications while avoiding unnecessary distractions. Obviously the acceleration $\ddot{h}$ predicted by the grand unification differs from the conventional (Newtonian) acceleration *g* by a factor of m/m'. Since $m' = m\left[1 + V_{nuclear\ binding}/(mc^2)\right]$ where $V_{nuclear\ binding}$ is negative, stronger nuclear binding energy leads to larger m/m', therefore faster falling acceleration. Lithium, wax, boron, water and lead should fall slower than iron, nickel and tin. This prediction uniquely differentiates the grand unification theory from the conventional theories by Galileo, Newton and Einstein because freefall acceleration is no longer the same for different atomic compositions – the phenomena that are not foreseen by Newtonian theories and specifically denied by the equivalence principle. The grand unification theory derives the motion of an object not through force, gravitational field, geodesics, or warped spacetime or field equations, but rather through the simple conservation of energy E^2 or *E*, meaning that the grand unification theory itself is capable enough to process the physical differences and consequences of different masses without relying on additional principles. The freefall experiment in vacuum represents the cleanest experimental setting because all interfering factors can be eliminated by vacuum while high-speed high-accuracy cameras can capture the miniature difference in the falling speed by continuously recording the entire process. Care must be taken to discharge the vacuum tube as well as the falling objects and shield the setup from electromagnetic fields to minimize uncertainties. The grand unification treats any natural forces equally as sources of potential energy and mass deficit, the above analysis can be extended to highly relativistic (in terms of the magnitude of potential energy compared to rest mass energy) electric or gravitational falling events. This includes astronomical freefalls where the gravitational mass deficit is comparable to or even greater than nuclear mass deficit. In this case, the inertial mass at rest is $m' = \zeta m = m[1 +$

$\left(V_{nuclear\ binding} - GMm/r\right)/(mc^2)\big]$, it diminishes further and further from gravitational mass as ζ deviates more and more from 1^- to 0 due to more negative $-GM/r/c^2$. Gravitational binding energy eventually dominates the mass deficit and brings the inertial mass close to zero. Due to the conservation of $E^2 = (\gamma\zeta mc^2)^2$, γ must be increased drastically approaching the speed of light in order to compensate for the loss of ζ, the falling object will behave more and more like massless particles such as photons and gravitons. It means that if the object is orbiting instead of free falling, its orbit should be exceptionally faster at lightning speed compared to Newtonian orbits. Astronomers who are unaware of this grand unification effect would have to come up with much larger theoretical masses to explain the observation of these relativistic orbits. (c) A less drastic alternative to freefall is the slide down a low friction slope. The fast process of freefall is difficult to measure precisely without costly instrumentation. Unlike the pendulum which can have slow periods and repetitive oscillations, the onetime freefall typically takes less than a second to finish, limiting the observable difference in falling time. The release of objects from rest is difficult to synchronize precisely, even minute uncertainties in initial speed and location at release can overwhelm the targeted tiny time differences between different materials. Although high-speed video cameras can eliminate these uncertainties by taking and analyzing submillisecond images of the entire process, it could be costly, and protective mechanism is always required to avoid damages to test objects and setups. The sliding motion is much less violent and more controllable, allowing precise time measurements, and the sliding mass can be made relatively large to minimize the uncertainties caused by friction and air drag. Objects with different atomic compositions with precisely the same weight (gravitational mass) can be placed inside the same sliding container and slid down the same slope to reduce the variation of uncertainties. The densities of test masses must be close in order to avoid the effect of air mass. (d) There is another completely different method to measure the inertial-to-gravitational mass ratio. It involves the evaluation of gravitational mass and inertial mass in two separate steps. The first step evaluates the gravitational mass m using a high precision scale or balance. The mass evaluated this way is quantified by the effect of gravity exerted on the mass, therefore is gravitational mass m. The second step uses a simple spring harmonic oscillator to evaluate the inertial mass m' by precisely measuring the period of oscillation T using a highspeed video camera. When the effect of the mass of the spring is neglected, $m' \approx kT^2/(2\pi)^2$ where k is the spring constant. Because the oscillator conserves energy between the kinetic energy of the mass and the potential energy of the spring, the oscillation period is therefore related to the inertial mass m'. By comparing the $\zeta = m'/m$ ratios measured with different atomic compositions, the inequivalence or equivalence between inertial mass and gravitational mass can be experimentally determined. To reduce the uncertainties involved in the experiment, samples of different atomic compositions can be made with the same gravitational mass and secured inside the same container for evaluations. Experimental design must carefully evaluate the appropriate density and amount of test mass, the effects of the spring due to its mass and nonlinearity, the air drag and the density of air. (e) Probably the most pertinent evaluation of inertial-to-gravitational mass ratio can be carried out by existing G factor measurement setups. Without changing the torque pendulum, spherical mass sources made of different atoms can be used, the precisely measured G factors can be compared with the theoretical values of $\zeta = m'/m \approx 1 + V_{nuclear\ binding}/(mc^2)$ where the nuclear binding energy is negative. (f) To further differentiate the grand unification theory from the equivalence principle, it is necessary to study the effect of kinetic energy on gravity because kinetic energy will increase the relativistic inertial mass. The gravitational effect can be detected by either an atomic clock or a torque balance. Direct measurement of such effect macroscopically in laboratory is impossible because reasonably achievable gravitational variation by temperature is on the order of 10^{-41} per kelvin. In comparison, most recent atomic and optical lattice clocks can reach a precision of 10^{-16} and can maintain a fractional uncertainty of approximately 10^{-19} after 3.3 hours of averaging, enough to resolve the fluctuation of gravity in laboratory due to a height variation of dr on the order of gdr/c^2 where g is the gravitational acceleration at sea level. The grand unification indicates that when the electrons in the atomic clocks vary by dr in altitude, their gravitational potential energies vary by $dV_{gravity} = mgdr$, and such potential energy variations are translated to fractional inertial mass variation of $dm'/m = dV_{gravity}/(mc^2) \approx mgdr/(mc^2) = gdr/c^2$ given the masses of electrons are much smaller than the mass of Earth. That is a gravitational variation of approximately 10^{-16}/meter, well within atomic clocks' resolutions. But such high precision is still inadequate to resolve the supposed temperature-induced gravitational effect implied by equivalence principle. There are two reasons for the non-detectability: the first is that the variation of relativistic inertial mass by temperature is weak, on the order of 10^{-14} per kelvin, the second is that the amount of mass that can have controlled temperature variation is limited, making the gravitational time-dilation effect very weak on the order of 10^{-27} to begin with. When a mass rise in temperature, the average kinetic energies of consisting atoms will increase, and their relativistic inertial masses will increase via mass-energy equivalence. For equivalence principle, their gravitational masses supposedly increase equivalently thereby increasing the gravitational pull to the atoms in a nearby atomic clock and slow it down due to gravitational time dilation or redshift. In contrast, for the grand unification theory, such variation of relativistic inertial mass will not change the gravitational potential energy because potential energy is not affected by thermal vibrations or relativistic inertial masses of atoms, the clock frequency should remain unchanged. The kinetic energies of the atoms inside the mass are $E_{kinetic} = m\langle v^2\rangle/2 = 3k_BT/2$ where k_B is the Boltzmann's constant. Such kinetic energies vary the overall relativistic inertial mass by a factor of $3k_BT/2/(mc^2)$, which is on the order of 10^{-14} per kelvin for light atoms. Unfortunately, the amount of mass M, its distance to the atomic clock r and the range of temperature variation are very limited, the total expected gravitational effect on variation of the clock frequency is $V_{gravity}/(mc^2) \cdot E_{kinetic}/(mc^2) = -(GM/r/c^2) \cdot 3k_B dT/2/(mc^2) \sim 10^{-41} dT$. For sure it is undetectable by any atomic clock or torque balance. But this investigation may be achieved by charged particles circling in a strong magnetic field at high relativistic speeds which can vary relativistic inertial masses by many folds. The accelerations and deaccelerations of such circling particles in gravity can be studied to identify whether the gravitational pull is proportional to original masses at rest or to the relativistic inertial masses. The grand unification theory predicts that such particles might rise and fall slower in gravity due to the relativistic circling speeds, the "might" condition is reserved for the possibility that these particles might have experienced reductions of inertial masses due to various mass deficits such as nuclear, electromagnetic and gravitational mass deficits. Grand unification theory has a very clear vision about potential energies and movements. Any potential energies including gravitational energy are conservative, they only depend on the amounts of involved charges (electric charges for electric forces, gravitational masses for gravitational forces and so on) and their geometries. Charges (including gravitational masses) are not affected by motions or potential energies themselves. Potential energies do change the inertial masses at rest through factors ζ just like kinetic energies change the relativistic inertial masses through factors γ. Motions are the combined consequences of both ζ factors representing the mass deficits that reduce inertial masses at rest and γ factors representing the relativistic kinetic effects that increase relativistic inertial masses. Potential energies and movements are interacting coherently through the conservation of energy concisely expressed as the extended mass-energy equivalence $E^2 = (\gamma\zeta mc^2)^2$. For the equivalence principle, such circling particles will rise and fall like regular objects because the motions of the particles in gravity are viewed as the geodesic properties of the warped spacetime regardless of the details of the particles, therefore their gravitational masses have to increase when the relativistic inertial masses increase due to the circling motions. By doing so, the equivalence principle makes the gravitational potential energies dependent on the speeds of particles, which effectively makes potential energy nonconservative. (g) Most of the above experiments involve large amounts of atoms and only observe the general gravitational effects due to the average mass deficit caused by overall nuclear binding energy. For a well-rounded theoretical framework, it is absolutely necessary to study the gravitational behavior of individual atoms and individual elementary particles instead of multiple atoms as a whole. For example, hydrogen isotopes from ^{1}H to ^{3}H have mass deficits ranging from 0 to 2.6 MeV per nucleon, while helium isotopes ^{3}He and ^{4}He have mass deficits of 2.2 and 6.8 MeV per nucleon respectively. These are all good candidates to study the mass deficit effects on gravitational behavior in microscopic domain. Moreover, elementary particles such as electron, proton, neutron and their antimatter counterparts also need to be studied. Given the recent technological advancements in penning traps, laser cooling and ion accelerators, precision magnetic fields can confine these ionized atoms in circular motions and allow them to drift up and down under the influence of gravity or to orbit in a fixed magnetic field at different altitudes while the timing difference can be resolved up to 1 part in 10^{16} with modern atomic clocks. Such high precision measurements not only can qualitatively resolve the difference between gravitational and inertial masses, but more importantly can also quantitatively evaluate the inertial mass at different altitudes. For example, the grand unification theory predicts that inertial mass at rest is $m' = \zeta m = m\left[1 + \left(V_{nuclear\ binding}/m - GM/r\right)/c^2\right]$, it varies when the mass is placed at different altitudes. When *r* varies by a distance of $dr = 10^3$ meters from sea level where $r_{sea\ level} = 6.378 \times 10^6$ meters, the inertial mass varies by a factor of $dm'/m \approx \left(GM/r_{sea\ level}^2/c^2\right)dr = gdr/c^2 \approx 1.1 \times 10^{-13}$ where *g* is the sea level gravitational acceleration constant. It requires a resolution better than 1 part in 10^{13} to qualitatively detect effects related to inertial mass such as the circling period of the charged mass in a fixed magnetic field, which is well within the accuracies of modern atomic clocks and optical frequency combs. Even better, if such measurement is performed far from the Earth, for example, in the geosynchronous equatorial orbit of $r = 3.58 \times 10^7$ meters, the inertial mass will be larger than that on Earth by a factor of 5.9×10^{-10}. Measurements of this difference offer much higher level of confidence and details. The importance of such space experiments can never be overstated because it can accomplish two fundamental theoretical inquisitions at the same time: ① will gravitational energy cause mass deficit like the nuclear binding energy? ② if the first inquisition is positive, then does the tested particle already have other intrinsic binding energy on top of the known nuclear binding energy? The inertial mass at rest taking other intrinsic binding energy into account should be $m' = \zeta m = m\left[1 + \left(V_{nuclear\ binding}/m + V_{intrinsic\ binding\ energy}/m - GM/r\right)/c^2\right]$. By varying *r* and observing physical quantities related to inertial mass m', it is possible to reveal the presence of $V_{intrinsic\ binding\ energy}/m$ which could be the binding energy of more fundamental particles that make up the test particle. For example, proton is supposed have zero nuclear binding energy because proton is all by itself. It is absolutely necessary to experimentally confirm if this is true or false. Of course, it might be very difficult to find possible intrinsic binding energy within the solar system because GM/r is very weak and narrowly ranged for realistic *r* and *M*, but in cosmic domain, such observations are possible because GM/r can have much greater range, and the observations of elementary particles under the influence of gravity might provide useful information to identify intrinsic binding energies. Electric potential energy is a much better choice in laboratory as it is much stronger than gravity. Electric potential energy can be quantized as $V_{electric} = c\hbar n_1 n_2 \alpha/r$ where $n_1 n_2$ is the product of numbers of involved unit charges, $\hbar$ is the reduced Planck constant and α is the fine structure constant. The more considerate inertial mass at rest thereby becomes $m' = \zeta m = m\left[1 + \left(V_{nuclear\ binding}/m + V_{intrinsic\ binding\ energy}/m + c\hbar n_1 n_2 \alpha/r/m - GM/r\right)/c^2\right]$. Theoretically, the second inquisition is related to the investigation of quantized gravitational masses (or possibly gravitational charges) carried by elementary particles. In other words, the gravitational potential energy can be quantized similarly to electric potential energy as $V_{gravity} = c\hbar m_1 m_2 \beta / r$ where $m_1 m_2$ is the product of the quantized gravitational masses (or gravitational charges) involved in the gravitational potential energy and β is the large structure constant for gravity parallel to the fine structure constant for electric force. The large structure constant β and unit gravitational masses need to be determined through extensive existing and future experiments. The inertial mass at rest can be then rewritten as $m_1' = \zeta m_1 = m_1[1 + (V_{nuclear\ binding} + V_{intrinsic\ binding\ energy} + c\hbar n_1 n_2 \alpha / r + c\hbar m_1 m_2 \beta / r) / m_1 c^2\ ]$. It is right now unclear if $V_{nuclear\ binding}$ and $V_{intrinsic\ binding\ energy}$ can be handled similarly, but it is definitely a possibility as $V_{nuclear\ binding}$ is found to be related to electric potential energy in the case of free neutron modeling. Further observation of this equation indicates that theoretically all the potential energies can possibly be quantized with corresponding charges, but such quantization will be incompatible with gauge theories where potential energy can have a constant offset without physical consequences. A careful review of the equations for extended energy-momentum relation and extended mass-energy equivalence shows that the grand unification theory intrinsically disagrees with the concept of gauge invariance because any offset in the scaler potential energy *V* corresponds to a change in rest mass, and any potential energy that doesn't change rest mass with an offset must be related to certain physical interaction which is not supposed to be arbitrarily added or removed. Gauge invariance is obviously unsupported by the equations used by the grand unification theory.

In addition to the experiments discussed above, there are many immediate applications or observations of grand unification related effects. For example, if a celestial body consists of mostly medium and heavy atoms with an average mass deficit of 8.2 MeV ($\zeta_{average} \approx 0.99128$), its orbit in space is determined by these average atoms. If there also exist accumulated light atoms such as those in ice ($\zeta_{water} \approx 0.99246$), the light atoms will experience a differential centrifugal force about one thousandth of the gravity that shapes the orbit. Such differential force can contribute to the tidal behavior of oceans on Earth, or the separation of hydrogen rich components from a loose comet or asteroid approaching a massive planet or separation of chemical components in the rings of planets. For example, the 6478 Gault asteroid approximately 3.7 kilometer in diameter located in the inner regions of the asteroid belt was found to show multiple tails probably started in 2013 or earlier. It is very possible that when the asteroid got close to Mars, some chemical components with light atoms experienced differential centrifugal forces relative to the stony main body and were separated from the asteroid's orbit and driven to separate orbits depending on their overall mass deficits. Notice that such force is completely different from the conventional tidal force because it originates from the difference in inertial-to-gravitational mass ratio caused by nuclear mass deficits rather than the nonuniformity of gravity at different locations in space. For another example, the grand unification theory predicts that inertial mass will be reduced by mass deficit due to gravitational potential energy. If two massive celestial bodies are close enough, the tremendous negative gravitational potential energy can induce mass deficits exceeding that by nuclear mass deficits. The inertial masses will become much more reduced than the gravitational masses while the gravitational interaction is unchanged as the gravitational masses (or gravitational charges) are unaffected by the gravitational potential energy. The consequent much lower inertial masses will make their orbits much faster than Newtonian orbits, and in extreme cases approaching the speed of light when inertial mass is reduced to near zero approaching massless particles such as photons and gravitons. An observer unaware of the difference between gravitational and inertial masses would be forced to significantly overestimate the involved masses based on Newtonian estimation to explain the fast orbits. Einstein's general relativity theory would think that time is much dilated (slowed down) so that the celestial bodies can move much faster (covering a fixed distance with much less time as time is dilated at the orbit), not realizing that it is caused by the greatly reduced inertial mass compared to gravitational mass. Moreover, the very low inertial mass predicted by the grand unification theory creates a condition for explosive release of energy by way of high energy gamma rays. For example, when a mass falls to a very dense celestial body like a neutron star or a black hole, its inertial mass $m' = \zeta m = m\left[1 + \left(V_{nuclear\ binding}/m - GM/r\right)/c^2\right]$. When r approaches GM/c^2, which is half of the Schwarzschild radius, m'/m will approaches 0, namely the inertial mass will approach zero, the energy carried by the nearly massless mass can be easily released as high energy gamma rays or gravitons which are also massless and are no longer pulled by the strong gravity that prevents any particles carrying positive masses from escaping. This energy release mechanism is much more efficient than nuclear fusion, therefore the emission of energy is extremely explosive typically seen in the fast gamma ray bursts or supernova explosions. Notice that in the grand unification theory, massless particles such as gamma rays are not affected by gravity, namely photons are not attracted, bent or redshifted or blueshifted by gravity, they will emit freely in any directions unless scattered or absorbed by massive particles.

9. Very important warnings and prospects about the calculation of ζ from mass deficit

Notice that the calculations of $m' = m\zeta$ used in this paper are very rough estimations and conceptual in nature taking rest mass values as what are presently known, and the calculations may subject to significant revisions because the true ζ might be much more complicated than the assumptions that only nuclear binding energy, electric potential energy and gravitational potential energy are involved. Currently there are no analyses done to verify how many gravitational charges are contained in each of protons, neutrons and electrons and what is the true initial mass without mass deficit corrections for any of them. Exact knowledge about underlying physics giving rise to the properties of various elementary particles are mostly unknown from the perspective of the grand unification framework, extensive future theories and experiments are required to identify various charge, mass and interactions for the underlying building blocks. The numerical treatments of m and m' presented in this paper only represent reasonable averages based on existing physical observations. For example, in this paper, a hydrogen atom is considered to have zero nuclear mass deficit (neglecting the electron for first order approximation) because there is no nuclear binding energy assuming that proton is free from mass deficit. This presumption is by itself precarious because it might be outright wrong as proton could be a compound particle. A free neutron is even more likely a compound particle given that its total electric charge is zero, but it possesses magnetic moment. It is therefore unclear if proton and neutron are free from intrinsic mass deficits. If protons and neutrons are compound particles, the questions become what subparticles and what potential energies should be used to calculate ζ under the framework of grand unification. The same considerations should also be given to leptons: are they also the product of more fundamental building blocks that suffered different binding energies so that they have different rest masses? This will be the subject of future elementary particle models in the framework of the grand unification theory to restructure the present Standard Model. In this restructured model, all elementary particles will be the results of gravitational charge, electric charge, any other possible charges and possible involved potential energies. In this framework, particles are built from the underlying masses and interacting potential energies. For example, the extended energy-moment relation can model a hydrogen atom as a compound particle using a proton and an electron as the building blocks for source of masses and the electric potential energy as the source of mass deficit. The model offers quantized atomic binding energy levels up to relativistic accuracy. Using these binding energies, the atom can be viewed as a compound particle with different rest masses at ground and excited states related to the different atomic binding energies. The same model has been used to calculate the free neutron's beta decay energies with a few assumptions leading to a value approximately 92% of experimental result, thereby serves as a viable model for neutron as well. Modeling of leptons similarly is at least conceptually sensible as the masses of leptons seem to indicate that electron, as the most stable one, has the lowest rest mass due to the most negative intrinsic binding energy. Obviously, without experimental investigations of initial elementary masses and theoretical modeling of binding interactions, it is difficult to predict how far such modeling can go. Regardless of the unknown outcome, the ability of the grand unification theory to equally handle all natural forces offers a wonderful possibility to establish a more fundamental theoretical framework than the largely empirical Standard Model that cannot even handle gravity.

The grand unification theory is still at its infancy, there are plenty of experimental and theoretical opportunities waiting to be explored, not only for well-equipped scientific communities, but also for citizen scientists and hobbyists who are open minded and imaginative. The extended energy-momentum relation

$E^2 = (mc^2 + V)^2 + (pc)^2$ and extended mass-energy equivalence $E^2 = (\gamma\zeta mc^2)^2$ do not require sophisticated mathematical knowledge and high level of abstraction, they are especially suitable for developing conceptual physics theories and practical applications. All efforts are welcome to contribute by reapproving, disapproving, improving and extending the grand unification theory experimentally and theoretically. Hopefully the initial quest for explaining the charge-to-mass ratio of electron that motivated the author to establish the extended energy-momentum relation can be accomplished within this generation.