## Phase-conjugating mirror with continuous-wave gain

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We demonstrate a phase-conjugating mirror that has a continuous-wave power reflectivity much greater than unity (gain ~100). This mirror uses nonresonant degenerate four-wave mixing in a single crystal of barium titanate (BaTiO<sub>3</sub>). With our mirror we have (1) observed cw self-oscillation in an optical resonator formed by this mirror and a normal mirror, (2) demonstrated a cw oscillator that, in spite of phase-distorting material placed inside the resonator, will always emit a TEM<sub>00</sub> mode, and (3) demonstrated an optical image amplifier. This mirror will work at any visible wavelength and with weak (milliwatt or weaker) pump beams.

Phase-conjugating mirrors were demonstrated previously with reflectivities that are greater than unity, and self-oscillation observed, but only for a few nanoseconds.<sup>1</sup> The largest reflectivity reported to date for a continuous-wave (cw) phase-conjugating mirror is only 17%.<sup>2</sup> In those experiments, either resonant degenerate four-wave mixing was necessary, which permitted operation only over a small frequency range ( $\sim 1$  GHz), or beams of megawatt power were needed. In this Letter, we report the first known demonstration of a cw phase-conjugating mirror with reflectivity greater than unity. We employ degenerate four-wave mixing of milliwatt beams, mediated by the photorefractive effect in a single crystal of barium titanate of 2.2-mm  $\times$ 2.8-mm  $\times$  4.2-mm dimensions at room temperature. The effect is nonresonant and operates over a large fraction of the visible spectrum. The main disadvantage of this phase conjugating (pc) mirror is its relatively slow response time, of the order of 1 sec at the nominal milliwatt-power levels of common lasers. (However, this response time shortens inversely with the pumpbeam power.)

By using our pc mirror, we have (1) observed cw self-oscillation in an optical resonator formed by this mirror and a normal mirror, (2) demonstrated wavefront correction when a phase-distorting medium is placed inside the self-oscillating resonator (that is, the pc mirror alters the transverse-mode structure of the resonator to compensate automatically for any phase distortions in the cavity), and (3) demonstrated optical image amplification.

To understand the operation of this mirror, consider two optical beams, with wave vectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , having nonorthogonal polarizations and the same angular frequency  $\omega$ . Call these beams the writing beams. Where they intersect in the crystal, they form an intensity-interference pattern with wave vector  $\mathbf{k} \equiv \hat{k}k =$  $\mathbf{k}_1 - \mathbf{k}_2$ . Electrical charges (of unknown origin) migrate in the crystal from the peaks into the troughs of the intensity-interference pattern and eventually reach a static-charge distribution. These charges create a strong, static, spatially periodic electric field equal to  $\hat{k} \operatorname{Re}[E \exp(i\mathbf{k} \cdot \mathbf{x})]$ . This field in turn modulates the index of refraction by the first-order electro-optic (Pockels) effect to create a refractive-index grating in the crystal.<sup>3</sup> A third reading beam, also at  $\omega$ , having a wave vector  $\mathbf{k}_3 = -\mathbf{k}_1$ , scatters from this grating to create a fourth signal beam of wave vector  $\mathbf{k}_4 = -\mathbf{k}_2$ , which is a phase conjugate of the second beam.<sup>4,5</sup> (See Fig. 1.)

Let  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$  be the incident intensities of the writing reference beam, the writing image beam, the reading beam, and the output intensity of the phaseconjugate signal beam, respectively. Consider the case in which all four beams are confined to the y-z plane, with the z direction taken along the c axis of the crystal. (See Fig. 1.) According to our previous theory<sup>6</sup> of grating formation in BaTiO<sub>3</sub>, when  $I_4 \leq 0.5I_3$ , the mirror reflectivity, here defined as ratio  $R = I_4/I_2$  of the intensities of the phase-conjugate beam to the image beam, is well approximated by

$$R_{\rm ord} = \left| \frac{\omega L E \eta}{4c} n_o^3 r_{13} \cos \theta \right|^2 \tag{1}$$

for a reading beam with ordinary polarization and by

$$R_{\text{ext}} = \left| \frac{\omega L E \eta}{4 c n_3} \cos \theta \left( n_e^4 r_{33} \sin \alpha_1 \sin \alpha_2 + 2 n_e^2 n_o^2 r_{42} \sin^2 \theta + n_o^4 r_{13} \cos \alpha_1 \cos \alpha_2 \right) \right|^2$$
(2)



Fig. 1. The reference writing beam (1) and the image writing beam (2) interfere in a crystal of undoped  $BaTiO_3$  to make a refractive-index grating with wave vector **k**. The reading beam (3) Bragg scatters off this grating to produce the phase-conjugate signal beam (4). The crystal is immersed in index-matching oil.

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for a reading beam with extraordinary polarization in the crystal. These expressions do not include the effects of two-beam energy coupling<sup>6-8</sup> or the phase mismatch that are due to the change in the index of refraction from each beam alone.<sup>9</sup> Here, c is the speed of light in vacuum,  $\eta \equiv (I_3/I_2)^{1/2}$ ,  $n_o$  and  $n_e$  are the ordinary and extraordinary indices of refraction in the crystal ( $n_o = 2.488$  and  $n_e = 2.424$  at 514 nm), and  $n_3$  is the index of refraction for the reading beam. L is the effective interaction length and is approximately l $\exp(-\frac{1}{2}\gamma l)$ , where l is the beam length in the crystal and  $\gamma$  is the optical intensity-attenuation coefficient. The  $r_{ij}$  are the conventional contracted electro-optic coefficients and in BaTiO<sub>3</sub> (in units of  $10^{-12}$  mV) (Ref. 10) are  $r_{13} = 8$ ,  $r_{33} = 23$ , and  $r_{42} = 820$ . In Eqs. (1) and (2),  $\boldsymbol{\theta}$  is the angle between the grating wave vector  $\mathbf{k}$  and the direction of the c axis, and  $\alpha_1$  and  $\alpha_2$  are the angles formed by each writing beam with the y axis. From Ref. 6 one sees that the intensity dependence of  $E\eta$  is contained in a factor  $I_1^{1/2}I_3^{1/2}/(I_1 + I_2 + I_3)$  so that, for  $I_2 \ll I_1$  or  $I_2 \ll I_3$ , the mirror reflectivity R given by either Eq. (1) or Eq. (2) is independent of the incident intensity  $I_2$  and depends only on the relative intensity  $I_3/I_1$  of the counterpropagating beams. Otherwise, the electric-field amplitude E depends only on the temperature of the crystal lattice, the charge and density of the migrating carriers, the dc dielectric constants of the crystal, and the relative orientation of the crystal and the optical beams.<sup>6</sup> In general, the writing image beam will form an intensity-interference pattern not only with the reference beam but also with the reading beam.<sup>6</sup> For the range of beam angles used below, we estimate that this grating contributes about 10% to the observed reflectivity R.

Inspection of Eq. (2) shows that, for a large range of angles, the reflectivity for extraordinary beams can be larger than unity, owing to the contribution from the unusually large  $r_{42}$  coefficient. (Previous experiments with  $BaTiO_3$  had k parallel to the crystal c axis, making  $\theta = 0$  and thereby precluding any contribution from the  $r_{42}$  term.) For example, with  $L \simeq 0.4$  cm ( $\gamma l \ll 1$ ),  $I_1 = I_3$ ,  $\theta = 22^{\circ}$ ,  $\alpha_1 = 18^{\circ}$ ,  $\alpha_2 = 26^{\circ}$ , and a calculated<sup>11</sup> value of  $E = 4.4 \times 10^2$  V/cm, we compute a mirror reflectivity of  $R_{\text{ext}} = 3.2$  for extraordinary polarizations at 514.5 nm. By approximating these conditions in an experiment with  $I_2 = 0.3 \text{ mW}$  and  $I_1 = I_3 = 5 \text{ mW}$ (beam area ~0.25 mm<sup>2</sup>), we observed  $R_{\text{ext}} \sim 2$ . However, in this instance we used ordinary rays for beams 1 and 2 to write the grating and used an extraordinary ray (beam 3) to read the grating [which does not alter Eq. (2) if  $\mathbf{k}_1 - \mathbf{k}_2 = \mathbf{k}_4 - \mathbf{k}_3$ ]. When we used extraordinary rays for all three incident beams, we observed even higher reflectivities ( $R_{
m ext} \sim 100$ ) because of the added contribution of energy coupling between beams 1 and 2. (With the above geometry, this coupling is less than 0.02 for ordinary rays but can exceed 50 for extraordinary rays, and the coupling greatly enhances the reflectivity by increasing the intensity of the image writing beam as it propagates through the crystal.)

The optical setup that we used to obtain these large reflectivities is shown in Fig. 1. The optical beams are incident upon the barium titanate at a glancing angle to the surface of the crystal. The crystal is immersed in index-matching oil (n = 1.51) in order to increase the



Fig. 2. A plot of the measured mirror reflectivity  $R_{\text{ext}} \equiv I_4/I_2$ as a function of the reading-beam intensity  $I_3$ . The objectbeam intensity was fixed at  $I_2 = I_1/4$ , and the angles of the incident beams were  $\alpha_1 = 16^\circ$ ,  $\alpha_2 = 24^\circ$ , and  $\theta = 20^\circ$  (see Fig. 1). In this plot,  $I_3$  has been normalized by the fixed intensity  $I_2$ .



Fig. 3. Optical setup for observing cw self-oscillation. The incident beams 1 and 3 are both linearly polarized in the plane of the figure and are extraordinary rays in the crystal. Self-oscillation is observed to grow between the crystal and a 94% reflectivity plane mirror M. Here L's are lenses with a focal length F = 100 mm, and P is a variable pinhole used to control the transverse-mode structure of the oscillation. The phase aberrator A is formed by bubbles of transparent glue on a microscope slide. The angles of the beams are about the same as in Fig. 2.

angle between the c axis and k. The argon laser produces a  $\text{TEM}_{00}$  Gaussian mode at 514.5 mm in a single longitudinal mode.

The following four experiments elucidate the mirror characteristics. In the first experiment, the intensities  $I_1$  and  $I_2$  of the writing beams are fixed, and the scattering efficiency of the grating is measured as the intensity  $I_3$  of the reading beam varies. All the beams are extraordinary rays in the crystal. From Fig. 2, it is seen that the intensity of the reflected beam (or signal beam) can be made to exceed the intensity of either of the writing beams. Since this signal beam is the phase conjugate of one of the writing beams (beam 2 in Fig. 1), the system acts as a phase-conjugate mirror with gain.

In the second experiment, two extraordinary counterpropagating beams (beams 1 and 3) are incident upon the crystal (beam 2 is blocked). A plane mirror is placed within view of the crystal, with the normal to the mirror directed approximately toward the crystal. Two new counterpropagating phase-conjugate beams are observed to grow between the crystal and the mirror with a time constant of the order of 1 sec. If the mirror is tilted, these new beams will fade away, only to reappear on whichever part of the mirror is closest to normal to the crystal. In the process of finding the cavity mode with the least loss in which to oscillate, the crystal finds and directs a beam at the most-reflective surface facing it. Oscillator output fades slowly (~1 sec) if beam 1 is blocked but extinguishes instantly if beam 3 is blocked, since beam 1 is helping to write the grating but beam 3 is reading it.

In the third experiment, two identical lenses (L) and an aberrator (A) are placed in the cavity formed by the phase-conjugating mirror (i.e., the crystal) and the mirror M, as in Fig. 3. Self-oscillation is allowed to build up, and its transverse-mode structure is photographed near both the real mirror and the wave-frontreversing mirror. Figure 4 shows these mode patterns, both of which are severely distorted by the aberrator. When a pinhole is placed at the focal length of the lens. in the manner suggested by AuYeung et al., <sup>12</sup> the mode pattern becomes uniform. If the crystal were acting just as an ordinary mirror, the light distribution, returning to the pinhole from this mirror, would be doubly distorted from having passed through the aberrator twice and would spill out and be blocked by the face of the pinhole, causing a large loss in the resonator. In fact, we observed that when the adjustable pinhole is made sufficiently small (800- $\mu$ m diameter) so as to reject high-order modes, the intracavity power in the resonator decreases at most by about 10% and sometimes increases, indicating that little light is lost on the walls of the pinhole and that the crystal is acting as a highquality phase conjugator.

In the fourth experiment, an image amplifier with an intensity gain of  $\sim 10$  is constructed by using two counterpropagating beams and an object beam, all extraordinary rays. When a resolution chart is placed in the object beam, an amplified real image of the resolution chart is observed. (See Fig. 5.)



Fig. 4. Photographs of far-field mode patterns of self-oscillation with a severe phase aberrator in the resonator cavity. (See Fig. 3.) With no aperture in the resonator cavity: mode pattern transmitted (a) through the back of the crystal and (b)through the 94% mirror. With a 1-mm-diameter pinhole in the cavity: mode pattern (c) from the back side of the crystal and (d) transmitted through the 94% mirror. These mode patterns were displayed on a white card 2 m from the cavity, photographed with Kodak Plus-X (ASA 125) film, and printed on high-contrast paper.



Fig. 5. Photograph of the real image of a resolution test chart (wheel diameter, 1 cm) formed in the object plane. The intensity of the image beam was measured to be  $\sim 10$  times the intensity of the object beam, demonstrating optical image amplification. The bright spot seen on the left-hand side is from self-oscillation between the crystal and one of the faces of the glass cuvette that holds the crystal. This image was photographed with Kodak Plus-X (ASA 125) film and printed on high-contrast paper. The angles of the beams are about the same as in Figs. 2 and 3.

In conclusion, we have demonstrated a cw phaseconjugating mirror with gain up to 100. We have used this mirror to construct an image amplifier and an optical resonator that self-oscillates. With the aid of a spatial filter, this oscillating resonator will correct phase aberrations inside the resonator cavity and emit a TEM<sub>00</sub> Gaussian mode.

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