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MAGNETOHYDRODYNAMICS OF THE EARTH'S DYNAMO

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1 INTRODUCTION

Albert Einstein once ranked the problem of the origin of the earth's magnetic field among the three most important unsolved problems in physics. In Einstein's time scientists went so far as to postulate new physical laws in order to explain the phenomenon of geomagnetism. Today it is generally accepted that the earth's magnetic field is generated by motions in the liquid part of the earth's core, but the details of this process are still unresolved. In this article an attempt is made to outline the theoretical problems associated with this process and to describe some of the advances that have been made in recent years.

The mathematical problem describing the generation of magnetic fields by motions in an electrically conducting fluid is called the dynamo problem. The second section of this article provides an introduction to dynamo theory and its geophysically pertinent results. The first mathematically convincing evidence that the dynamo process is indeed possible in a singly connected volume of a homogeneous fluid was derived only two decades ago by Backus (1958) and Herzenberg (1958). Although the dynamo problem is akin to the problem of hydrodynamic instability, it did not become widely known among fluid dynamicists until recently. That technical applications appear to be remote seems to be the main reason for the fact that the dynamo problem is usually not mentioned in textbooks on fluid mechanics or even magnetohydrodynamics.

The dynamo process converts mechanical energy into magnetic energy and dissipates it in the form of ohmic heat. The question of the energy source of the earth's magnetic field is therefore of primary importance. This question is considered in Section 3 in connection with a discussion of the physical state of the earth's core. Most geophysicists regard convection driven by thermal or chemical buoyancy as the most likely source of energy for the geodynamo, but the possibility of a dynamo driven by the earth's precession cannot be entirely excluded.

Fortunately the unresolved question of the energy source is not a major obstacle for the theory. As discussed in Section 4 the dynamics of motions in the earth's core are dominated by the Coriolis force rather than the energy-providing forces.

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This allows us to bypass some of the uncertainties caused by the lack of knowledge about the earth's core and proceed to the magnetohydrodynamic problem of the geodynamo. In Section 5 steps towards a solution of this problem are discussed. The ultimate goal of any theory of the geodynamo is to develop models that are sufficiently detailed that the form and the observed secular variations of the geomagnetic field can be interpreted in terms of the physical parameters of the earth's core. The development of an efficient numerical dynamo model is needed to attain this goal.

The earth is not the only planet exhibiting a magnetic field. Jupiter's strong magnetic field was revealed by peculiar radio signals long before it was measured by the Pioneer 10 and 11 space probes. The discovery of the magnetic field of Mercury by the Mariner 10 space probe has surprised planetary scientists in view of the finding that the moon does not exhibit a large-scale magnetic field. In spite of large differences in the properties of the three planets for which an active dynamo appears to be required the physical mechanisms may be similar. This suggests the possibility of a general theory of planetary magnetism, as discussed in Section 6. The constraints offered by such a theory may ultimately provide the key for the understanding of the origin of the earth's magnetic field.

2 DYNAMO THEORY

2.1 *Basic Equations*

The kinematic aspects of the dynamo problem are described by Ohm's law for a moving conductor and by Maxwell's equations in the magnetohydrodynamic approximation in which the displacement current is neglected. It is convenient to eliminate all variables but the magnetic induction $\mathbf{B}$ according to the following scheme:

Ohm's Law — Maxwell's equations (magnetohydrodynamic approximation)

$$\mathbf{j} = \sigma(\mathbf{v} \times \mathbf{B} + \mathbf{E}) \qquad \nabla \times \mathbf{B}/\mu = \mathbf{j}, \quad \frac{\partial}{\partial t}\mathbf{B} = -\nabla \times \mathbf{E}, \quad \nabla \cdot \mathbf{B} = 0$$

$$\frac{1}{\mu\sigma}\nabla \times \mathbf{B} = \mathbf{v} \times \mathbf{B} + \mathbf{E}$$

$$\frac{\partial}{\partial t}\mathbf{B} + \nabla \times (\lambda \nabla \times \mathbf{B}) = \nabla \times (\mathbf{v} \times \mathbf{B}) \tag{2.1a}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{B} - \lambda \nabla^2 \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} \tag{2.1b}$$

In the last step the general form (2.1a) of the dynamo equation has been simplified by the assumption that the velocity field $\mathbf{v}$ satisfies $\nabla \cdot \mathbf{v} = 0$. Since the effects of compressibility are of minor importance in the earth's core, the dynamo problem is usually considered for a solenoidal velocity field. The inverse of the product of the electrical conductivity σ and the permeability μ is called the magnetic diffusivity λ. In deriving Equation (2.1b) it has been assumed that λ is constant. For liquid metals like mercury λ is of the order of 1 $m^2\ sec^{-1}$, and a value of the same order of magnitude is usually assumed for the liquid iron core of the earth, since the effects of rising temperature and rising pressure tend to compensate each other.

In order to complete the description of the dynamo problem, boundary conditions for the magnetic field must be added. It is often assumed that the conductivity σ differs from zero only within a finite volume V while the outside is electrically insulating. Since the magnetic field **B** cannot have any sources it must decay towards infinity at the rate of at least the third power of the inverse of the distance r from V,

$$|\mathbf{B}| \approx 0(r^{-3}) \quad \text{for} \quad r \rightarrow \infty. \tag{2.2}$$

An important property of the dynamo equation (2.1) is that it remains invariant with respect to a transformation to a rotating frame of reference, since **B** remains unchanged with respect to such a transformation within the framework of the magnetohydrodynamic approximation. Another important property of Equation (2.1) is that the flux of **B** through any material surface of the fluid is conserved in the limit $\lambda \rightarrow 0$, corresponding to an infinitely conducting fluid. This property represents an analogue to Kelvin's theorem and follows from the similarity of Equation (2.1) to the vorticity equation. Since the generation of a magnetic field is meant to imply the generation of magnetic flux, it is evident that finite effects of magnetic diffusion are required. The dynamo process cannot occur in a superconductor.

In general two forms of the dynamo problem are distinguished. The kinematic dynamo problem is concerned with the conditions under which growing solutions **B** of Equation (2.1) exist for arbitrarily given solenoidal velocity fields **v**. In the case of time-independent fields **v** an exponential time dependence exp $\{pt\}$ can be assumed for **B**. A dynamo process occurs when a solution **B** exists of the boundary-value problem defined by (2.1) and (2.2) for which the eigenvalue p has a positive real part. One also speaks of dynamo action in this case.

Exponentially growing solutions are clearly unphysical beyond a finite range of time, since the magnetic energy cannot grow indefinitely. In reality the velocity field **v** cannot be prescribed independently of the magnetic field. Instead **v** must obey the equations of motion, which include the Lorentz force $(\nabla \times \mathbf{B}) \times \mathbf{B}/\mu$. As the magnetic field grows the Lorentz force modifies the velocity in such a way that the growth of the magnetic field is reduced and an asymptotic equilibrium value for the magnetic energy is attained, at least in an average sense. Accordingly, the second form of the dynamo problem, called the magnetohydrodynamic dynamo problem, is based on the equations of motion in conjunction with the dynamo equation (2.1). The coupling of the equations by the Lorentz force introduces a nonlinearity, which has prevented any simple solutions of this problem. For geophysical applications, however, it is hard to avoid the solution of the full magnetohydrodynamic dynamo problem, since the choice of forces is much better constrained than the choice of the velocity fields and since the nonlinearity of the dynamo process is a necessary ingredient for the determination of the amplitude of the magnetic field.

The kinematic dynamo problem and the magnetohydrodynamic dynamo problem have the same mathematical relationship as the linear problem of hydrodynamic instability and the nonlinear postinstability problem. In fact, the dynamo problem represents a particular kind of magnetohydrodynamic instability in which the motion of an electrically conducting fluid becomes unstable to a growing disturbance that manifests itself in the form of a magnetic field. It is not unusual in problems of hydrodynamic instability that the growing disturbance is characterized by a new degree of freedom or even the appearance of a new physical quantity. In the case of the onset of thermal convection in a layer heated from below, for instance, a motionless state becomes unstable to a state with motion. The dynamo problem, however, possesses a number of peculiar properties that are described briefly in the following sections. Whenever possible attention is drawn to similarities with ordinary problems of hydrodynamic instability.

2.2 *The Disk Dynamo*

In view of the simplicity of the dynamo equation (2.1) it may appear surprising that the question of the existence of growing solutions has been answered only fairly recently. The dynamo principle of the generation of electromagnetic energy from mechanical energy has been known for a long time, of course, and has become one of the most commonly used technical processes. Technical dynamos differ from planetary dynamos, however, in that they require a complicated distribution of electrical conductivity. The simplest example of a technical dynamo is the disk dynamo shown in Figure 1. A metal disk is rotating about its axis of symmetry in a weak initial magnetic field $\mathbf{B}_0$. The field induces an electromotive force between the axis and the rim of the disk that can be used to drive a current in a non-rotating circuit. When the winding of the circuit has the appropriate sense the field $\mathbf{B}$ generated by the current has the same direction as the initial field $\mathbf{B}_0$. When

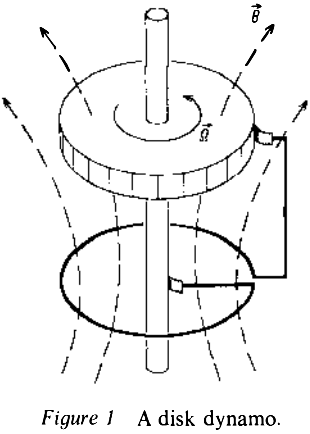

Figure 1 A disk dynamo.

the rotation rate Ω is high enough, $\mathbf{B}$ exceeds $\mathbf{B}_0$ and a self-excited dynamo is obtained. The condition for dynamo action or self-excitation is given by

$$M\Omega/R > 2\pi, \tag{2.3}$$

where M is the mutual inductance between circuit and disk and R is the resistance of the circuit.

The multiply connected distribution of conductivity of the disk dynamo is not available in planetary cores, and complicated velocity fields must compensate for an essentially uniform distribution of conductivity within the singly connected domain of the core. In other respects the fluid dynamo, or homogeneous dynamo as it is often called, is rather similar to the disk dynamo. The latter has been studied in considerable detail as the simplest example of a dynamo. Oscillations (Bullard 1955, Lebovitz 1960), reversals (Rikitake 1958, Allan 1962, Robbins 1975), and other nonlinear properties have been investigated by using suitably modified disk dynamos or systems of coupled disk dynamos. Although properties such as the reversals of the magnetic field resemble reversals of the geomagnetic field shown by the paleomagnetic record, the theory of the disk dynamo cannot easily be generalized to the case of the homogeneous dynamo. The origin of geomagnetic reversals has thus remained one of the most intriguing problems of dynamo theory.

2.3 *Necessary Conditions for Dynamos*

A shear flow can become unstable when a disturbance velocity field gains more energy from the shear by the stretching of vortex lines than it loses by viscous dissipation. The nondimensional parameter describing the ratio of the two terms is the Reynolds number $\mathrm{Re} \equiv UL/\nu$, where U is a typical velocity of the shear flow, L is a characteristic length, and ν is the kinematic viscosity. Similarly, growing solutions of the dynamo equation (2.1b) require that the term on the right-hand side describing the stretching of magnetic-field lines becomes comparable with the magnetic diffusion term on the left-hand side. The magnetic Reynolds number $Rm \equiv UL/\lambda$ gives a measure of the ratio of the two terms, and the condition for dynamo action in general requires that Rm exceeds a certain finite value Rm_c,

$$Rm \geqq Rm_c. \tag{2.4}$$

Relationship (2.3) is an example of such a condition.

The comparison of the magnetic-diffusion term and the stretching term in Equation (2.1b) provides the basis for necessary conditions for dynamo action. Considering an arbitrary volume V of fluid with constant diffusivity λ, Backus (1958) derived the inequality

$$\frac{1}{2}\frac{d}{dt}\int_{V+V'} |\mathbf{B}|^2 \leqq \{m(t)-\pi^2\lambda/r_0^2\} \int_{V+V'} |\mathbf{B}|^2, \tag{2.5}$$

where V' denotes the insulating space outside V, $m(t)$ is the largest principal value of the rate-of-strain tensor $S_{ij} \equiv \frac{1}{2}(\partial_j v_i + \partial_i v_j)$ of the velocity field in V, and r_0 represents the radius of the smallest sphere enclosing V. An obvious consequence of the inequality is that the condition

$$Rm^* \equiv \frac{m(t)r_0^2}{\lambda} \geqq \pi^2 \tag{2.6}$$

is necessary for dynamo action. Using different estimates Childress (1969) derived the condition

$$Rm \equiv Ur_0/\lambda \geqq \pi, \tag{2.7}$$

where U is the maximum velocity in V. Conditions (2.6) and (2.7) are mathematically equivalent to the necessary conditions for hydrodynamic instability derived by Serrin (1959). Other more specialized conditions can be derived and are known in fluid mechanics as energy stability limits. Joseph (1976) gives a comprehensive account of this subject.

A general relationship that does not have an analogue in hydrodynamic stability theory is given by (Busse 1975a):

$$\frac{1}{2}\frac{d}{dt}\int_V |\mathbf{B}\cdot\mathbf{r}|^2 \leqq \{-\lambda+\max(\mathbf{v}\cdot\mathbf{r})Q\}\int_{V+V'} |\nabla\mathbf{r}\cdot\mathbf{B}|^2, \tag{2.8}$$

where $\mathbf{r}$ is the position vector and Q is defined by

$$Q = \left(\int_V |\mathbf{B}|^2 \bigg/ \int_{V+V'} |\nabla\mathbf{r}\cdot\mathbf{B}|^2\right)^{1/2}. \tag{2.9}$$

Relationship (2.8) is particularly useful for the application to the earth's core, which may be regarded in first approximation as a sphere of constant diffusivity surrounded by an insulating medium. It is convenient to use the following general representation for the solenoidal vector field $\mathbf{B}$,

$$\mathbf{B} = \nabla \times (\nabla \times \mathbf{r}h) + \nabla \times \mathbf{r}g, \tag{2.10}$$

where the origin of $\mathbf{r}$ is assumed at the center of the sphere. The first term on the right-hand side of (2.10) is called the poloidal component and the second the toroidal component of $\mathbf{B}$, and h and g are arbitrary scalar functions that can be chosen such that their average over any surface $|\mathbf{r}| = \text{const}$ vanishes, since the addition to g or h of any function depending only on $|\mathbf{r}|$ does not affect $\mathbf{B}$. Only the poloidal component of $\mathbf{B}$ intersects the surface of the conducting sphere. The toroidal component vanishes outside the sphere, which is the main reason for the uncertainty about the strength of the magnetic field inside the earth's core. Estimates for the toroidal field strength vary by two orders of magnitude between $5 \cdot 10^{-4}T$ and $0.1T$ ($10^{-4}T = 1$ G), while the estimate of $5 \cdot 10^{-4}T$ for the poloidal field can be obtained easily by extrapolation from the observed value at the earth's surface.

Since the radial component of $\mathbf{B}$ depends only on h, a lower estimate for the integral in the denominator of expression (2.9) can be obtained from the energy E_p of the observed poloidal component of the geomagnetic field. The integral in the numerator describes the total magnetic energy, E_M, in the earth's core [except for the factor $(2\mu)^{-1}$], which can be bounded from above by estimates of ohmic dissipation. The lower bound for the radial velocity in the earth's core implied by relationship (2.8),

$$\max (\mathbf{v}\cdot\mathbf{r}) \geqq \lambda(2E_p/E_M)^{1/2}, \tag{2.11}$$

can thus serve as a useful criterion for the feasibility of proposed dynamo processes (Busse 1975a, Gubbins 1975).

2.4 *Negative Theorems*

The dynamo equation possesses some interesting properties without parallel in other areas of fluid dynamics. The most famous one is expressed by Cowling's theorem, which states that Equation (2.1b) does not permit axisymmetric or two-dimensional steady solutions for **B**. The original proof by Cowling (1934) has been extended by Backus & Chandrasekhar (1956) and by Lortz (1968a). Cowling's theorem has played an important historical role. Since the earth's magnetic field is approximately axisymmetric it seemed doubtful for a long time whether the dynamo process was possible in a nearly homogeneous body of fluid like the earth's core. Today it is well known, mainly because of the work of Braginsky (1965a,b, 1975), that arbitrary small deviations from axisymmetry are sufficient to generate magnetic fields if the magnetic Reynolds number is high enough.

Even more important for practical applications than Cowling's theorem is the toroidal theorem, which was found by Elsasser (1946) and has been proven rigorously by Bullard & Gellman (1954). The theorem states that any solution **B** of Equation (2.1a) decays if the velocity field **v** is toroidal, i.e. if it can be written in the form

$$\mathbf{v} = \nabla \times \mathbf{r}\psi, \tag{2.12}$$

and if the diffusivity λ is a function of the radial coordinate $r \equiv |\mathbf{r}|$ only. The toroidal theorem would be implied by the general condition (2.8) if it could be proven that any steady or growing solution **B** of the dynamo equation must have a radial component. Such a proof is highly desirable because it would exclude the possibility of a magnetic field inside a planetary core that could not be noticed from the outside. Even though the equation for the toroidal potential g is relatively simple, it has not yet been possible to prove that all solutions for g must decay if $h \equiv 0$. There are a number of similar conjectures that have remained without proof, although they are generally believed to be true. [See Busse (1977a) for further details].

The toroidal theorem can also be formulated for a plane geometry where the diffusivity λ depends only on the coordinate in the direction of the constant unit vector **k**. In this case the theorem states that all solutions **B** of Equation (2.1a) decay in time if **v** is of the form

$$\mathbf{v} = \nabla \times \mathbf{k}\psi \tag{2.13}$$

and if λ is a function of $\mathbf{k}\cdot\mathbf{r}$ only. The toroidal theorem in this form clearly demonstrates why it has been difficult to demonstrate the feasibility of the homogeneous dynamo process. Most simple velocity fields can be written in the form (2.13) and are therefore excluded from dynamo action. The closest analogue to the toroidal theorem in hydrodynamic stability theory is the fact that some flows, such as plane Couette flow or Hagen-Poiseuille flow in a circular pipe, are stable with respect to infinitesimal disturbances. It is worth emphasizing that the absence of dynamo action in the case of toroidal velocity fields does not imply that mechanical energy could not be converted into magnetic energy. A velocity field of the form (2.13) may lead to an initial increase of the magnetic energy, but ultimately all magnetic fields must decay because of the unbalanced ohmic dissipation of the poloidal component of the magnetic field.

2.5 *Simple Dynamos*

To illuminate some typical features of the dynamo process we consider a simple example. The simplest velocity field not excluded by the toroidal theorem is a two-dimensional field of the form

$$\mathbf{v} = \nabla \times \mathbf{k}\psi(x,y) + \mathbf{k}w(x,y), \tag{2.14}$$

where $\mathbf{k}$ is the unit vector in the z direction of the Cartesian system of coordinates. The work of Childress (1970) and G. O. Roberts (1970, 1972) has shown that spatially periodic dynamos can be constructed easily because the boundary conditions for the magnetic field at the surface of the conducting fluid are avoided. Following G. O. Roberts (1972) we assume

$$\psi = A \sin \alpha x \sin \alpha y = A\, w/C, \tag{2.15}$$

with constants A and C. The solution for the magnetic field must have a z dependence according to Cowling's theorem. Since the dynamo equation does not depend explicitly on t or z we may assume

$$\mathbf{B} \propto \exp\{i\gamma z + pt\}. \tag{2.16}$$

The magnetic field can be separated into a mean part $\langle \mathbf{B} \rangle$ and a fluctuating part $\mathbf{B}' \equiv \mathbf{B} - \langle \mathbf{B} \rangle$, where the brackets indicate the average over the x, y plane. By taking the average over the dynamo equation (2.1b) and subtracting it from Equation (2.1b) we obtain two equations for $\langle \mathbf{B} \rangle$ and $\mathbf{B}'$,

$$(p + \lambda\gamma^2)\langle \mathbf{B} \rangle = i\gamma \mathbf{k} \times \langle \mathbf{v} \times \mathbf{B}' \rangle, \tag{2.17}$$

$$(p - \lambda\nabla^2)\mathbf{B}' = \langle \mathbf{B} \rangle \cdot \nabla \mathbf{v} + i\gamma w \langle \mathbf{B} \rangle + \dots. \tag{2.18}$$

Assuming that the fluctuating field $\mathbf{B}'$ is small in comparison with the mean field $\langle \mathbf{B} \rangle$ we have neglected terms involving $\mathbf{B}'$ on the right-hand side of (2.18). This assumption is justified in the limit $\gamma^2 \ll \alpha^2$, as can be confirmed from the results. Equation (2.17) suggests that p will be of the same order of magnitude as $\lambda\gamma^2$. Assuming this and neglecting terms of the order γ in Equation (2.18) we find as the approximate solution $\mathbf{B}' = \langle \mathbf{B} \rangle \cdot \nabla \mathbf{v}/2\lambda\alpha^2$. After inserting this solution in Equation (2.17) we obtain

$$(p + \lambda\gamma^2)\langle \mathbf{B} \rangle = i\gamma \mathbf{k} \times \langle \mathbf{B} \rangle AC/4\lambda\alpha^2, \tag{2.19}$$

which can be satisfied only if

$$p = -\lambda\gamma^2 + \gamma AC/4\lambda\alpha^2. \tag{2.20}$$

The solution itself can be written in the form

$$\langle B_x \rangle + i\langle B_y \rangle = B_1 \exp\{i\gamma t + pt\}, \tag{2.21}$$

where B_1 is a complex constant to be determined by the initial conditions. A dynamo process exists if p is positive, i.e. the condition for dynamo action is

$$(\alpha A/\alpha\lambda)(C/\gamma\lambda) > 4. \tag{2.22}$$

The right-hand side is written in this condition as the product of two magnetic Reynolds numbers, one of which is based on the scale of the fluctuating field, while the other is based on the wavelength of the mean field. The dependence of the dynamo condition on the product of two differently defined magnetic Reynolds numbers is characteristic for nearly all homogeneous dynamos.

The field lines of the magnetic field (2.21) can be visualized as the edges of the steps in a winding staircase, as shown in Figure 2 for the case $\gamma > 0$. The same sense of winding characterizes the velocity field. Some illumination of the dynamo process may be gained by neglecting the effect of diffusion and considering the distortion of magnetic field lines by the velocity field. In Figure 3 the distortions owing to the two components of the velocity field have been separated. It can be seen that an x component results from the field lines that are originally aligned with the y direction and vice versa. It must be kept in mind, though, that magnetic diffusion is required for the generation of magnetic flux.

There are two properties of the above example that are typical for the majority of the homogeneous dynamos considered in the literature:

1. The velocity field (2.14), (2.15) possesses the same sense of winding everywhere. Mathematically this is expressed by the property that the helicity $\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle$ is

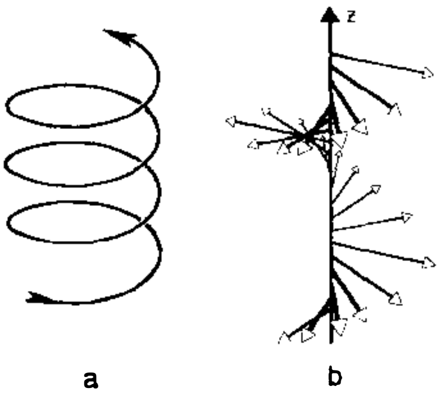

Figure 2 (*a*) Streamline of the velocity field (2.14) for $AC > 0$. (*b*) The direction of the mean magnetic field $\langle \mathbf{B} \rangle$ as a function of z.

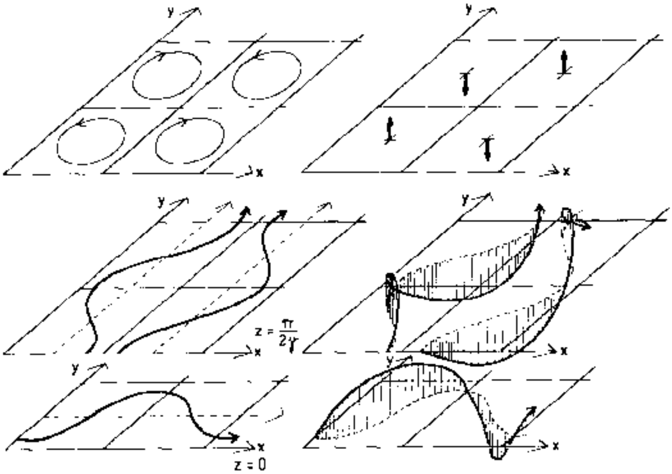

Figure 3 A sketch of dynamo process. The upper part of the figure shows the components $\nabla \times \mathbf{k}\psi$ (left) and $\mathbf{k}w$ (right) of the velocity field. Below the distortions of the magnetic field caused by the velocity field are shown. The distortion of the field in the y direction at the level $z = \pi/2\gamma$ results in a component of the field in the x direction at the level $z = 0$, and vice versa. The magnetic fields at the two levels can thus reinforce each other if diffusion is taken into account.

finite. The dynamo condition (2.22) can actually be expressed in terms of the helicity, since the latter is proportional to AC. There is no law that a finite helicity is required for dynamo action. It is known, however, that the dynamo process is most direct in the case of a finite helicity and that velocity fields without helicity require higher values of the magnetic Reynolds number for dynamo action.

2. The magnetic field separates into a mean and a fluctuating part. The separation of scales between the velocity field and the large-scale component of the magnetic field is not solely a matter of mathematical convenience. From the investigation of dynamos by numerical methods (Gubbins 1973, Bullard & Gubbins 1977) it is known that a sufficient separation of scales is necessary to obtain dynamo action.

The separation of scales is the basis for the *mean field electrodynamics* (MFE) formulated by Steenbeck, Krause & Rädler (1966). These authors make statistical assumptions about a small-scale turbulent velocity field and obtain in the simplest case an equation of the form

$$\frac{\partial}{\partial t}\langle \mathbf{B}\rangle + \lambda\nabla^2\langle \mathbf{B}\rangle = \alpha\nabla \times \langle \mathbf{B}\rangle, \tag{2.23}$$

where the brackets indicate a three-dimensional local average. Equation (2.23) is based on the assumption of isotropic homogeneous turbulence without the property of mirror symmetry. This lack of symmetry can be expressed by a finite helicity and leads to a nonvanishing coefficient α. When α is sufficiently large, dynamo action occurs, as has been demonstrated by numerous solutions of Equation (2.23) (see Steenbeck, Krause & Rädler 1966, Krause & Steenbeck 1967, P. H. Roberts 1972).

Equation (2.23) can be generalized by the addition of a large-scale motion, as for example a differential rotation in a rotating system. Models based on these equations have been applied to the earth's core and to the sun (Deinzer & Stix 1971, Krause & Rädler 1971, Levy 1972). In the latter case the models appear to be more appropriate because the statistical assumptions, in particular the property of isotropy, are better satisfied than in the earth's core where the Coriolis force plays a dominant role. Because of its simplicity the concept of MFE has become very popular. However, "this seeming simplicity is a delusion. Beneath the surface of turbulent dynamo theory lies a morass of difficulties on which the subject rests uneasily" as Soward & Roberts (1976) have pointedly remarked. A recent review of MFE has been given by Roberts & Soward (1975). Some of the mathematical problems involved in this approach can be analyzed in more detail by restricting attention to the case of random inertial waves, as was done by Moffatt (1970a,b, 1972) and Soward (1975).

Among the large number of other approaches for the solution of the kinematic dynamo problem I mention here only the elegant solution by Lortz (1968b), which is based on a helical system of coordinates. For an application of Lortz' model to the case of a sphere see Benton (1975). A nearly complete review of solutions up to 1970 has been given by P. H. Roberts (1971). More recent reviews have been given by Gubbins (1974) and Moffatt (1976). Some of the geophysically relevant solutions of the kinematic dynamo problem, in particular numerical solution of the problem in a sphere, are discussed in Section 5.2.

2.6 *The Magnetohydrodynamic Dynamo Problem*

The kinematic dynamo problem describes the dynamo process in its initial stages, when a particular mode among the small random magnetic disturbances that permeate the universe becomes amplified by dynamo action. When the amplitude of the magnetic field reaches a value such that the Lorentz force in the equation of motion is no longer negligible, the magnetohydrodynamic dynamo problem must be considered. The quadratic dependence of the Lorentz force on the amplitude of the magnetic field suggests a perturbation approach based on an expansion in powers of the nondimensional magnetic energy density M,

$$\begin{aligned} \mathbf{v} &= \mathbf{v}_0 + M\mathbf{v}_1 + \ldots, \\ \mathbf{B} &= M^{1/2}(\mathbf{B}_0 + M\mathbf{B}_1 + \ldots). \end{aligned} \tag{2.24}$$

To obtain a finite-amplitude solution a minimum of four steps is required. In the first step the equations of motion are solved without Lorentz forces. This is by no means a trivial problem, because solutions of the form (2.13) must be avoided. Since nearly all known exact solutions of the Navier-Stokes equations can be written in the form (2.13), $\mathbf{v}_0$ is usually determined by approximation methods. In the second step the kinematic dynamo problem for $\mathbf{v}_0$ is solved. The strongest growing magnetic field is accepted as the physically relevant solution and the corresponding Lorentz force is calculated. The resulting modification $M\mathbf{v}_1$ of the velocity field is derived in the third step. In the fourth step the dynamo equation of the order $M^{3/2}$ is considered. In general it is sufficient to consider the solvability condition to determine the equilibrium value M of the magnetic energy density. When the initial growth of the magnetic field is exponential an expansion analogous to (2.24) can be used for the growth rate p. The balancc for the equilibrium valuc M of the magnetic energy density is given in this case by real part $\{p_0 + Mp_1\} \approx 0$, which indicates that the perturbation approach is restricted to the region of the parameter space of the problem where the real part of p_0 is sufficiently small.

The above perturbation approach has been used by Busse (1973) in the case of convection in a layer heated from below with a plane parallel shear flow and by Soward (1974, see also Childress & Soward 1972) in the case of a rotating convection layer. The most important result of these studies is that the equilibrium amplitude of the magnetic field is determined by diffusion properties of the system rather than by an equipartition of magnetic and kinetic energy. Although it is known from problems of finite-amplitude convection that the perturbation approach often provides a qualitatively correct description far beyond the region where converging expansions can be expected, there are undoubtedly effects of the Lorentz force that cannot be captured by the perturbation approach. This problem is discussed again in Section 6.1 in connection with the discussion of the equilibration in the case of the geodynamo.

Studies on the extension of turbulent dynamos into the nonlinear magnetohydrodynamic regime have been made by a number of authors. Malkus & Proctor (1975) and Proctor (1977) have analyzed on the basis of MFE the effects of large-scale flows generated by Lorentz forces. For a statistical treatment of the nonlinear magnetohydrodynamic problem we refer to Pouquet, Frisch & Léorat (1976).

Before closing this brief survey of dynamo theory I want to mention the laboratory investigation of a homogeneous dynamo by Lowes & Wilkinson (1963, 1968). These authors used the high permeability of iron rotors in their experimental realization of the Herzenberg dynamo in order to achieve a sufficiently high magnetic Reynolds number. Because of the nonlinear properties of the permeability and the effects of hysteresis, the experimental dynamo exhibits a complex behavior and is not well suited for a comparison with theoretical predictions. An experimental dynamo without ferromagnetic material is highly desirable but rather costly because of the large dimensions required.

3 THE PHYSICAL STATE OF THE EARTH'S CORE

3.1 *Properties of the Core*

It has been said that we know more about the interior of stars than about the interior of our own planet. This is somewhat exaggerated. The radii of the liquid outer core and the solid inner core have been measured by seismic methods to an accuracy of a few kilometers (see Figure 4). It is also generally accepted that the core consists mainly of iron, which is alloyed with nickel in the inner core, while the density of the outer core requires the admixture of a lighter element, most

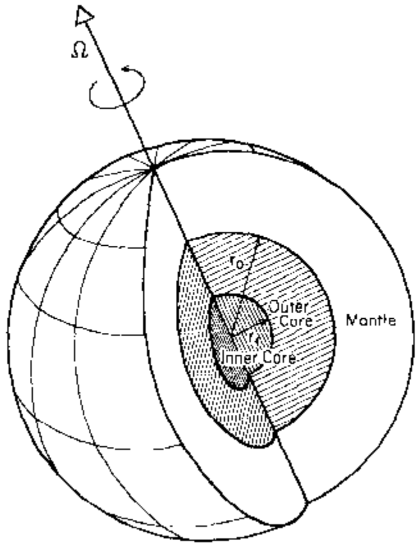

Figure 4 The earth's interior. The mean radii r_i and r_0 have the values 1210 km and 3480 km, respectively.

likely sulphur or silicon. But the physical properties of iron at core pressures of a few megabars are scarcely known, and estimates for the viscosity in the outer core vary by several orders of magnitude. The electrical and thermal conductivities are somewhat better known even though a factor 3 must be regarded as a realistic estimate for the uncertainty (Malkus 1968). Some of the commonly used values for the properties of the earth's outer core are listed in Table 1, and we refer to the recent monograph by Jacobs (1975) for a more detailed discussion.

The form and the secular variation of the geomagnetic field represent a large potential source of information about the core. If a sufficiently detailed theory of the earth's dynamo were available, constraints on the properties of the earth's core could be derived by comparing theoretical predictions and observations. As in many other areas of geophysics, however, the relationship between the development of theoretical models and the interpretation of observational data is one of mutual dependence. Thus the progress in the theory of the geodynamo is hampered nearly

Table 1 Commonly assumed properties of the earth's core

kinematic viscosity	$\nu = 10^{-6}$ m^2 sec^{-1}
thermal conductivity	$k = 60$ WK^{-1} m^{-1}
electrical conductivity	$\sigma = 5 \cdot 10^5$ mho m^{-1}
thermal diffusivity	$\kappa = 7 \cdot 10^{-6}$ m^2 sec^{-1}
magnetic diffusivity	$\lambda = 1.6$ m^2 sec^{-1}
coefficient of thermal expansion	$\beta = 4.5 \cdot 10^{-6}$ K^{-1}

as much by the lack of knowledge about the earth's core as by mathematical difficulties.

A still unresolved problem is the question of the energy source of the geodynamo. The discussion by Bullard (1949) of various external and internal forces acting on the core still holds. Convection driven by thermal or chemical buoyancy appears to be the most likely cause of motions. Turbulent motions induced in the core by the precession of the earth is another possibility that requires consideration.

3.2 *Convection*

Assuming that the core is cooling down, thermal energy is available from the latent heat liberated by the growth of the inner core (Verhoogen 1961, Malkus 1973) and from radioactive elements distributed throughout the core. The latter proposal has been a subject of controversy (Brett 1976). Yet there appears to be the possibility that a significant fraction of the observed surface heat flux could be produced in the earth's core by the radioactive potassium isotope ^{40}K (Goettel 1976). Chemical buoyancy is provided by the fact that the light component in the outer core stays in solution and becomes enriched near the growing inner core (Braginsky 1963, Loper & Roberts 1977). The gravitational energy released by the upward motion of the enriched material may not be large. The energy is not subject to the constraints of thermal efficiency, however, when it is converted into ohmic dissipation by the dynamo process (Gubbins 1976).

Much of the recent discussion of the physical state of the earth's core has been stimulated by the hypothesis of Higgins & Kennedy (1971) that the outer core is at the melting temperature of iron or of an iron-sulphur mixture (Usselman 1975). According to Higgins and Kennedy the adiabat of liquid iron is a steeper function of the pressure than the melting temperature, with the consequence that the core would be stably stratified. A more detailed inspection of the problem (Kennedy & Higgins 1973) shows that this hypothesis must be restricted to the outer part of the outer core. The exact distance from the inner core up to which thermal convection could occur depends on the Grüneisen parameter, the value of which at high pressures is not known. The assumptions made by Higgins and Kennedy have been disputed by other scientists (Frazer 1973, Verhoogen 1973), and it is also possible to remove the discrepancy between the isentrope and the melting temperature by assuming an iron slurry distribution in the outer core (Busse 1972, Elsasser 1972). Each of the alternatives to the Higgins and Kennedy proposal poses its own problems, however, and the stable stratification of the outermost core must be regarded as a serious possibility.

3.3 *Motions Induced by Precession*

The feasibility of a precession-driven geodynamo has been investigated by Malkus (1963, 1968). The advantage of this proposal is that the forces are well known, since they derive from the astronomical phenomenon of the 26,000-year precession of the earth's axis about the normal of the ecliptic. Since the precessional torques of moon and sun acting on the mantle are larger than those acting on the core because of the smaller ellipticity of the latter, the co-precession of mantle and core requires a coupling torque exerted by the mantle on the core. Poincaré (1910) discussed this problem on the basis of the solution for the motion of a homogeneous ideal fluid in a precessing spheroidal cavity, which he rederived after it had been derived by Hough (1895) fifteen years earlier. The Hough–Poincaré solution is based on the assumption of constant vorticity and suggests a negligible effect of precession on motions in the earth's core. It can be shown, however, that this solution does not represent the solution of the problem for a viscous fluid in the limit where the viscosity vanishes (Busse 1968). Instead of a constant vorticity a singularity in the form of a cylindrical vortex sheet with a radius of $(\frac{3}{4})^{1/2}$ of the core radius develops when the viscosity tends to zero. Malkus (1968) has demonstrated by laboratory experiments that the shear flow becomes unstable to wavy disturbances and that a fully turbulent state is realized when the rate of precession is sufficiently large.

It is doubtful whether the precession of the earth is strong enough to induce a turbulent state in the core. If the outer part of the outer core is stably stratified, as proposed by Kennedy & Higgins (1973), the differential rotation induced by precession would be stabilized, since the region of strongest shear is close to the core–mantle boundary. The problem becomes more complex when Lorentz forces are included, and the heuristic arguments by Malkus in favor of a precession-driven magnetohydrodynamic turbulence as the cause of geomagnetism have been criticized by Rochester et al (1975). On the other hand, a precessional origin of geomagnetism is an attractive hypothesis in terms of energy considerations, since the vast reserve of the rotational energy of the earth would be available. Only a fraction of the energy lost by tidal dissipation would be needed to supply the energy lost by ohmic heating. I conclude that the question of the feasibility of a precession-driven geodynamo must be regarded as open at this time and that a detailed theoretical analysis of the problem is desirable.

4 HYDRODYNAMICS OF THE EARTH'S CORE

4.1 *General Remarks*

From the discussion in Section 2 it is evident that the dynamo process can be regarded as an instability of the solution of the basic equations of motion without magnetic field. An understanding of the hydrodynamics of the earth's core in the limit of vanishing Lorentz forces is therefore a prerequisite for the solution of the full magnetohydrodynamic problem of the geodynamo. The solutions of interest in the cases of convection as well as of precession are instabilities themselves of a basic state. In the language of bifurcation theory the dynamo process thus represents a secondary or higher order branching in the solution space of the respective boundary-value problem.

This section is restricted to the classical problem of thermal convection in a rotating, uniformly heated, self-gravitating sphere. The solution for this problem exhibits the characteristic features of the dynamics in rotating planetary cores without magnetic fields. Because of the dominating influence of the Coriolis force, the nature of the energy-providing forces has relatively little influence on the form of the motions, and general conclusions reached in the case of thermal convection apply in other cases as well. It is apparent from the results that the presence of an inner core does not affect the solution significantly. It must be remembered that in spite of its sizeable diameter the earth's inner core occupies only about $\frac{1}{20}$ of the volume of the core. For the same reason the inner core is usually neglected in the analysis of the dynamo problem.

4.2 *Thermal Convection in a Rotating Sphere*

A detailed analysis of the problem of convection in a sphere in the asymptotic limit of large rotation rates has been given by P. H. Roberts (1968). The physically realized solution was determined by Busse (1970a). Figure 5 shows a qualitative sketch of this solution, which exhibits the characteristic alignment of the convection rolls with the axis of rotation. To minimize the stabilizing influence of the Coriolis force the realized convection mode obeys as far as possible the Proudman-Taylor theorem, which states that steady small-amplitude motion in a rotating fluid of vanishing viscosity must be independent of the z coordinate in the direction of the axis of rotation. Because of the boundary condition the velocity field cannot be entirely z-independent unless the radial component vanishes, in which case no work is done by buoyancy forces. To accommodate the deviations from the Proudman-Taylor theorem the convective motion becomes time-dependent and assumes the form of columns propagating slowly in the prograde azimuthal direction, like Rossby waves. In addition, viscous friction becomes large enough, owing to a small azimuthal wavelength, to balance part of the Coriolis force. The relevant parameter is the Ekman number $E = \nu/\Omega r_0^2$ based on the kinematic viscosity ν, the rotation rate Ω, and the radius of the sphere r_0. The wavelength decreases with $E^{1/3}$ and the phase velocity divided by Ω decreases with $E^{2/3}$ as E tends to zero.

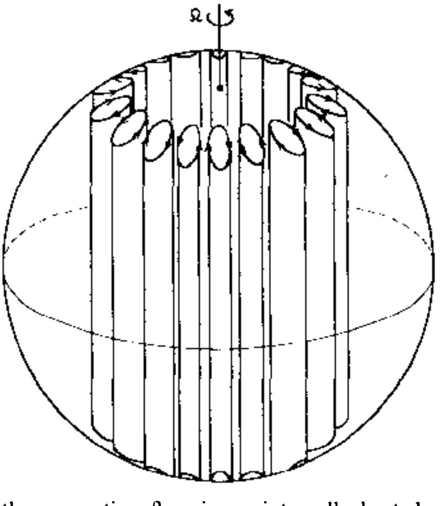

Figure 5 Sketch of the convection flow in an internally heated self-gravitating sphere (after Busse 1970a).

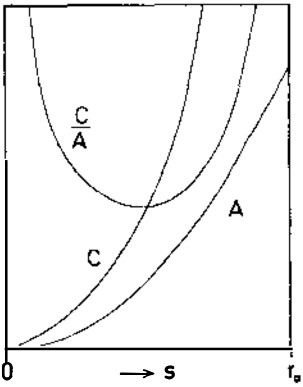

Figure 6 The stabilizing component C of the Coriolis force and the buoyancy force A as a function of distance s from the axis. The onset of convection occurs at the distance $s \approx r_0/2$ from the axis, where C/A reaches a minimum.

The most remarkable property of the solution is that the convective instability sets in at a distance of about $r_0/2$ from the axis. This distance corresponds to the minimum of the ratio between the nongeostrophic part C of the Coriolis force and the buoyancy force A. Since the convection motion is predominantly parallel to the equatorial plane, only the buoyancy provided by the components of gravity and the temperature gradient perpendicular to the axis of rotation enter the dynamics in first approximation. Since both components increase linearly with distance s from the axis, A increases quadratically. The stabilizing force C increases with the inclination of the boundary, as shown in Figure 6, such that C/A reaches a minimum near $s = r_0/2$.

The property that the component of gravity perpendicular to the axis of rotation is the predominant source of buoyancy is the basis for a laboratory simulation experiment (Busse & Carrigan 1976). Since the buoyancy depends on the product of temperature gradient and gravity, thermal convection in a rotating planetary core can be modeled experimentally by using the centrifugal force in place of gravity and by reversing the temperature gradient. The onset of convection in a rotating spherical shell cooled from the inside and heated from the outside occurs in the form of regularly spaced columns, as shown in Figure 5, except that the onset occurs at the equator of the inner sphere. At higher temperature differences convection columns tend to fill the spherical shell. Even though the spacing becomes irregular, the columns retain their perfect alignment with the axis of rotation (see Figure 3 of Busse & Carrigan 1976).

4.3 *Differential Rotation Generated by Convection*

The concept of the generation of differential rotation owing to the advection of momentum by thermal convection has played an important role in the theory of the earth's dynamo. In analogy to the tendency of convection to homogenize temperatures it has been argued that angular momentum tends to be equalized in rotating convecting systems. This hypothesis has been used by Bullard (1949) and Bullard et al (1950) to explain the westward drift of the nondipole part of the geomagnetic field. It can be easily demonstrated, however, that the hypothesis of angular-momentum mixing is false (Busse 1971). Convection in a tall cylindrical annulus rotating about its vertical axis provides a good example. When the annulus is cooled from the inside and heated from the outside, convection driven by centrifugal buoyancy occurs in the form of two-dimensional columns similar to those discussed in the preceding section. Since the Coriolis force is entirely balanced by the pressure when the convective motion is exactly two-dimensional, the effects of rotation vanish from the dynamics of the problem. The solution becomes identical to that of two-dimensional convection rolls in a layer heated from below, and the possibility of the generation of a mean flow can be excluded in this case. A more detailed analysis of the problem in the case of an annulus of finite height reveals that the sign of the differential rotation depends on the radial curvature of the top and bottom boundaries (F. Busse and L. Hood, in preparation). The problem of the generation of differential rotation has been investigated in more detail in the astrophysical context. The sun actually offers the best argument against the hypothesis of angular-momentum mixing, since the highest angular velocity is observed at the solar equator. Although solar convection represents a highly turbulent system, the equatorial acceleration can be explained qualitatively by the nonlinear solution for convection in a thin rotating spherical shell (Busse 1970b). Recent numerical calculations by Gilman (1972, 1977) indicate that the maximum angular velocity shifts to higher latitudes as the Ekman number is decreased. It is evident from the numerical results that the problem of differential rotation in the earth's core defies a simple solution.

5 MODELS OF THE GEODYNAMO

5.1 *Historical Developments*

The understanding of the origin of the earth's magnetic field has always been the main motivation for the development of dynamo theory. It is thus not surprising that a large number of dynamo models have been constructed that take into account the spherical configuration of the earth's core even though rather unrealistic assumptions are made about the velocity field. The mathematical complexity of the full magnetohydrodynamic problem was the rationale given for the attempts to understand the geodynamo on the basis of kinematic theory alone. In addition, the belief was widely held that dynamo action depended on rather special properties of the velocity field. It was tacitly assumed that if a solution describing the main features of the earth's magnetic field could be found, the associated velocity field would approximate the flow actually realized in the core. The development of dynamo theory in the last decade has shown that this belief is unfounded. The work by G. O. Roberts (1970) and others has demonstrated quite convincingly that nearly all velocity fields can give rise to dynamo action if the magnetic Reynolds number is high enough. The main features of the geomagnetic field can undoubtedly be reproduced by a variety of very different velocity fields, especially since so little is known about the toroidal field in the core. Today it is evident that the solution of the full magnetohydrodynamic dynamo problem cannot be avoided. Although the forces causing motions in the earth's core are not well known, there appear to be only a few reasonable choices. It can therefore be expected that the construction of sufficiently detailed magnetohydrodynamic models will provide the constraints required to isolate the mechanism of the geodynamo. Since the kinematic problem is always a part of the full problem, the methods used in kinematic dynamo theory will be of continuing interest. Some of them are discussed briefly below.

5.2 *Kinematic Models*

Kinematic dynamo theory does not provide constraints for the choice of velocity fields, so heuristic arguments have often been introduced to justify a particular selection. In many kinematic models a strong azimuthal flow is assumed that provides a simple way to generate a toroidal magnetic field from a poloidal field. This idea was expressed in the early work by Elsasser (1947, 1956) and Bullard (1949) and is the basis for Parker's (1955) dynamo process. The observed westward drift of the nondipole component of the geomagnetic field can be interpreted as a manifestation of a retrograde differential rotation with an amplitude of 10^{-3} m sec^{-1} in the outer parts of the core. On the other hand the westward drift may represent the propagation of magnetohydrodynamic waves in the earth's core, as proposed by Braginsky (1965b, 1967), Hide (1966), and Malkus (1967). A mechanism for the selective excitation of westward-drifting waves based on the instability of a mean toroidal field was discovered by Acheson (1972). Unfortunately it is not possible, as Backus (1968) has shown, to decide on observational grounds whether the westward drift represents a material velocity or a phase- or group-velocity of waves in the core. It must therefore be concluded that the observational evidence provides little help for an appropriate choice of the velocity field.

The generation of the poloidal field from the toroidal field requires a non-axisymmetric process according to Cowling's theorem. Parker (1955, 1970, 1971) in his pioneering work assumed turbulent cyclonic eddies driven by thermal buoyancy and was led to equations that anticipated the more formally derived equations of *mean-field electrodynamics* mentioned in Section 2.5. Bullard & Gellman (1954) assumed a poloidal velocity field with a cos 2ϕ dependence on the azimuthal coordinate ϕ. The latter authors were the first to apply numerical methods for the solution of the dynamo problem. With their limited computational facility Bullard & Gellman were unable to demonstrate numerical convergence of their results. Later authors found that the critical magnetic Reynolds number of the Bullard–Gellman dynamo increased as more terms of the expansion in spherical harmonics were taken into account. Gibson & Roberts (1969) in particular showed that it is unlikely that a stationary dynamo of the Bullard–Gellman type exists. Ever since, the problem of convergence has been a major issue in numerical investigations of dynamos. Part of the problem stems from the restriction of Bullard and Gellman's analysis to stationary solutions even though in general the growth rate p does not move through zero but intersects the imaginary axis at some finite value as the magnetic Reynolds number increases. Recent numerical calculations of dynamos in a sphere have been more successful. G. O. Roberts (see P. H. Roberts 1971) and Gubbins (1973) considered cases of axisymmetric velocity fields and obtained particularly simple solutions with an azimuthal dependence of the form exp $\{im\phi\}$. Pekeris, Accad & Shkoller (1973) used Beltrami velocity fields that are characterized by $\mathbf{v} \times (\nabla \times \mathbf{v}) = 0$ and thus satisfy the stationary equations of motion for an ideal flow subject to a conservative body force. The arguments for the geophysical significance of these velocity fields are not convincing, however. More recently Kumar & Roberts (1975) obtained a large class of spherical dynamo solutions that have been used for a test of the analytical dynamo approach of Braginsky (1965a).

Among the kinematic models Braginsky's dynamo (1965a,b, 1975) is of particular interest because it is based on the limit of high magnetic Reynolds number. In addition to the use of an expansion in powers of $Rm^{-1/2}$ Braginsky assumed a large toroidal magnetic field in zeroth order and showed that the nested system of equations resulting from the expansion could be solved in a consistent manner. Tough & Roberts (1968) and Soward (1972) generalized and clarified Braginsky's approach and took the equations of motion into account. Braginsky himself (1967, 1975) has interpreted the fluctuating part of the velocity field in his dynamo as buoyancy-driven Alfvén waves, which he christened MAC waves because of the combined magnetic, buoyancy (Archimedean), and Coriolis forces. A closed solution of the magnetohydrodynamic problem has not yet been given, however. For a review of the various possibilities of wave motions and other general aspects of the magnetohydrodynamics of the earth's core see the review articles by Roberts & Soward (1972), Hide & Stewartson (1972), and Acheson & Hide (1973).

5.3 *Magnetohydrodynamic Models*

The ultimate goal of the magnetohydrodynamic theory of the geodynamo is to solve the coupled system of the equations of motion and the dynamo equation for geophysically reasonable forces and to attain amplitudes of the magnetic field comparable to those observed. Only partial steps towards the realization of this goal have been achieved. In this section we consider a particularly simple magnetohydrodynamic model that includes most of the important physical ingredients of the geodynamo even though the geometrical configuration is not that of a sphere.

It is evident from the discussion in Section 4.2 that the problem of convection in a rotating sphere is essentially identical to the problem of convection in a rotating annulus that has conical boundaries and is subject to a radial gravity (Busse 1970a). This model can be extended to the investigation of a dynamo process driven by convection. To obtain a simple boundary condition for the magnetic field the radial extent D of the annulus is assumed to be large compared with the height in the axial direction. The small-gap assumption $D \ll s_0$, where s_0 is the mean radius of the annulus, is retained. This allows the use of a Cartesian system of coordinates and the application of analytical methods.

The solution of the dynamo equation turns out to be rather similar to the example discussed in Section 2.5. The toroidal part of the velocity field (2.14) corresponds to the columnar convection motion and the motion along the columns is generated by the Ekman suction at the top and bottom boundaries or by higher-order effects introduced by the finite inclination of the boundaries (for details see Busse 1975b). A minor difference is that the function w is z dependent, in contrast to expression (2.15). Because of the finite height of the annulus there exists a smallest value of the convection amplitude for which dynamo action is possible. A rough estimate for this value can be obtained from condition (2.22) if γ is set equal to 2π divided by the height of the annulus. It can be shown that the corresponding amplitude of convection is consistent with the assumption that only a small fraction of the observed heat flux at the earth's surface originates in the core. The form of the mean magnetic field for the strongest growing mode is sketched in Figure 7. Both the azimuthal and the radial component are antisymmetric with respect to the equatorial plane. The radial component gives rise to an axisymmetric dipolar field outside the annulus. Thus the model reproduces the three primary features of the geomagnetic field: the dipolar nature, the approximate alignment with the axis of rotation, and the stationarity of the dynamo process.

Except for the secular variations the geomagnetic field indicates a stationary geodynamo. This must be viewed in contrast to the oscillatory solar dynamo, with a period of 22 years corresponding roughly to the time scale of turbulent magnetic diffusion in the solar convection zone. Reversals of the geomagnetic field occur on a time scale of a few 10^5 years and must be regarded as a secondary

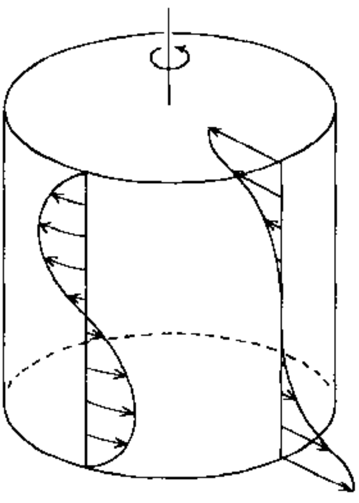

Figure 7 A qualitative sketch of the mean magnetic field in the annulus model of the geodynamo. The poloidal field (right) continues as a dipolar field outside the conducting region.

phenomenon of the dynamo process, since that time scale is much longer than the time scale of magnetic diffusion. There is little known about the causes of reversals. They could represent excursions from normal oscillations of the dipole strength about its equilibrium value, as suggested by the analogous phenomenon found in the case of disk dynamos (see Section 2.2). Or they may be triggered by external events, as suggested by the surprising correlation between reversals and the impacts of large meteorites (Glass & Heezen 1967, Pohl 1976).

The analysis of the annulus model of the geodynamo discussed above has been extended into the nonlinear regime using the perturbation method mentioned in Section 2.6. An interesting phenomenon is the property that the Lorentz force can actually enhance dynamo action (Busse 1977b). This property contradicts Lenz' rule which describes the normal effect of the Lorentz force, but which does not have the character of a rigorous law in magnetohydrodynamics. The property of the Lorentz force to relax dynamic constraints, in particular in a rotating system, cannot be fully comprehended within the framework of a perturbation analysis (Soward 1974). It is generally believed that the relaxation of the dynamical constraint imposed by the Coriolis force is the basic reason for the generation of magnetic fields in planets and stars and that the amplitude of the magnetic field is determined by this property. Only speculative suggestions about the appropriate balance have been proposed so far. This theme is discussed in the next chapter.

An important constraint that the Lorentz force must satisfy in a rotating fluid contained by axisymmetric boundaries is Taylor's (1963) condition

$$\int_{s=\text{const}} [(\nabla \times \mathbf{B}) \times \mathbf{B}]_{\phi}\, d\phi\, dz = 0. \tag{5.1}$$

The integral is extended over an arbitrary concentric cylindrical surface intersecting the contained fluid and can be readily derived from the equations of motion in the limit of vanishing viscosity. In the annulus discussed above the ϕ component of the Lorentz force becomes small in the limit when D is very large compared with the height of the annulus and viscous forces can be invoked to avoid constraint (5.1). Otherwise a differential rotation will be generated that tends to modify $\mathbf{B}$ in such a way that (5.1) is satisfied. For an example of this process in the case of a sphere we refer to Proctor (1977).

6 PLANETARY DYNAMOS

6.1 *The Equilibration of Magnetic Energy*

The most interesting question of dynamo theory—and the most difficult to answer—is the question of the relationship determining the amplitude of the geomagnetic field. That a definite relationship exists is indicated by the fact that in spite of many reversals the amplitude has not changed much throughout geologic history. I have already emphasized that simple criteria, for example the equilibration between kinetic and magnetic energy, are not relevant in dynamo theory, and that the knowledge of the dipole strength of other planetary magnetic fields may play a key role in the determination of equilibration criteria. To start the discussion let us consider three possible relationships between Coriolis force (C), Lorentz force (L), and viscous friction (F),

(*a*) C $\gg$ F $\gg$ L; (*b*) C $\gg$ L $\gg$ F; (*c*) L $\approx$ C $\gg$ F.

In cases (*a*) and (*b*) it is assumed that the major part of the Coriolis force is balanced by the pressure gradient. Case (*a*) is almost certainly not realized in the earth's core. I mention it only because it is accessible to the magnetohydrodynamic perturbation approach discussed in the preceding section.

Traditionally case (*c*) has been assumed to describe the balance of the geodynamo. The investigation of the simple problem of convection in a layer heated from below and rotating about a vertical axis indicates that the presence of a homogeneous magnetic field minimizes the critical temperature difference for the onset of convection when case (*c*) is realized (Eltayeb 1972, Roberts & Stewartson 1975). Case (*c*) requires a toroidal field of several hundred Gauss, and although the westward drift of the nondipole field has been interpreted in favor of such a high field strength (Braginsky 1965b, Hide 1966), there are strong arguments against this hypothesis. First, there is no need to balance the entire Coriolis force by the Lorentz force, as in the example of the horizontal convection layer, since the gravity vector is not parallel to the axis of rotation in most of the earth's core. The pressure gradient offers an energetically preferred balance for the Coriolis force. Second, the high ohmic dissipation associated with a large toroidal field imposes considerable strain on any proposal for the energy source of the geodynamo. Thermal convection would be an inadequate source of energy in this case because of its low efficiency (Verhoogen 1961) and the upper bound on the heat flux from the earth's core provided by the observed surface heat flux. Third, there are alternative interpretations possible for the westward drift that do not require the assumption of a large toroidal field. Balance (*b*) seems to me to be the most likely one. Arguments in support of such a balance come from the model discussed in Section 4.3, which suggests that poloidal and toroidal components of the magnetic field in the earth's core do not differ by more than a factor 10, say, and from the following more speculative considerations.

6.2 *Dynamical Constraints for the Strength of Planetary Magnetic Fields*

The annulus model of the geodynamo discussed in Section 4.3 can be easily extended into the regime (*b*) in which the Lorentz force exceeds viscous friction if the poloidal magnetic field is neglected. Since the latter is presumably somewhat smaller than the toroidal field anyhow, the mathematical benefits of the neglect of the poloidal field outweigh the advantages of an accurate but more cumbersome analysis. The analysis of convection in a rotating annulus in the presence of an azimuthal magnetic field assumes the geostrophic balance, as discussed in Section 3. The Lorentz force, however, now becomes the major effect in balancing the deviations from the Proudman-Taylor condition caused by the inclination of the conical boundaries. There are two major results (Busse 1976, see also Eltayeb & Kumar 1977). The wave number of convection columns in the azimuthal direction is no longer of the order $E^{-1/3}$ but decreases as the magnetic field strength increases.

Second, there is an upper limit of the magnetic field strength beyond which solutions approximately satisfying the geostrophic balance can no longer exist. For the case of planetary interest the upper bound assumes the form

$$2(\lambda/\kappa)M \equiv (\lambda/\kappa)B_0^2/\Omega^2 r_0^2 \rho_0 \mu \lesssim \eta^2/a^4, \tag{6.1}$$

where κ is the thermal diffusivity, B_0 is the strength of the magnetic induction, and η is the tangent of the angle of inclination. The annulus model does not provide a reasonable lower bound for the dimensionless wavenumber a of the convection columns. This must come from the fact that dynamo action ceases if the number of eddies becomes too small. Gubbins (1973) and Bullard & Gubbins (1977) found that at least two eddies are required in the radial direction in a spherical shell resembling the earth's core. If we therefore assume a lower bound of the order 10 for a we find that the right-hand side of condition (6.1) yields a value somewhat less than 10^{-4}.

At the present time two planets, Jupiter and Mercury, are known to exhibit a large-scale magnetic field. The existence of these fields is difficult to explain unless a dynamo process occurs in the respective electrically conducting planetary cores. Both magnetic fields are similar to the geomagnetic field in that they are predominantly dipolar and nearly aligned with the axis of rotation. This together with the fact that thermal convection appears to be the most likely energy source in all cases suggests that the strengths of the magnetic fields may be determined by a common constraint. Condition (6.1) could express such a constraint. Indeed, if the observed polar field strength extrapolated to the core is used in place of B_0 and the values of Table 2 are used a surprisingly similar value of the expected order of magnitude is obtained in all three cases. Clearly, a more detailed analysis is needed to establish this hypothesis. At least it suggests that a general theory of a convection-driven dynamo in a sphere may be applicable to a variety of planetary cores.

An alternative possibility has been discussed by Dolginov (1976), who considers precession as the common origin of planetary magnetism. Assuming heuristically that the magnetic field strength is proportional to $|\mathbf{\Omega} \times \mathbf{\Omega}_p| \rho r_0^3/\lambda$ where $\mathbf{\Omega}_p$ is the angular velocity of precession, Dolginov finds reasonable agreement for various

Table 2 Properties of planetary cores

		Earth	Jupiter	Mercury
angular velocity	Ω (10^{-5} sec^{-1})	7.3	17.5	0.12
characteristic dimension	l (10^6 m)	3.5	10	1.8
magnetic diffusivity	λ (m^2 sec^{-1})	1.6	1.2	1
thermal diffusivity	κ (10^{-6} m^2 sec^{-1})	7	40	4
magnetic induction	B (10^{-4} T)	3.7	23	0.019
energy parameter	M (10^{-10})	0.85	4.6	0.33
	$\frac{\lambda}{\kappa}M$ (10^{-5})	1.9	1.4	0.85

planets. Since no physical justification for the assumed balance is given and since the actual torque exerted by the planetary mantle on the core has not been taken into account, the comparison is not entirely satisfactory.

7 CONCLUSION

Like hydrodynamic turbulence the generation of magnetic fields in an electrically conducting fluid originates in the form of an instability. Since this instability is characterized by the appearance of an additional physical quantity, the magnetic field, it might be expected that the theory of planetary magnetism is even more complex than the theory of hydrodynamic turbulence. The intimate relationship between planetary magnetism and rotation and the distinct regular features exhibited by planetary magnetic fields suggest, however, that the mechanism of the generation of the magnetic fields in planetary cores is actually simpler than ordinary hydrodynamic turbulence. Whether or not this belief is overly optimistic will become clear within the next decade if the progress of dynamo theory continues at the same rate as in the past.

At the present time dynamo theory is entering a new area. The basic dynamo mechanism has been understood well enough that the problem of quantitative models for the geodynamo can be attacked. The solution of this problem will require considerable numerical efforts and it is likely that a number of new nonlinear phenomena will be discovered in the course of the computations. It is therefore advisable to continue the general exploration of the magnetohydrodynamic dynamo problem by analytical as well as numerical methods. The investigation of nonlinear oscillations and the possibility of reversals of the magnetic field appears to be of particular interest. Even the kinematic dynamo problem still offers numerous mathematical challenges, as was indicated in Section 2.4. A concise classification of the different forms of dynamo action is desirable and the influences of different boundary conditions needs further study. The history of the dynamo problem has been full of surprising results and if the past is any indication of the future, there are many more fascinating properties waiting to be discovered.

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