LASER LEVEL EXPERIMENTS

MEASURING CURVATURE OVER WATER SURFACE

Terrestrial Laser Targeting (TLT) Method to Measure Curvature of Water Surface on Lake Balaton, Hungary and Lake Ijssel, Netherlands.

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EXPERIMENT GOALS

FECORE Inc. intends to achieve the greatest distance laser optical measurement over water surfaces in Hungary and The Netherlands connected to our series of refraction experiments.

The main principles of the experiments are:

- To study the effects of terrestrial refraction in different ambient conditions at very small incidence angles close to the Non-Uniform Density Transition Zone (NUDTZ) above water surfaces.
- To determine the shape of the surface of large bodies of water.

MOTIVATION

Through our previous experiments, we've found inconsistencies with the curvature of the geoid model over water surfaces, and we hypothesize the theoretical calculation of curvature on the surface of Lake Balaton and Lake Ijssel is non-existent.

We are researching whether these lakes have a geopotential surface anomaly, or are all water surfaces non-uniform with the geoid surface.

ABSTRACT

The Terrestrial Laser Targeting (TLT) method was used in order to achieve long-distance curvature measurements over water surfaces. A Super Accurate Laser Aiming Device (SALAD) and a high-precision laser device with a collimation of 0.08mRad were developed. Through the analysis of the error source models of curvature testing, the optical configuration of the testing devices was optimized. Several target distances in different ambient conditions in different locations were tested. Environment readings are referenced and calculated to reduce measurement errors caused by ambient conditions.

Through the above processes, the relative accuracy of the measurements meet the experiment design requirements. The TLT method used in the experiments has high accuracy and practical advantages.

INTRODUCTION

The geoid is defined as a more smoothed representation of the Earth and is described as the surface that would be assumed by the undisturbed surface of the sea. Therefore, the water surface follows the geopotential surface, and by that we have the common understanding water surfaces follow the curvature of Earth. The topographic surface is measured with different surveying methods all based on the assumption of the WGS84 model.

INTRODUCTION

The required accuracy depends on the needed deliverable output. Accuracy refers to how closely a measurement or observation compares to a true or established value, since measurements and observations are subject to errors. By analyzing the error-source models of curvature testing, the optical configuration design of the testing device was optimized.

The precision of the TLT measurement is expected to be within 1% of the volume compared to the target hidden height calculated on each measurement distance to arrive at a definitive result. The accuracy of the Terrestrial Laser Targeting method is affected by the angle of sight, distance from the object, and weather conditions. Considering those limitations, the research will evaluate and compare accuracy with the volume of expected target hidden height calculation.

INTRODUCTION

The Terrestrial Laser Targeting (TLT) survey method can be used with sufficient accuracy on large distance curvature measurements. Compared with other curvature test methods, the method used in this paper has proper accuracy and practical advantages.

In this document, we are introducing TLT measurements up to 40 km (25 miles) conducted by our research team as late as the 23rd of April, 2018.

OBJECTIVE OF RESEARCH

The general objective of this research is to evaluate and compare the results of TLT measurements over surfaces of lakes with the calculation of the geoid curvature to determine the shape of the lake surface.

Our secondary objective is to study the effects of terrestrial refraction in different ambient conditions at very small incidence angles close to the Non-Uniform Density Transition Zone (NUDTZ) above the surface of the lakes.

SCOPE AND LIMITATION OF THIS STUDY

The scope of this study is limited within evaluating and comparing the curvature of the surface on Lake Balaton and Lake Ijssel and the effects of small incidence angle refraction. Determining and evaluating the accuracy of the measurements requires favorable weather conditions. During measurements taken on Lake Balaton, there were many limitations, especially adverse weather conditions (cold, humidity, snow, and wind). Due to the unstable weather, all goals set forth were not accomplished in the timeframe allowed, so our measurements were continued on Lake Ijssel in favorable weather conditions.

SIGNIFICANCE OF THE STUDY

This document can be used as a spring board for further studies for those who are interested in the research of water surface model measurements.

IN THIS DOCUMENT

We will provide our measurements and guide you through:

- The experimental process
- Environmental conditions
- Curvature calculations
- Optical and geodesic correction factors
- The references and the terms used, as well as an explanation of results.

MEASUREMENT PROCEDURE - LAKE BALATON

The initial aiming of the SALAD occurred during daytime to the direction of the target by visual observation with a precision of ±1 degree using a Nikon P900 camera and a Celestron Powerseeker 70EQ telescope.

The laser beam was difficult to see from a side view as it was well collimated, therefore it had to be within a few degrees facing the observer to be detectable.

After sunset, the laser beam was adjusted parallel to the water surface using the horizon line and visible city lights on the opposite shore as a reference. An observation team was placed on the opposite shore spread along the coast line to locate the laser beam while maintaining communication with the laser operators in order to fine tune the beam's direction.

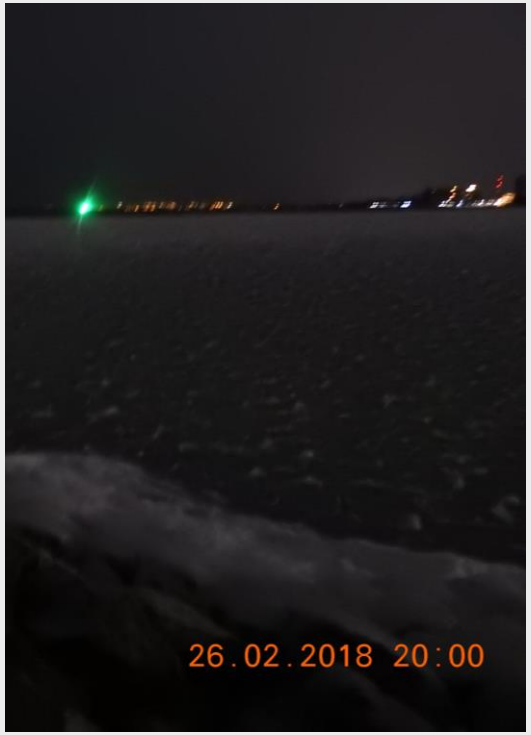
MEASUREMENT PROCEDURE - LAKE IJSSEL

Based on our experience with aiming difficulties at Lake Balaton, Mike Cavanaugh made changes in the software of the SALAD and developed an automatic GPS targeting system. The new aiming precision of the SALAD is ±0.01 degrees. We used a closer distance reference target with GPS coordinates to calibrate the laser heading, and the software was then capable of automatic laser targeting to any position based on GPS coordinates.

The observation team placed on the opposite shore shared their GPS coordinates with the laser team, and the laser was automatically pointed to that location. The laser beam then was fine tuned by the teams through GSM communication.

MEASUREMENT PROCEDURE

Once the beam was directly on the target, we took the measurement readings. We used optical visualization with cameras and a measurement board.



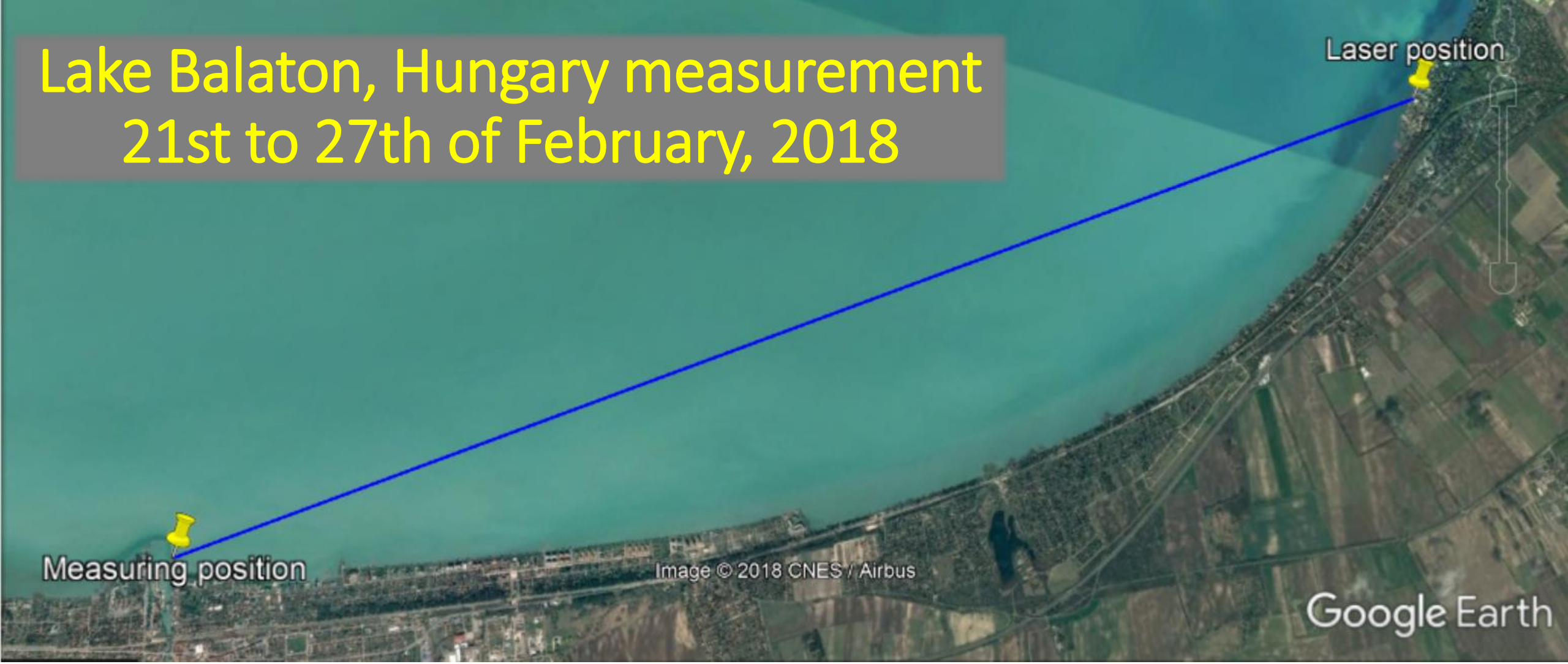
LASER BEAM HEIGHT CALCULATION

We determined the lowest minimum altitude of the laser beam on the curved surface model at the target location using Autocad with 14 digits precision and compared the results with generally accepted curvature calculators.

We then calculated the corrections for the height above MSL (Mean Sea Level), the WGS84 ellipsoid model, and the difference in geoid undulation.

REFRACTION

Laser light travels in a straight line through a homogeneous medium. Light angles due to different refraction indexes in a non-homogeneous atmosphere. We calculated the direction and amount of refraction to show how much laser-beam deviation affects the measurement outcome.
In a non-homogeneous atmosphere where the index of **refraction increases** with height, rays of sufficiently small initial elevation angles are **refracted upward**. This curvature is proportional to the rate of increase of the bi-directional index of refraction with height.

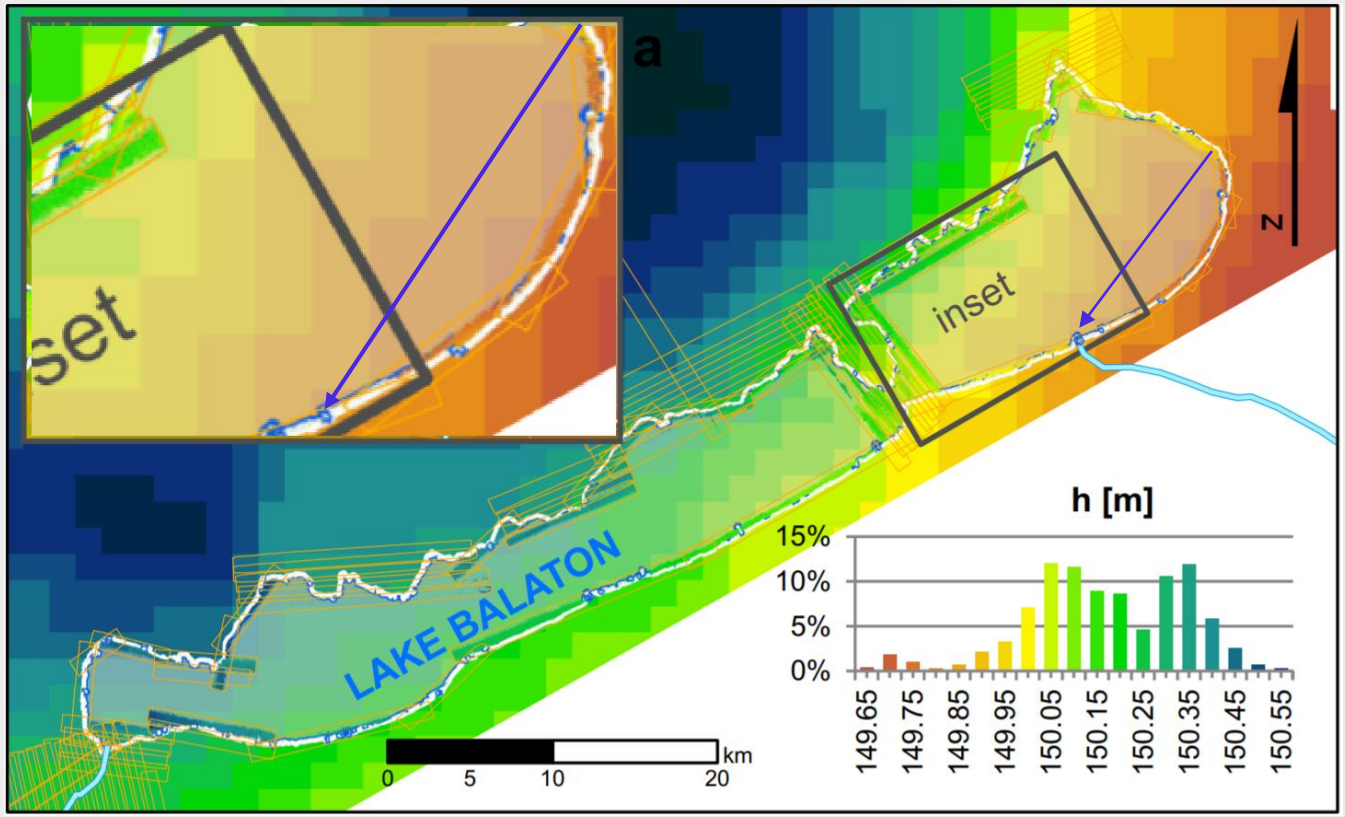


Map of Lake Balaton showing the measuring position and laser position.

LASER POSITION
Lake Balaton


CELESTRON

GEOID UNDULATION – LAKE BALATON



Geoid undulation is the term used to describe the distance of the geoid above (positive) or below (negative) the mathematical reference ellipsoid (WGS84).

Geoid undulation was at the **same level** in both positions.

[1]

EFFECT OF TIDES - LAKE BALATON

"Lake tides are known to have amplitudes of up to 10 cm (0.33 feet) in larger lakes (Trebitz, 2006). In case of Lake Balaton, the shallow depth and the relatively low water volume of the lake suggests this effect would be smaller. During long-term investigations of water movement conducted in the 1970's, no evidence of lake tides were observed (Muszkalay, 1973), therefore we do not expect tides to have influenced the lake level."

[1]

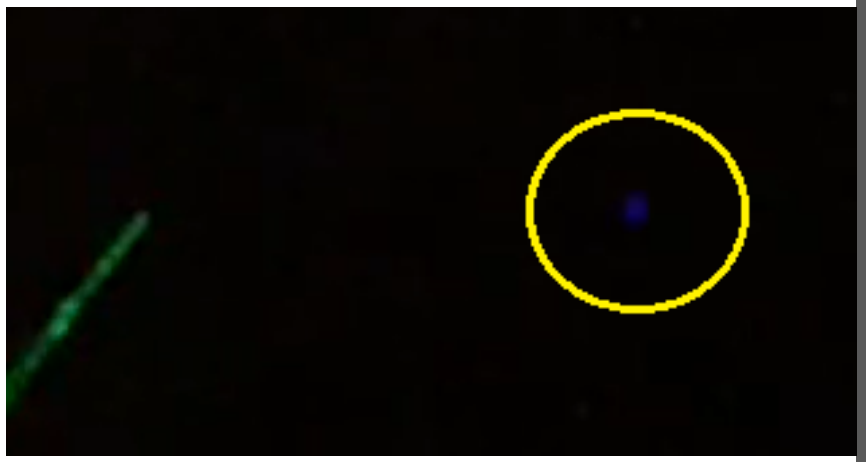
REFRACTION CONDITIONS

The lake was partly frozen, therefore walking along the beam on the ice or using a boat was not possible. As no data on the ambient conditions along the laser beam was available, we measured the conditions at the measuring position and laser position. The data showed there were no significant differences in temperature and humidity between the start and end points of the laser beam.

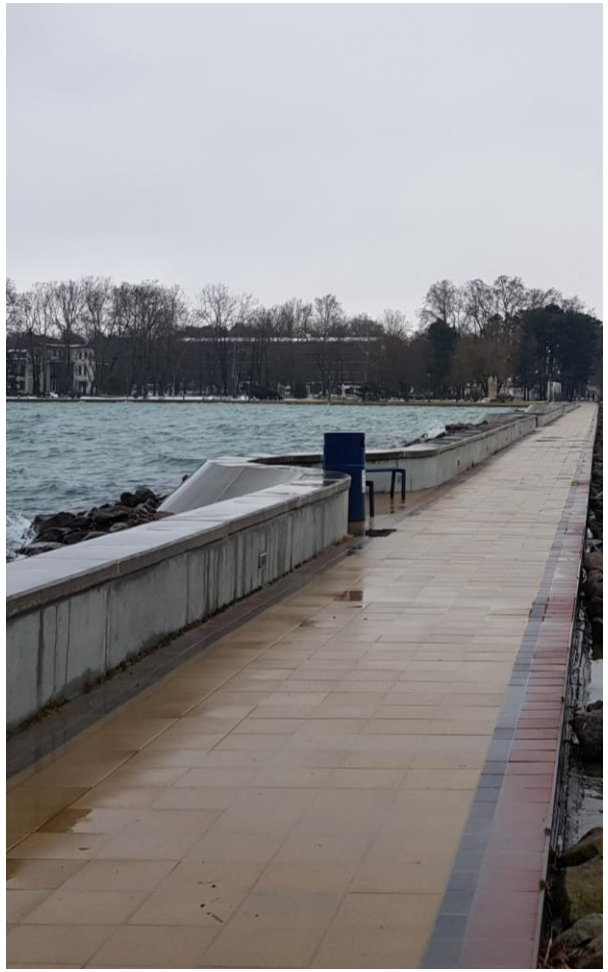
MEASUREMENT 1 - LAKE BALATON

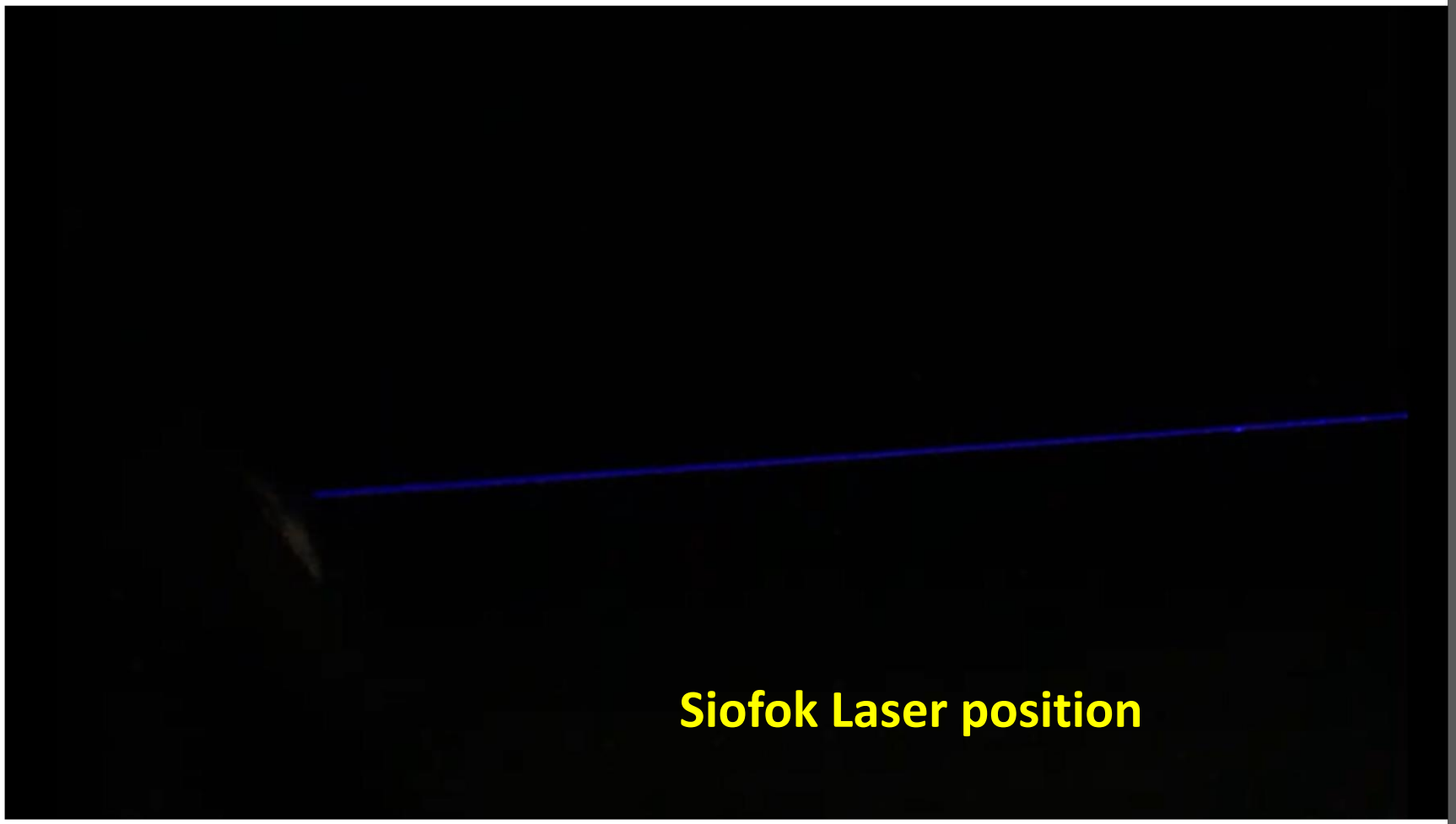
On the 22nd of February at 22:44 PM, the blue laser pointer was at the opposite shore (Siofok) to help the targeting of the SALAD at the laser position. The pointer was held in hand at 1.5 meters (4.92 feet) above the lake surface level at Siofok. The team at the laser position was able to see the beam and record it from 12 km (7.46 miles) at 1.6 meters (5.25 feet) above the water level.

MEASUREMENT 1 22nd February 2018 at 22:44 PM



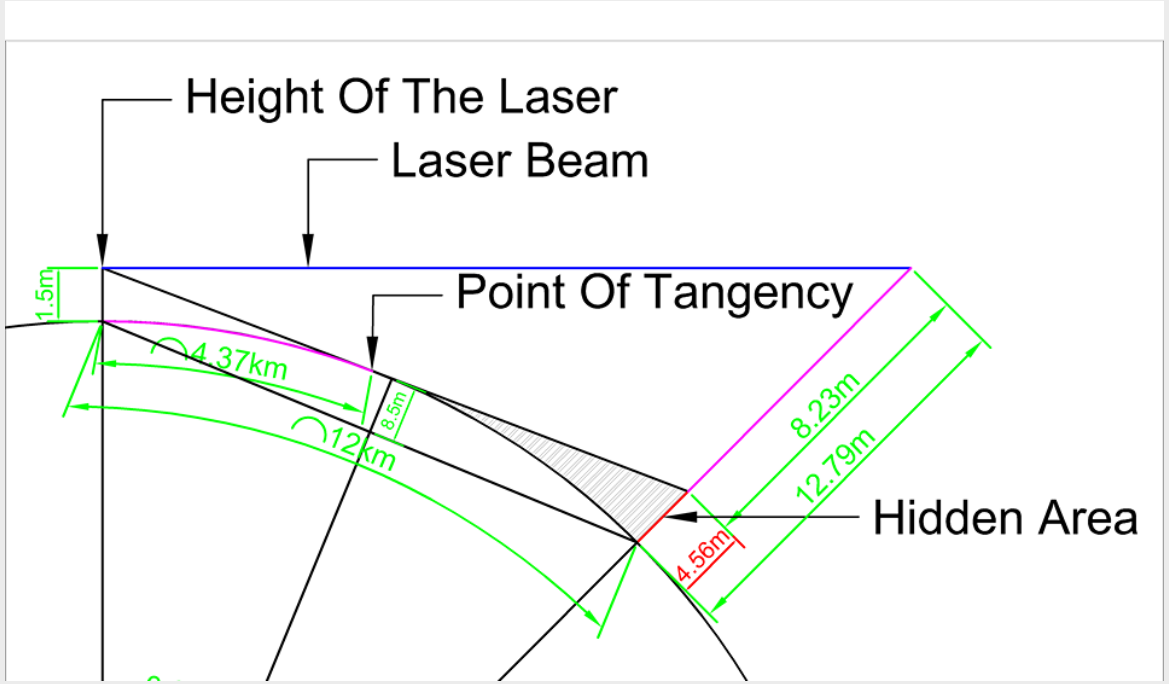
MEASUREMENT 1 22nd February 2018 at 22:44 PM





LASER BEAM HIDDEN HEIGHT CALCULATION

The 12km (7.46 miles) distance calculations based on a spherical model results in a target hidden height of the measurement position of 4.56 meters (14.96 feet)



CURVATURE CORRECTION WGS84 AND MSL

The radius of Earth is 6366.776 km (3956.131 miles) at the 46.911702° latitude (Siofok) on the WGS84 ellipsoid model, and 6366.75 km (3956.115 miles) at the 46.982097° latitude (Balatonvilagos Target). The measurement direction heading is 131.09°

$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

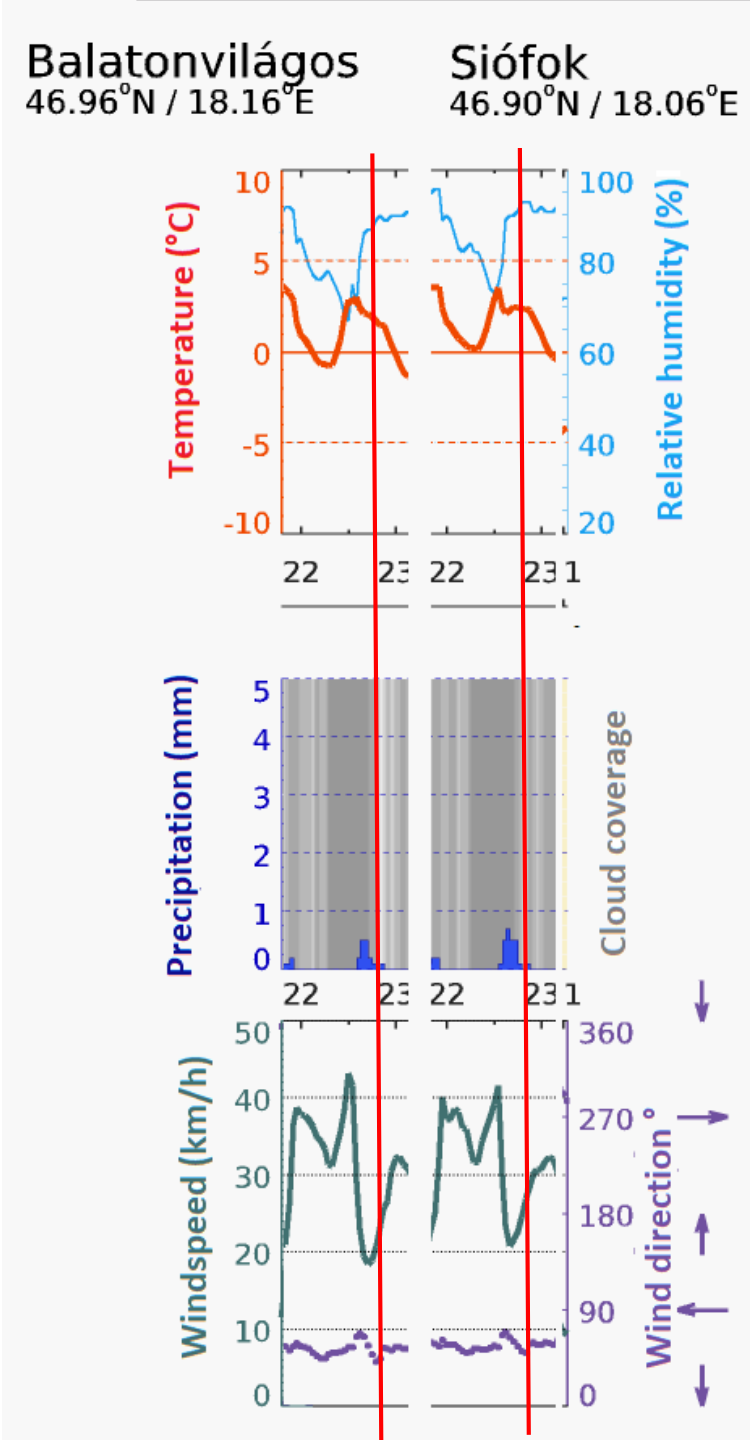
The calculated difference of curvature drop on WGS84 to the spherical model from Siofok to Target: **+0.046 mm** (+0.0018 inches)

The height of Lake Balaton above Mean Sea Level is +105 meters (+344.49 feet) that gives a calculated difference of **-0.187 mm** (+0.0074 inches) of curvature drop.

MEASUREMENT 1

ambient conditions
22nd February 2018 at 22:44 PM

Balatonvilagos
Hum. 90%
Temp. +2°C (35.6°F)
Wind 32km/h (19.9 mph)
waves 50cm high (1.64 feet)



Siofok
Hum. 92%
Temp. +3°C (37.4°F)
Wind 32km/h
Waves 50cm high

REFRACTION CALCULATION OF 1ST MEASUREMENT

The lake temperature was 2°C (35.6°F) and the air at night was around 3°C (37.4°F) above the lake.

The humidity is always higher the closer you are to the lake surface, which indicates a **lower refractive index**.

The ambient conditions showed that the refractive indexes were about the same at the two sides of the lake. The difference in temperature was marginal. The level of humidity decreases as you rise in altitude above lake surface.

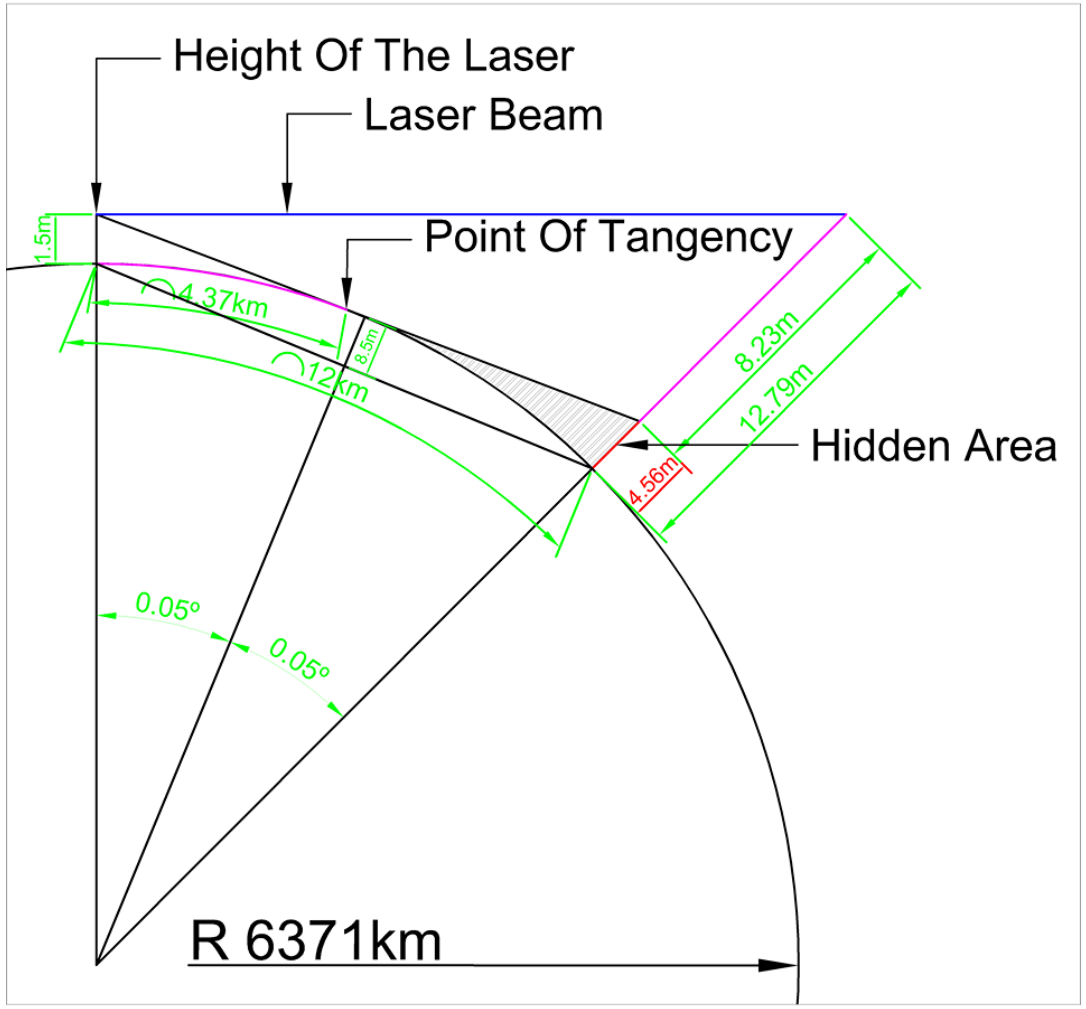
We concluded the gradients above the lake surface did not cause any significant refraction of the laser beam.

POSITIONS AND HEIGHT DATA AT LAKE BALATON

Balatonvilagos (laser position):
Latitude = 46.9820972222222° N = 46° 58' 55.55" N
Longitude = 18.162325° E = 18° 9' 44.37" E
GPS ellipsoidal height = 149.85 meters (491.6 feet)
Geoid height = 44.918 meters (147.367 feet)

Siofok (measurement position):
Latitude = 46.911702777778° N = 46° 54' 42.13" N
Longitude = 18.044494444444° E = 18° 2' 40.18" E
GPS ellipsoidal height = 149.85 meters (491.6 feet)
Geoid height = 45.047 meters (147.790 feet)

Difference of geoid height: -129 mm (-5.08 inches)



Laser beam was recorded at 1.6 meters (2.79 feet)

Measurement 1
Target hidden height spherical model:
4.570 meters (14.993 feet)

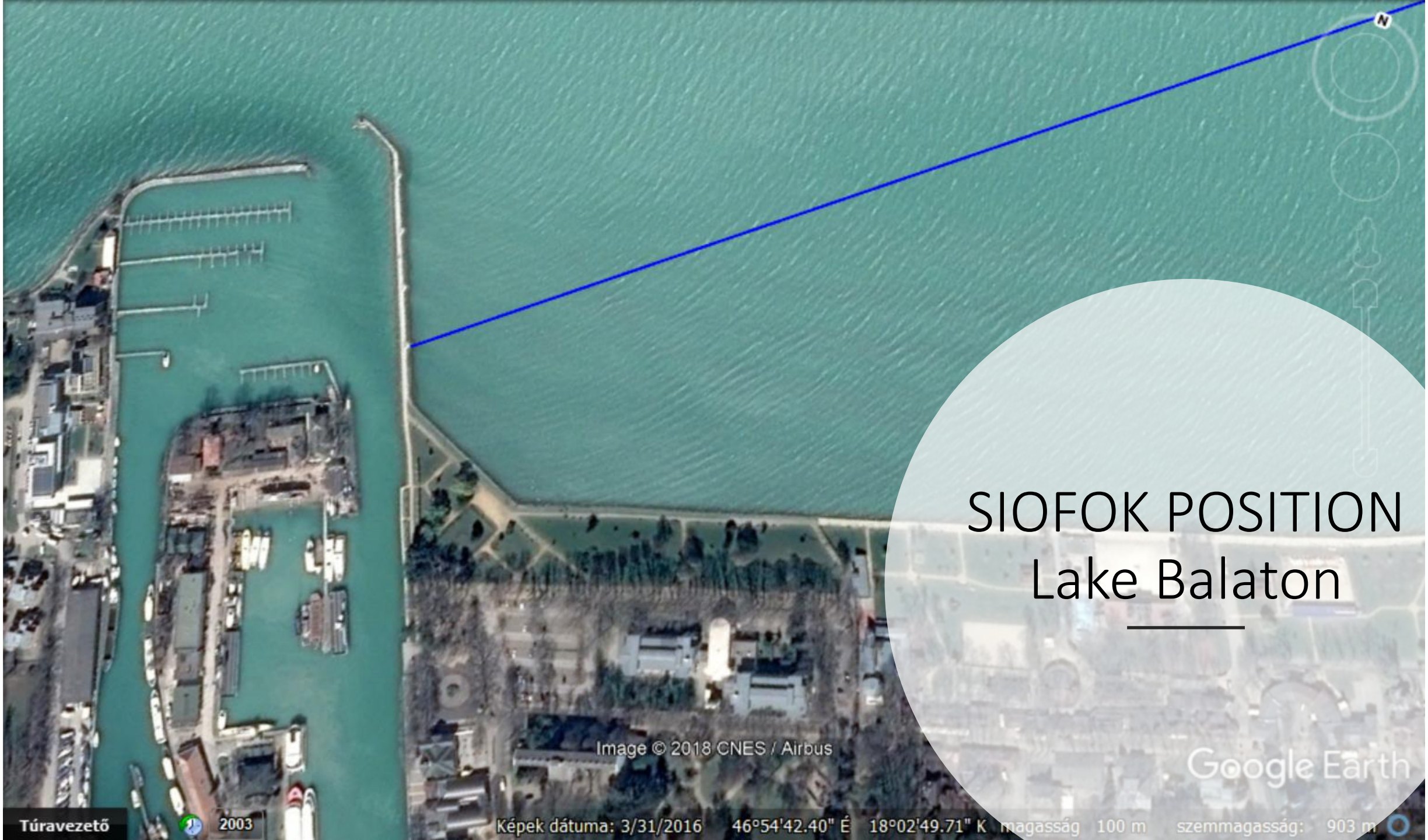
WGS84 laser to target correction:
+0.029 mm (0.0011 inch)

MSL correction:
-0.119 mm (-0,0047 inch)

Refraction correction
+0 mm (+0 inch)

EXPECTED target hidden height
4.5699 meters (14.9929 feet)

Difference of geoid height:
129 mm (-5.08 inches)



SIOFOK POSITION
Lake Balaton


19
18
17
16
15
14
13

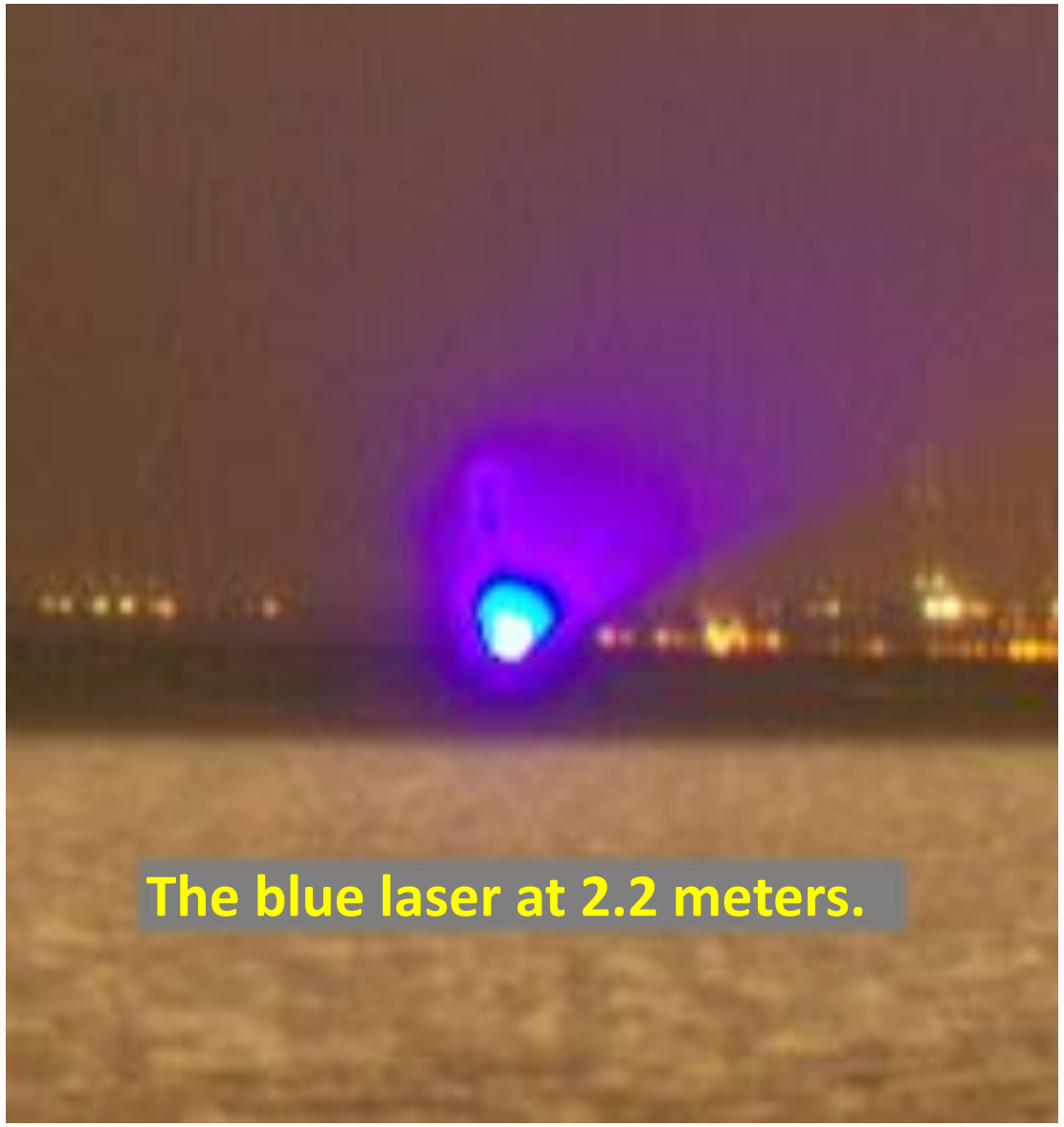

26.02.2018 20:03

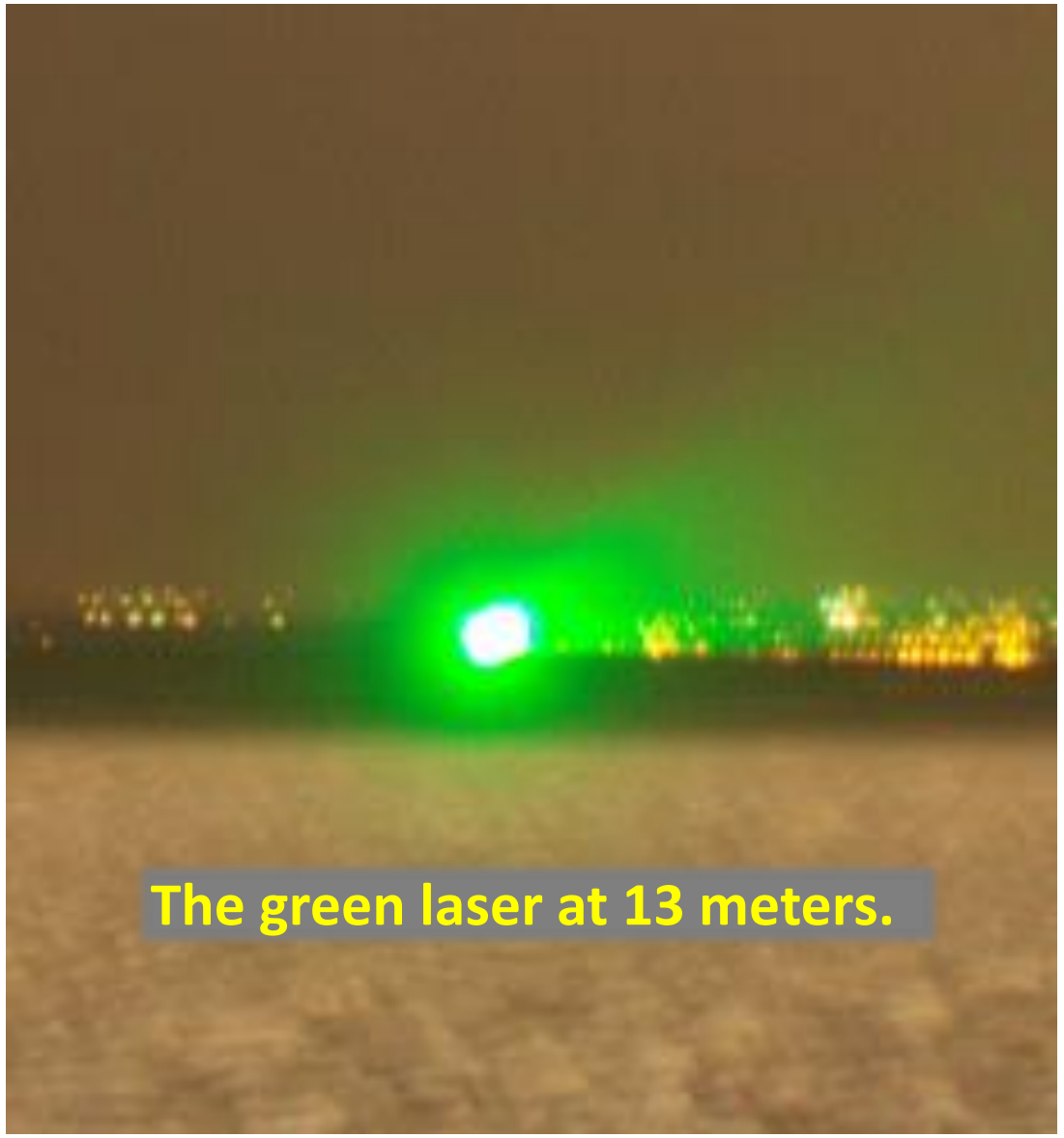
MEASUREMENT 2
LAKE BALATON

On the 26th of February at 20:00, the blue laser pointer was placed at Balatonvilagos on the SALAD at 2.2 meters (7.2 feet) above the lake surface. The source of the beam was seen on the opposite shore at 12km (7.46 miles) distance (Siofok) at 1.6 meters (5.25 feet) above the lake surface.

MEASUREMENT 2

26th of February 2018 at 20:00


The blue laser at 2.2 meters.


The green laser at 13 meters.

CURVATURE CORRECTION WGS84 AND MSL

The radius of Earth is 6366.776 km (3956.131 miles) at the 46.911702° latitude (Siofok) on the WGS84 ellipsoid model, and 6366.75 km (3956.115 miles) at the 46.982097° latitude (laser).
The measurement direction heading is 228.91°

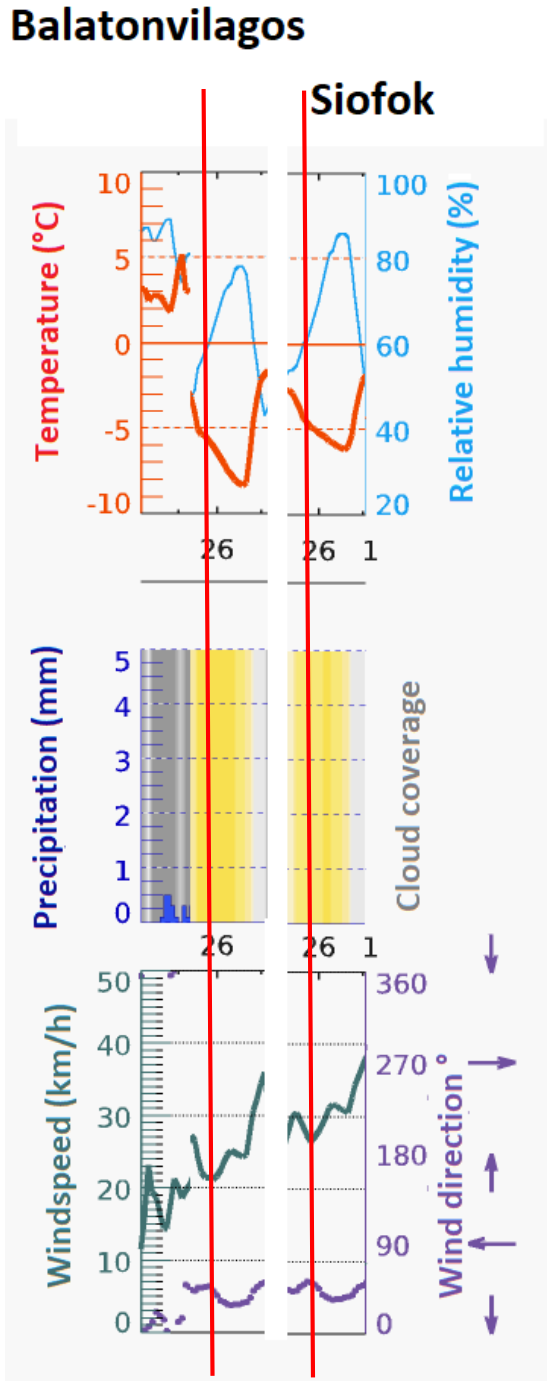
$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

The calculated difference of curvature drop on WGS84 to the spherical model from Laser to Siofok is: **-0.046 mm** (-0.0018 inches)

The height of Lake Balaton above Mean Sea Level is +105 meters (+344.49 feet) that gives a calculated difference of **-0.187 mm** (+0.0074 inches) of curvature drop.

MEASUREMENT 2

26th of February 2018 at 20:00



Balatonvilagos

Hum. 58%

Temp. -6°C (21.2°F)

Wind 22km/h (13.7 mph)

waves 1m high (3.3 feet)

Siofok

Hum. 60%

Temp. -5°C (23°F)

Wind 28km/h (17.4 mph)

waves 1m high (3.3 feet)

REFRACTION CALCULATION OF 2ND MEASUREMENT

The lake temperature was 0°C (32°F) and the air at night was down to a minimum of -11°C (12.2°F) above the lake. Air temperature at the time of the measurement was -6°C (21.2°F).

The ambient conditions showed the refractive indexes were about the same at the two sides of the lake. The change in temperature was decreasing and humidity was increasing above the lake surface versus the air above – resulting in a slight upward refraction.

REFRACTION CALCULATION OF 2ND MEASUREMENT

Angle of incidence (θ1): 0.0029°

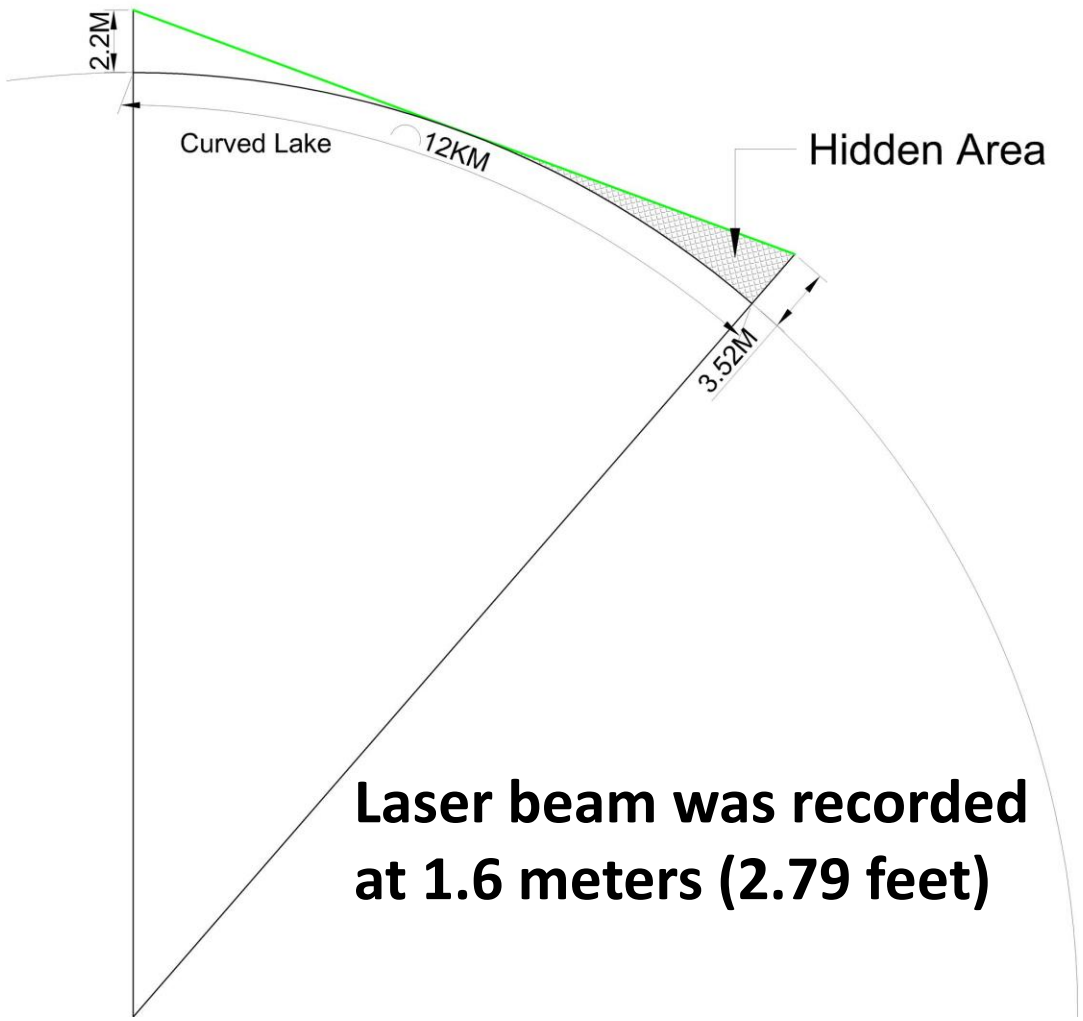
Refractive index calculation
(based on Modified Edlén Equation)

n1 = 1.000302762 (445nm, -6°C, 60%)

n2 = 1.000296015 (445nm, 0°C, 70%)

Angle of refraction is calculated with Snell's law:
sin θ2 = (n1 * sinθ1) / n2 = 0.00290002 degrees
Angle of deviation = 0.000000020°

We conclude the ambient conditions refracted the laser beam upward by a maximum of
0.235 mm (0.00924 inches)



Laser beam was recorded at 1.6 meters (2.79 feet)

Measurement 2
Target hidden height spherical model:
3.52 meters (11.5484 feet)

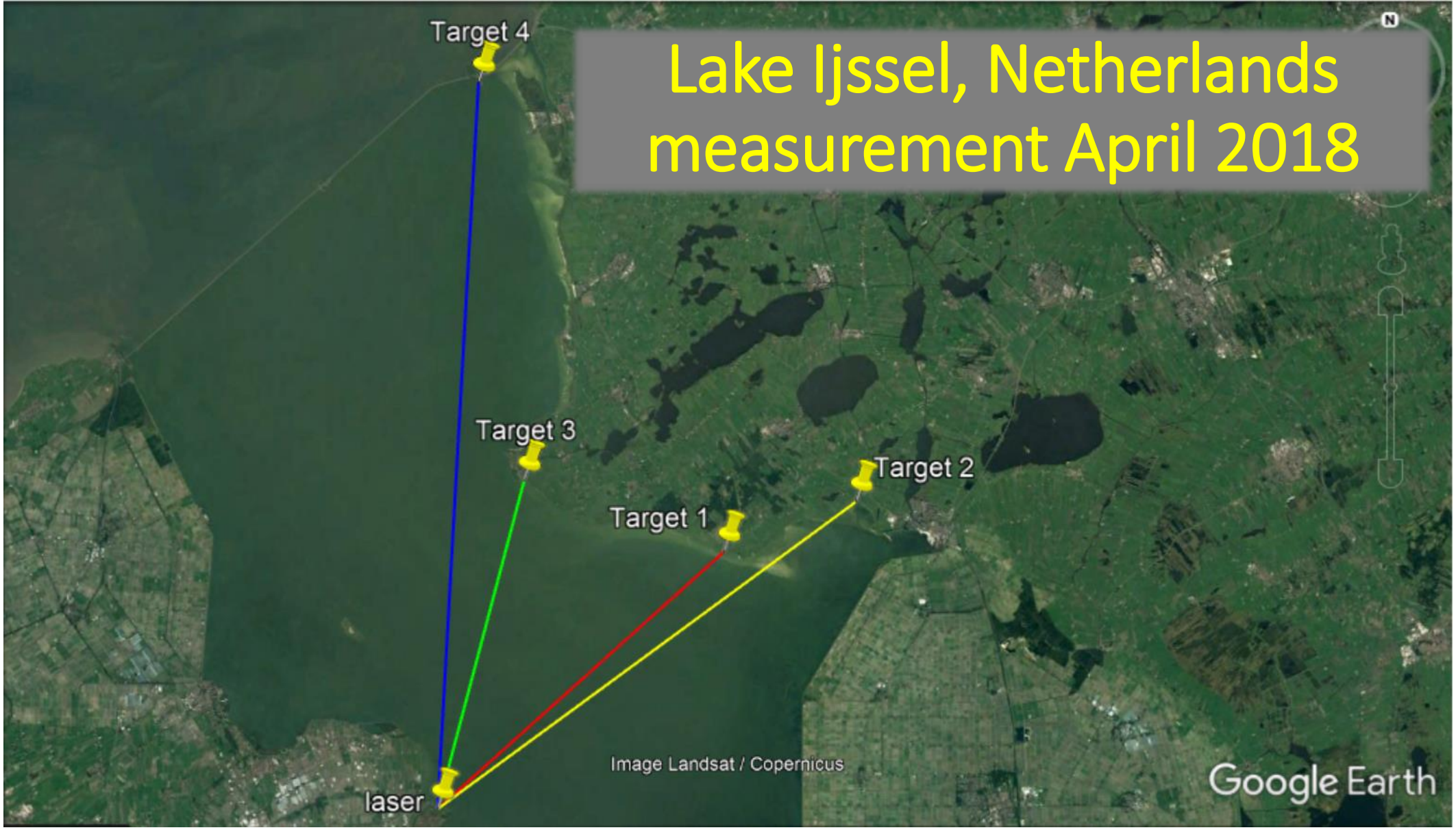
WGS84 laser to target correction:
+0.026 mm (0.001 inches)

MSL correction is
-0.119 mm (-0.0047 inches)

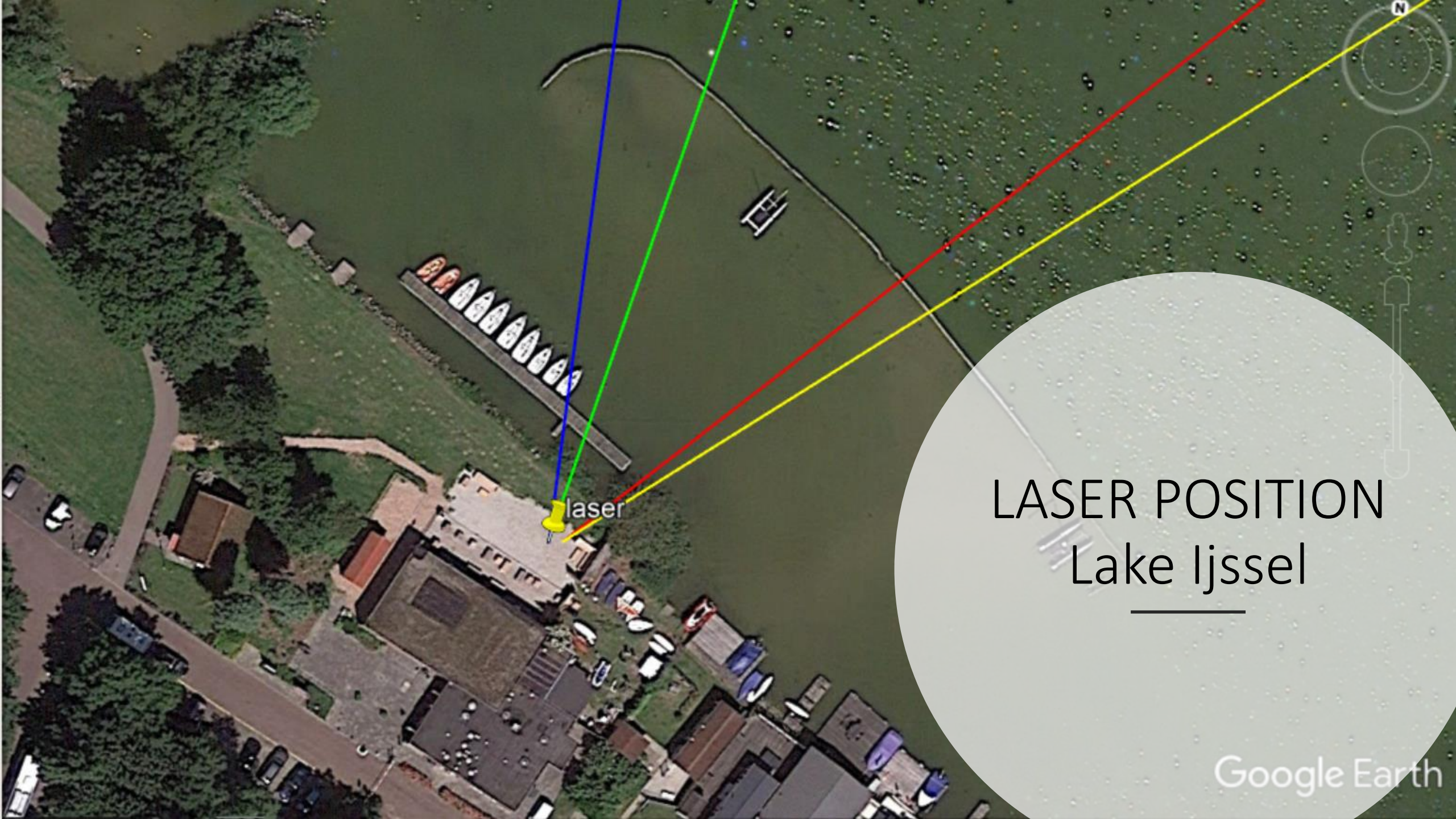
Refraction correction (max)
+0.235 mm (+0.00924 inches)

EXPECTED target hidden height
3.52009 meters (11.5487 feet)

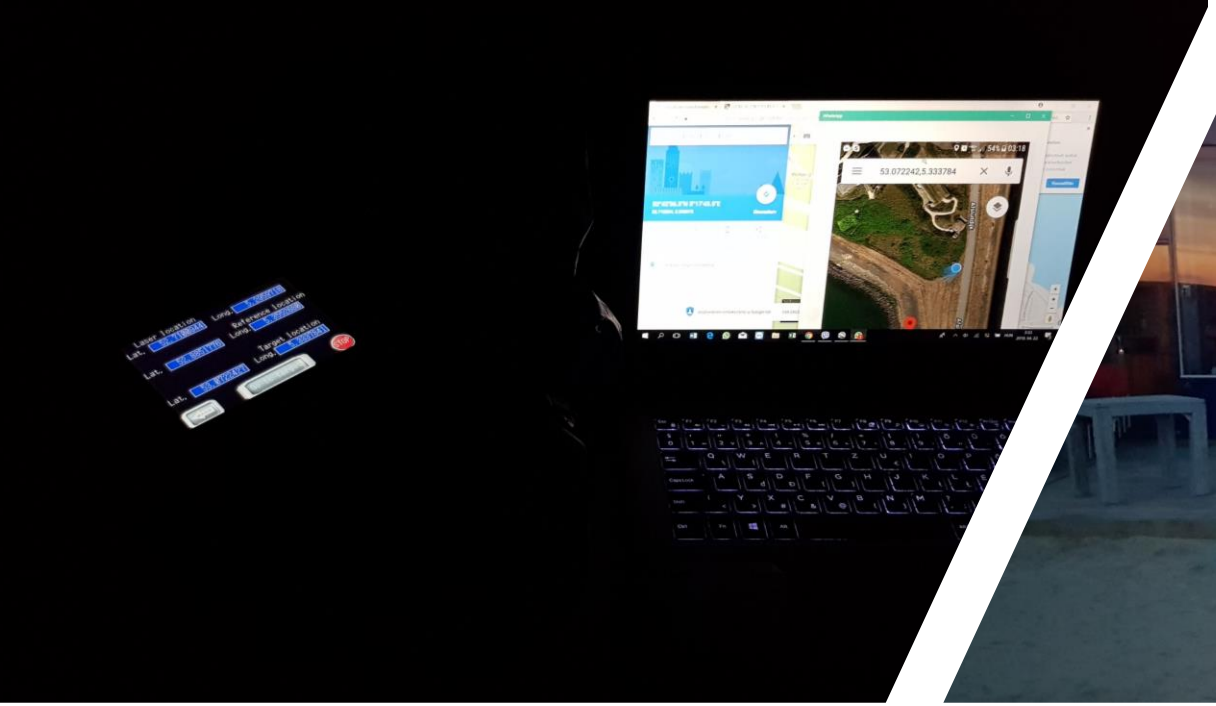
Difference of geoid height:
-129 mm (-5.08 inches)



Map of Lake Ijssel between the four measurement positions and laser position

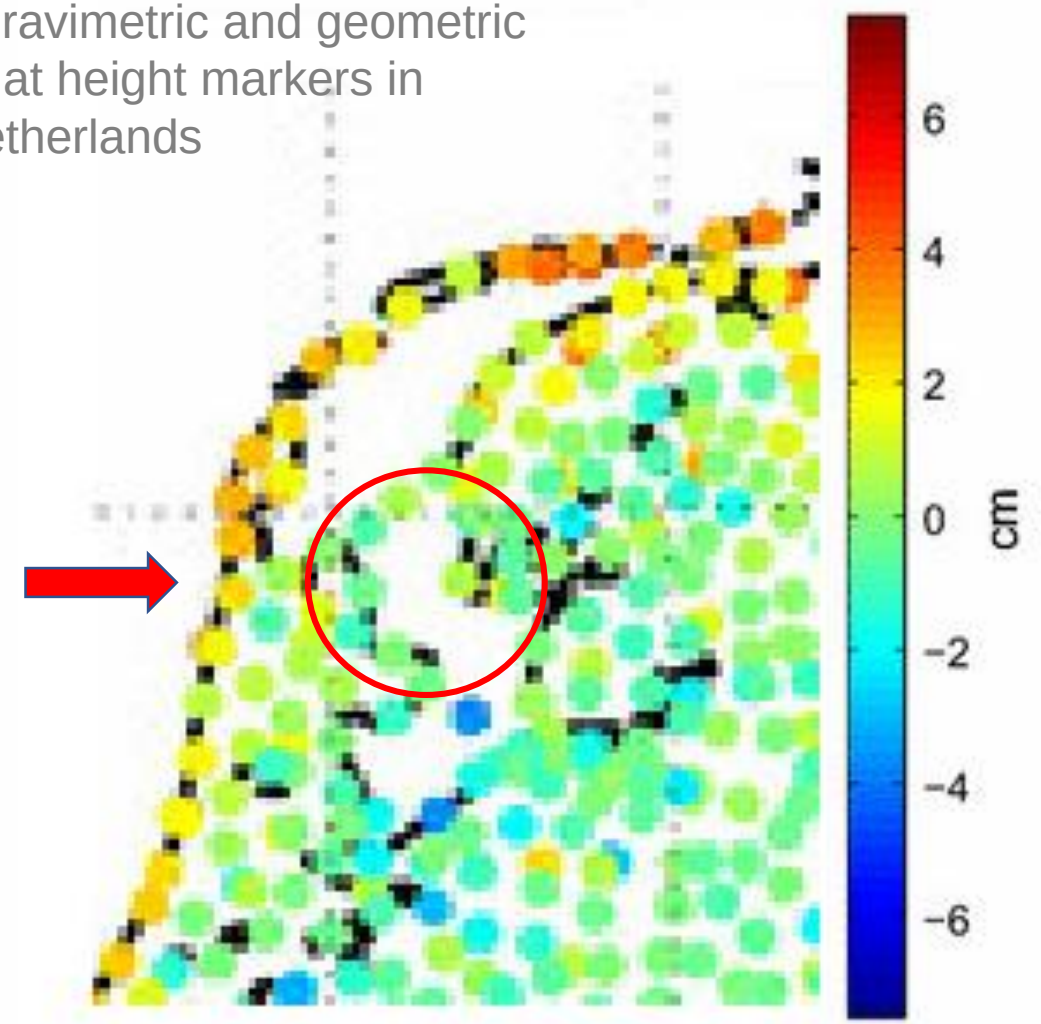


LASER POSITION
Lake Ijssel


Laser location
Lat.
Long.
Reference location
Lat.
Long.
Target location
Lat.
Long.
53.072242,5.333784

GEOID UNDULATION AT LAKE IJSSEL

Differences between gravimetric and geometric height anomalies at height markers in The Netherlands



Geoid undulation is at the same level in all positions.

[2]

EFFECT OF TIDES AT LAKE IJSSEL

Lake tides are known to have amplitudes of up to 10 cm (0.33 feet) in larger lakes (Trebitz, 2006). In case of Lake Ijssel, the average shallow depth of 5.5 meters (18 feet) and the relatively low water volume of the lake suggests this effect would be even smaller and within our measurement error margin.

REFRACTION CONDITIONS

The lake temperature ranged from 7 - 14 Celsius (45F - 57F).

As no data on the ambient conditions along the laser beam was available, we used data at the two end positions. The data showed there were no significant differences in temperature and humidity between the start and end points of the measurements.

TARGET 1 POSITION
Lake Ijssel
(Measurement 3)

MEASUREMENT 3
LAKE IJSSEL TARGET 1

On the 8th of April at 1:00 AM, the blue laser pointer was placed on the SALAD at 2.85 meters (9.35 feet) above the lake surface.

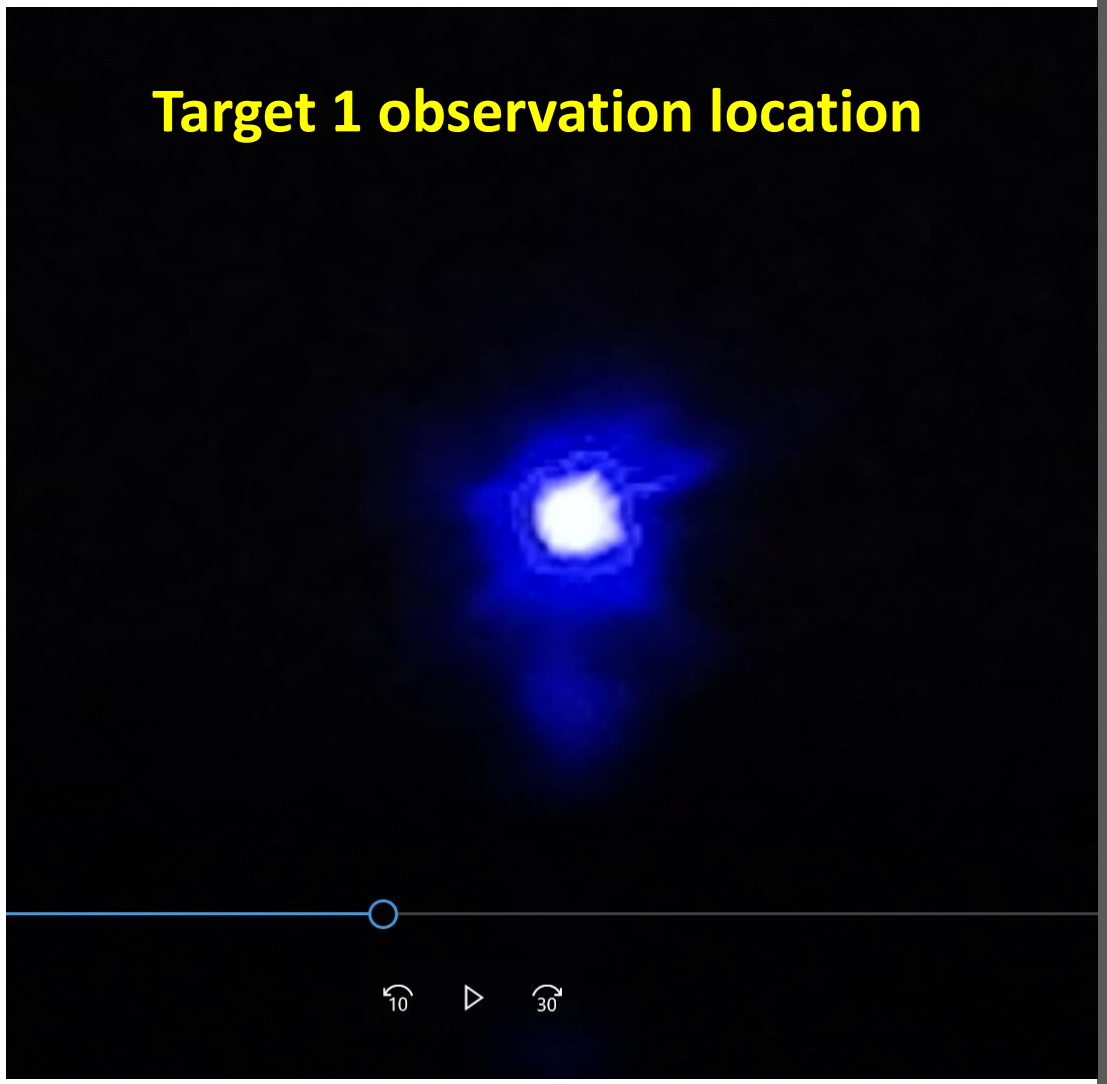
It was seen on the opposite shore at 21.26 km (13.21 miles) to Target 1 at 1.2 meters (3.94 feet) above the lake surface.

MEASUREMENT 3

Lake Ijssel 8th April 2018 at 1:00 AM

MEASUREMENT 3

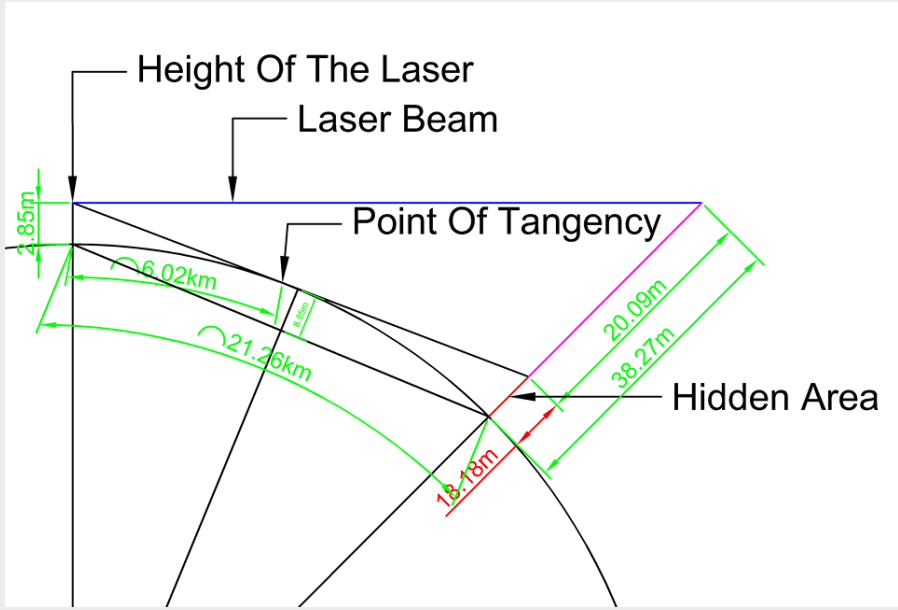
Lake Ijssel 8th April 2018 at 1:00 AM



MEASUREMENT 3

LASER BEAM HIDDEN HEIGHT CALCULATION MEASUREMENT 3 TARGET 1

The 21.26km (13.21 Miles) calculations based on a spherical model results in a target hidden height of the Measurement 3 Target 1 position of 18.18 meters (59.6 feet)



CURVATURE CORRECTION WGS84 AND MSL

The radius of Earth is 6364.598 km (3954.778 miles) at the 52.836208° latitude (Target 1) on the WGS84 ellipsoid model, and 6364.643 km (3954.806 miles) at the 52.710094° latitude (laser).
The measurement direction heading is 48.51°

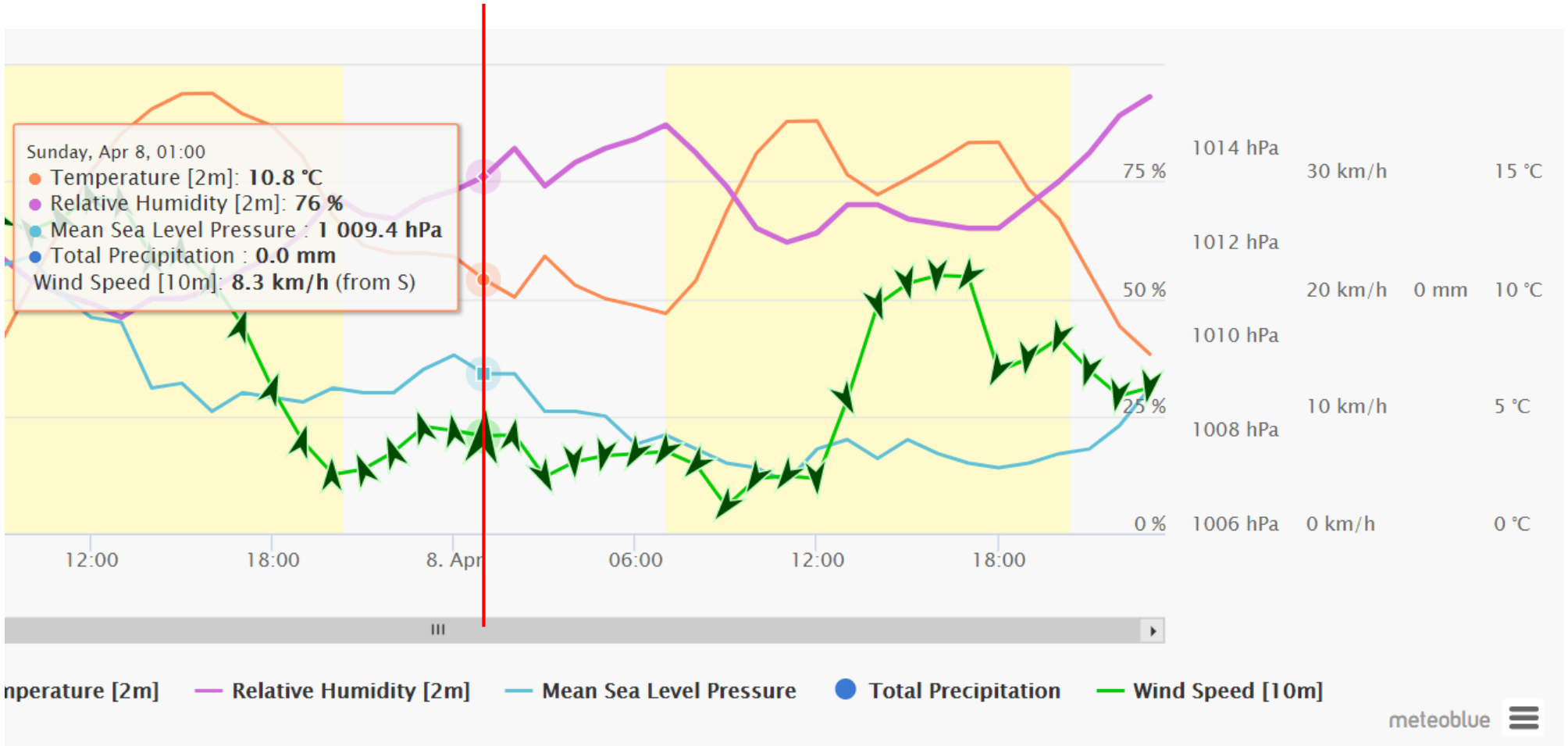
$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

The calculated difference of curvature drop on WGS84 to the spherical model from Laser to Target 1: **+0.251 mm** (+0.0099 inches)

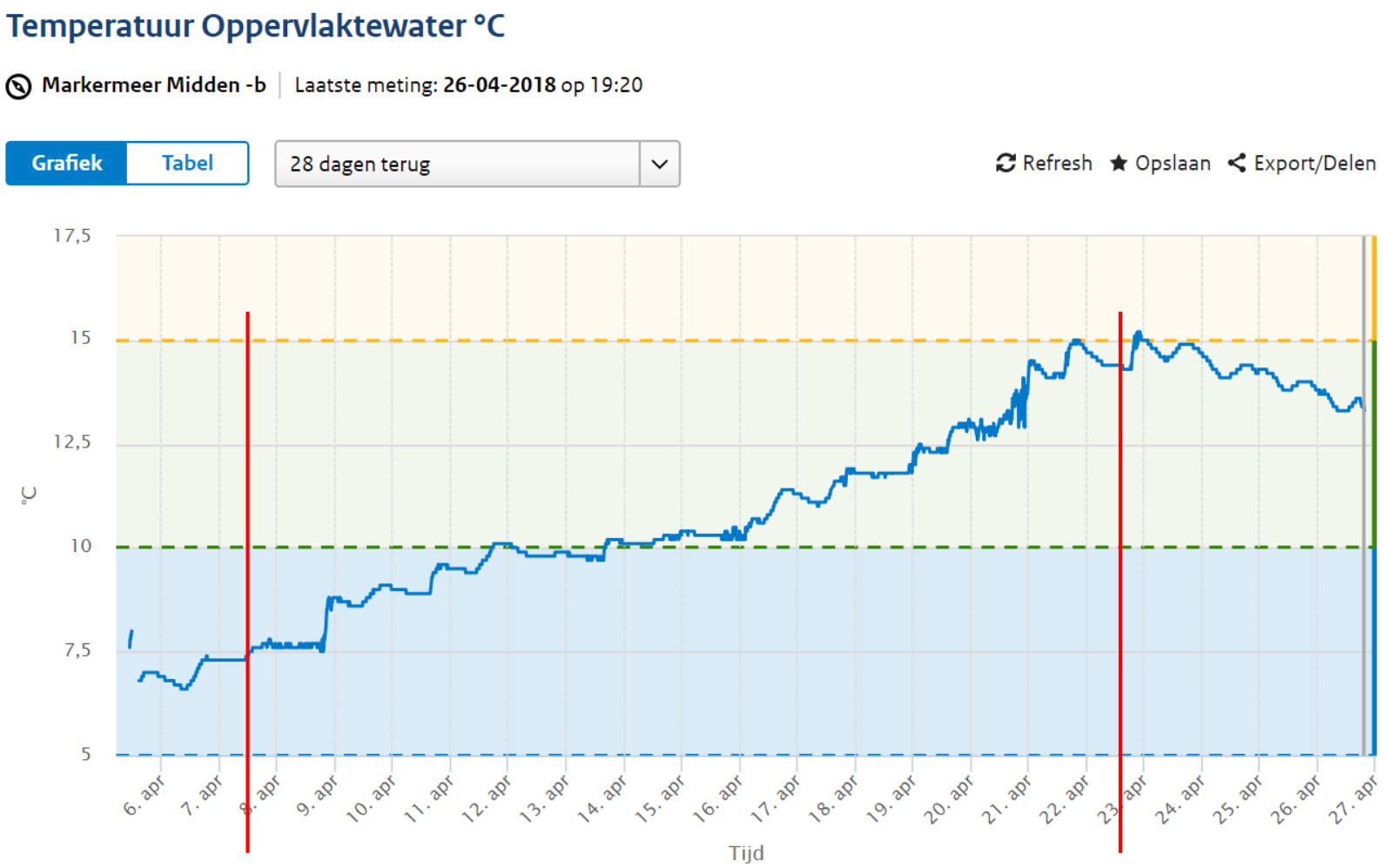
The height of Lake Ijssel above sea level is 0 meters.

MEASUREMENT 3

8th April 2018 at 1:00 AM



Lake Ijssel water temperature chart, April 2018



REFRACTION CALCULATION OF 3RD MEASUREMENT

The lake temperature was 7.6 Celsius (45.7 F) and the air was 10.8 Celsius (46.4 F) above the lake.

The ambient conditions showed the refractive indexes were about the same at the two sides of the lake. The difference in temperature was marginal. The level of humidity decreases as you rise in altitude above lake surface.

REFRACTION CALCULATION OF 3RD MEASUREMENT

Angle of incidence (θ1): 0.0047°

Refractive index calculation
(based on Modified Edlén Equation)

n1 = 1.000284523 (445nm, 10.8°C, 76%)

n2 = 1.000287811 (445nm, 7.6°C, 85%)

Angle of refraction is calculated with Snell's law:
sin θ2 = (n1 * sinθ1) / n2 = 0.004699985 degrees
Angle of deviation = 0.000000015°

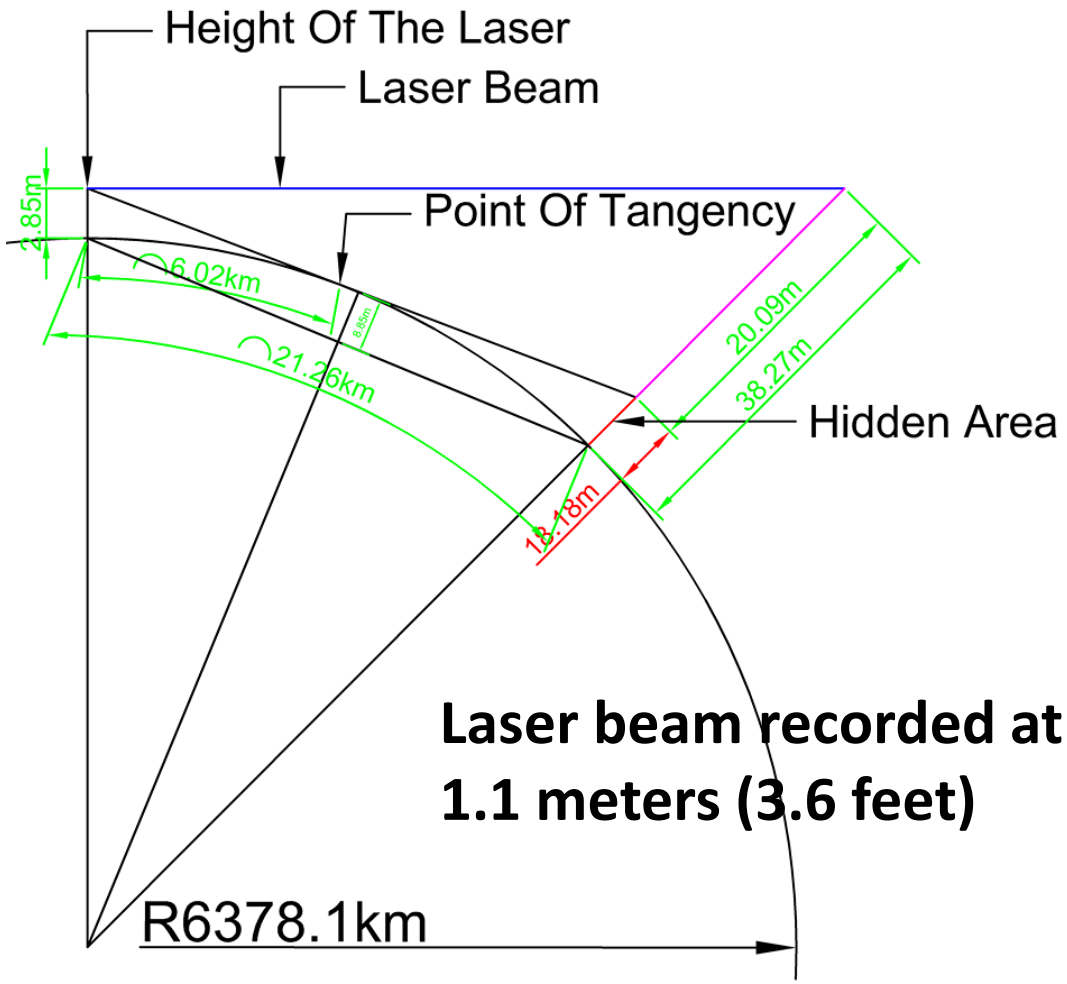
We concluded the ambient conditions refracted the laser beam downward by maximum of
0.329 mm (0.01295 inches)

POSITIONS AND HEIGHT DATA AT TARGET 1 POSITION

Enkhuizen (laser position):
Latitude = 52.7100944444445° N = 52° 42' 36.34" N
Longitude = 5.2959722222222° E = 5° 17' 45.5" E
GPS ellipsoidal height = 0 meters (0 feet)
Geoid height = 42.409 meters (139.137 feet)

Target 1 (measurement position):
Latitude = 52.8362027777778° N = 52° 50' 10.33" N
Longitude = 5.5325416666667° E = 5° 31' 57.15"E
GPS ellipsoidal height = 0 meters (0 feet)
Geoid height = 42.176 meters (138.373 feet)

Difference of geoid height: +233 mm (9.17 inches)



Laser beam recorded at 1.1 meters (3.6 feet)

Measurement 3
Target hidden height spherical model:
18.18 meters (59.6449 feet)

WGS84 laser to target correction:
+0.18 mm (0.007 inches)

MSL correction is **0mm**

Refraction correction (max)
-0.329 mm (-0.01295 inches)

EXPECTED target hidden height
18.17985 meters (59.64445 feet)

Difference of geoid height:
+233 mm (+9.17 inches)

TARGET 2 POSITION
Lake Ijssel
(Measurement 4)

MEASUREMENT 4
LAKE IJSSEL TARGET 2

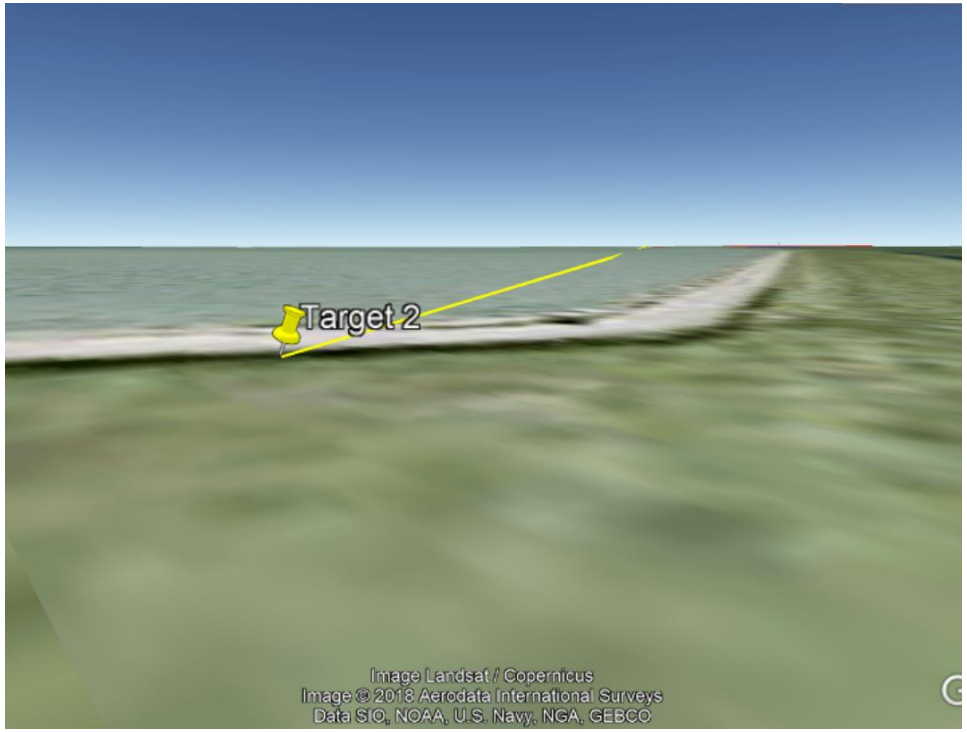
On the 8th of April at 3:00 AM, the blue laser pointer was placed on the SALAD at 2.85 meters (9.35 feet) above the lake surface. It was seen on the opposite shore at 28.68 km (17.82 miles) to Target 2 at 0.85 meters (2.79 feet) above the lake surface.

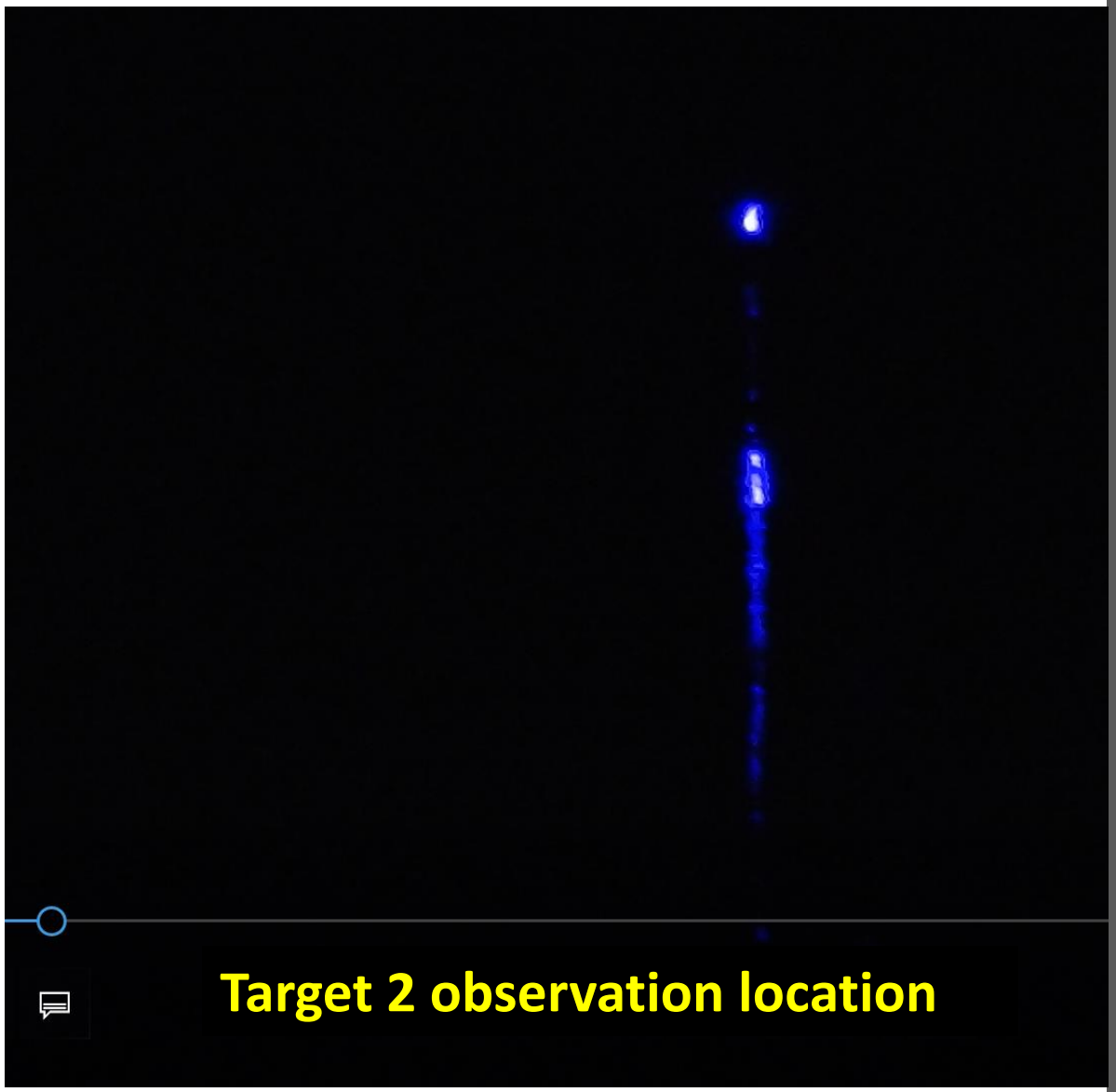
MEASUREMENT 4

Lake Ijssel 8th April 2018 at 3:00 AM

MEASUREMENT 4

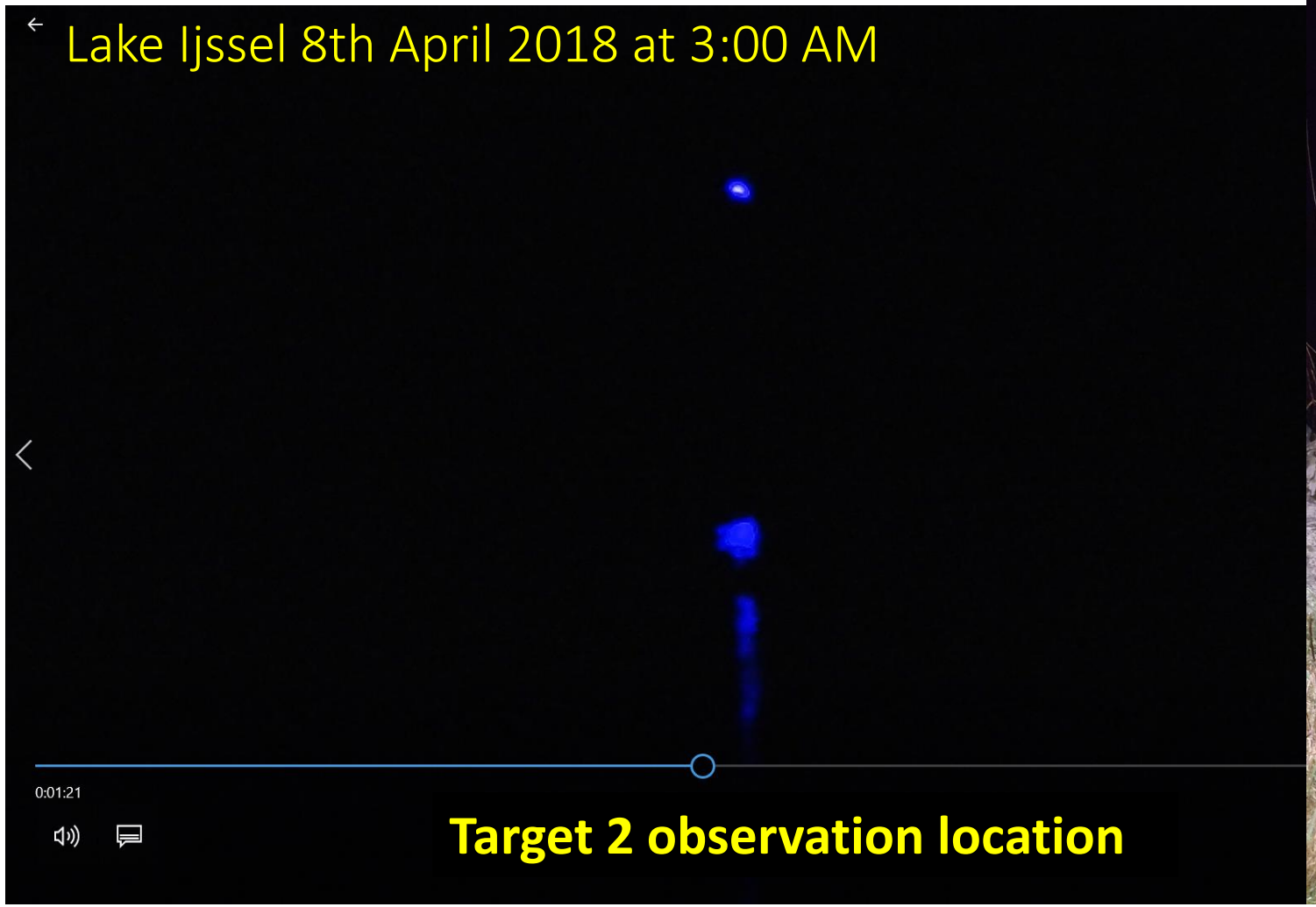
Lake Ijssel 8th April 2018 at 3:00 AM





MEASUREMENT 4

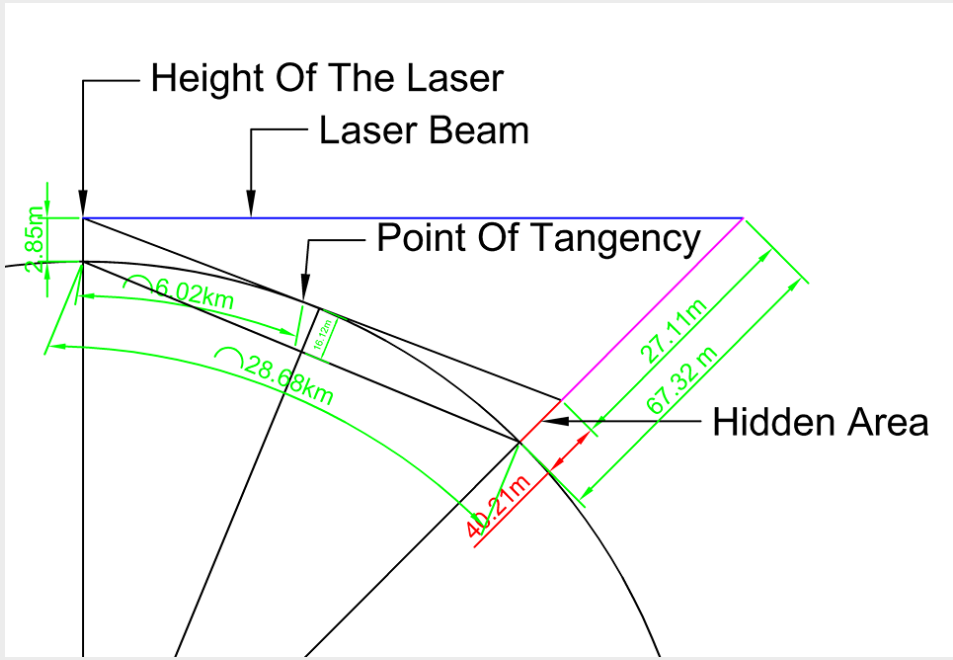
Lake Ijssel 8th April 2018 at 3:00 AM



Target 2 observation location

LASER BEAM HIDDEN HEIGHT CALCULATION MEASUREMENT 4 TARGET 2

The 28.68km (17.82 miles) distance calculations based on a spherical model results in a target hidden height of the Measurement 4 Target 2 position of 40.21 meters (131.9 feet).



CURVATURE CORRECTION WGS84 AND MSL

The radius of Earth is 6364.589 km (3954.772 miles) at the 52.860294° latitude (Target 2) on the WGS84 ellipsoid model, and 6364.643 km (3954.806 miles) at the 52.710094° latitude (laser).
The measurement direction heading is 54.15°

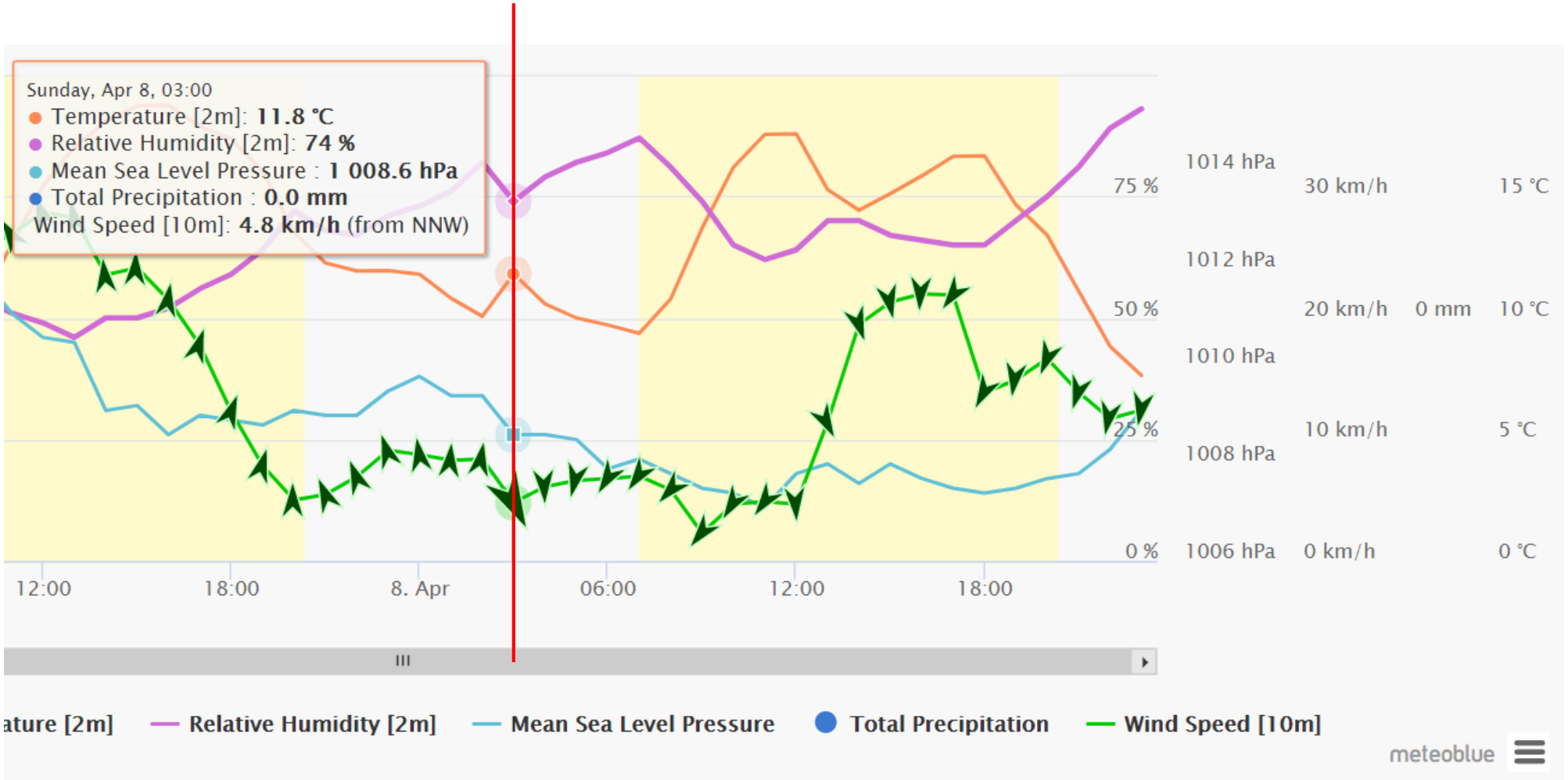
$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

The calculated difference of curvature drop on WGS84 to the spherical model from Laser to Target 2:
+0.458 mm (+0.018 inches)

The height of Lake Ijssel above sea level is 0 meters.

MEASUREMENT 4

8th April 2018 at 3:00 AM



REFRACTION CALCULATION OF 4TH MEASUREMENT

Angle of incidence (θ1): 0.0040°
Refractive index calculation
(based on Modified Edlén Equation)

n1 = 1.000283508 (445nm, 11.8° C, 74%)

n2 = 1.000287811 (445nm, 7.6° C, 85%)

Angle of refraction is calculated with Snell's law:
sin θ2 = (n1 * sinθ1) / n2 = 0.003999983 degrees

Angle of deviation = 0.000000017°

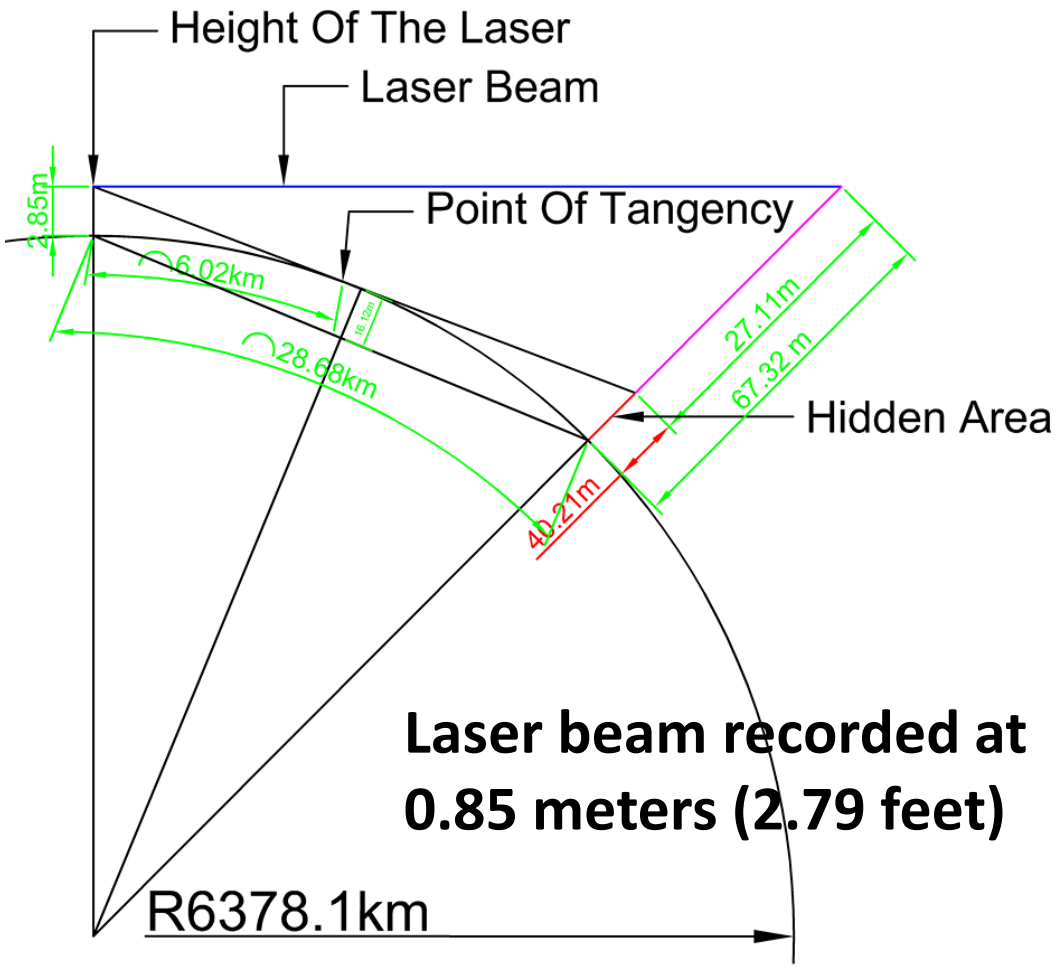
We concluded the ambient conditions refracted the laser beam downward by maximum of
0.494 mm (0.0194 inches)

POSITIONS AND HEIGHT DATA AT TARGET 2 POSITION LAKE IJSSEL

Enkhuizen (laser position):
Latitude = 52.7100944444445° N = 52° 42' 36.34" N
Longitude = 5.2959722222222° E = 5° 17' 45.5" E
GPS ellipsoidal height = 0 meters
Geoid height = 42.409 meters (139.137 feet)

Target 2 (measurement position):
Latitude = 52.8602638888889° N = 52° 51' 36.95" N
Longitude = 5.64140555555556° E = 5° 38' 29.06"E
GPS ellipsoidal height = 0 meters
Geoid height = 42.103 meters (138.13 feet)

Difference of geoid height: +306 mm (12.05 inches)



Measurement 4
Target hidden height spherical model:
40.21 meters (131.921 feet)

WGS84 laser to target correction:
+0.439 mm (0.0173 inches)

MSL correction is **0mm**

Refraction correction (max)
-0.494 mm (-0.0194 inches)

EXPECTED target hidden height
40.20995 meter (131.9208 feet)

Difference of geoid height:
+306 mm (+12.05 inches)

TARGET 3 POSITION
Lake Ijssel
(Measurement 5)

MEASUREMENT 5
LAKE IJSSEL TARGET 3

On the 22nd of April at 0:00 AM, the blue laser pointer was placed on the SALAD at 2.92 meters (9.58 feet) above the lake surface.
It was seen on the opposite shore at 18.73 km (11.64 miles) to Target 3 at 1.6 meters (5.25 feet) above the lake surface.

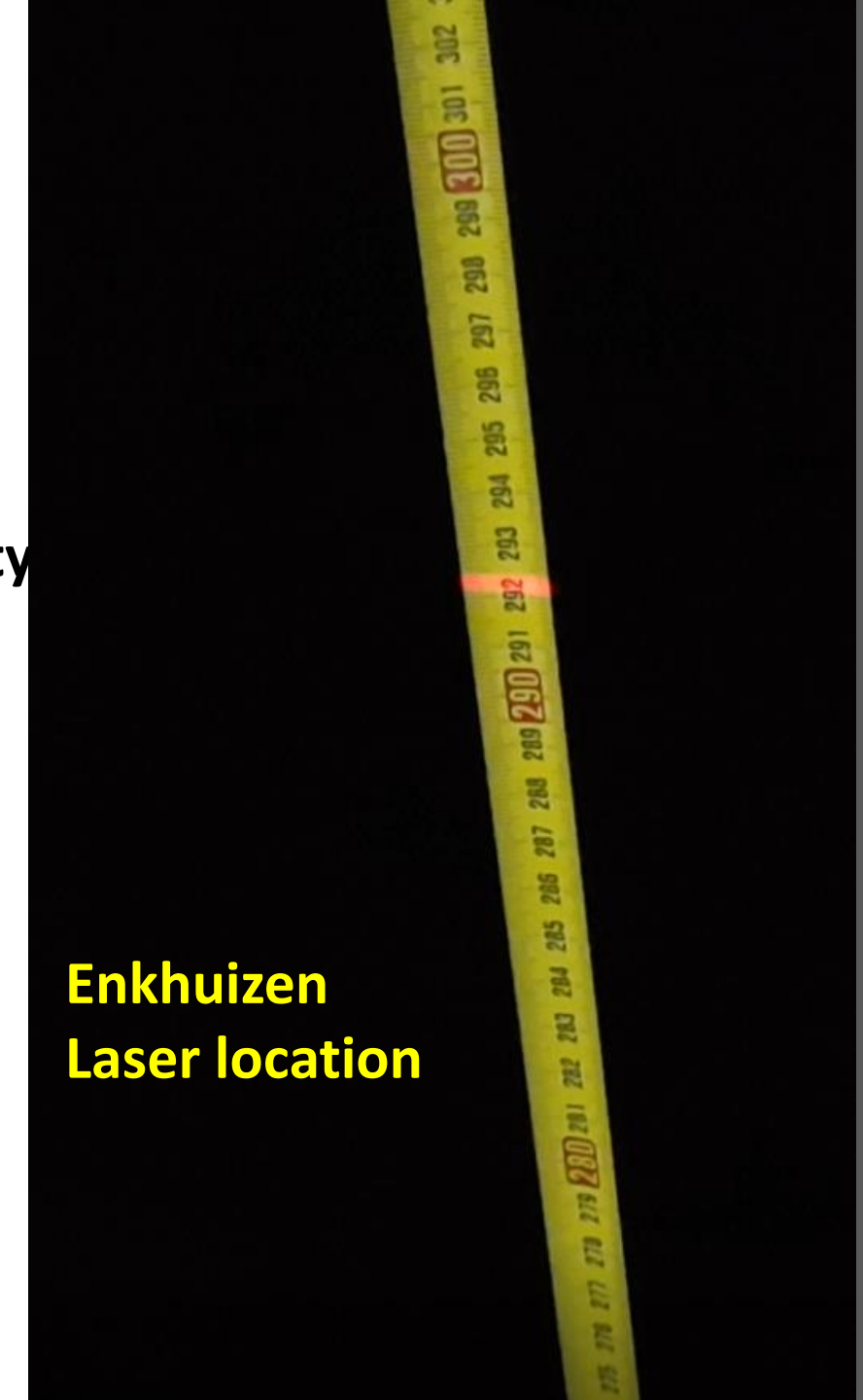
MEASUREMENT 5

Lake Ijssel 22nd April 2018 0:00 AM

Temperature & humidity

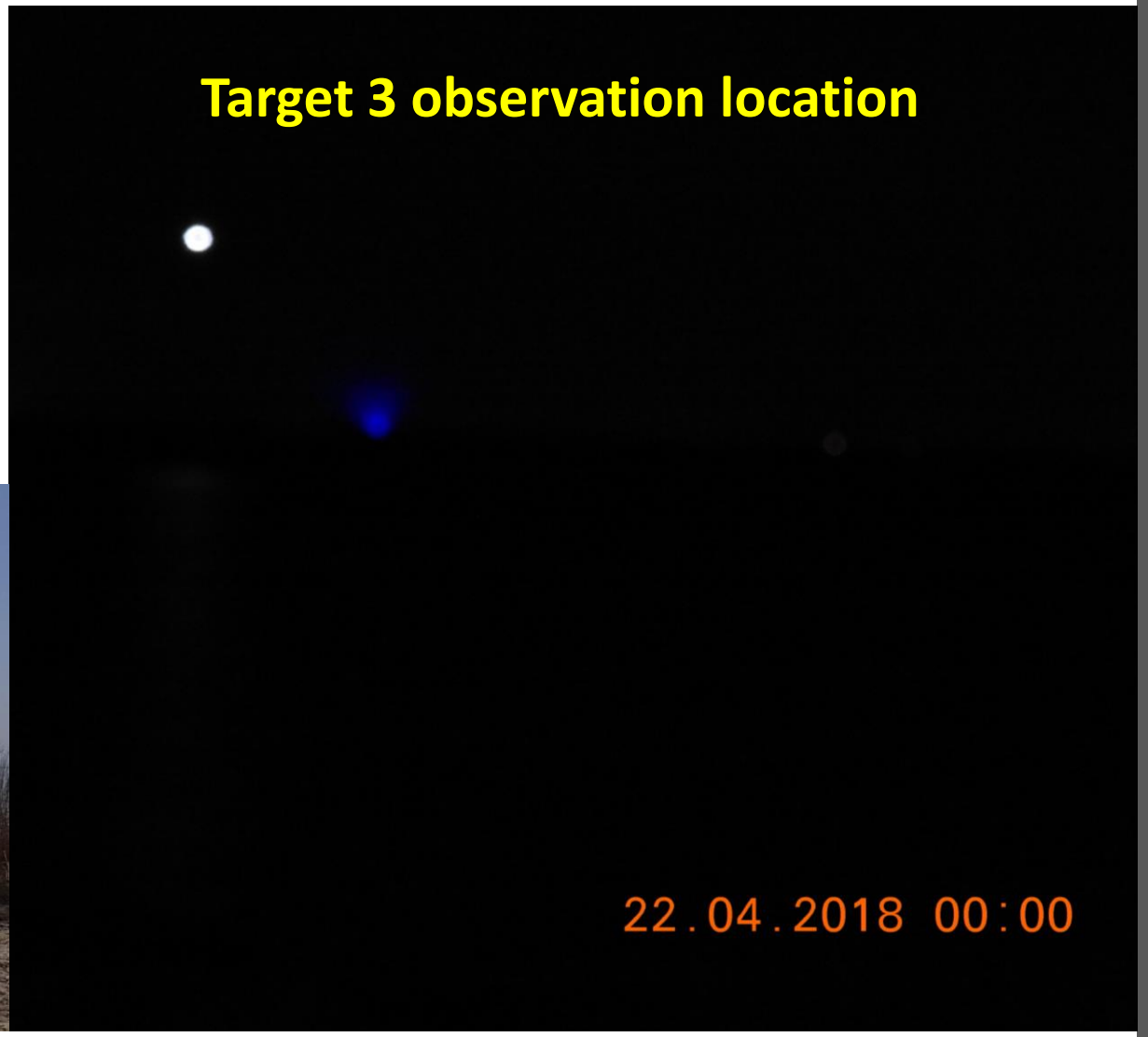
At water level
11.4°C (52.5°F)
89%

At 2 meters above
(6.56 feet)
13.3°C (56°F)
77%



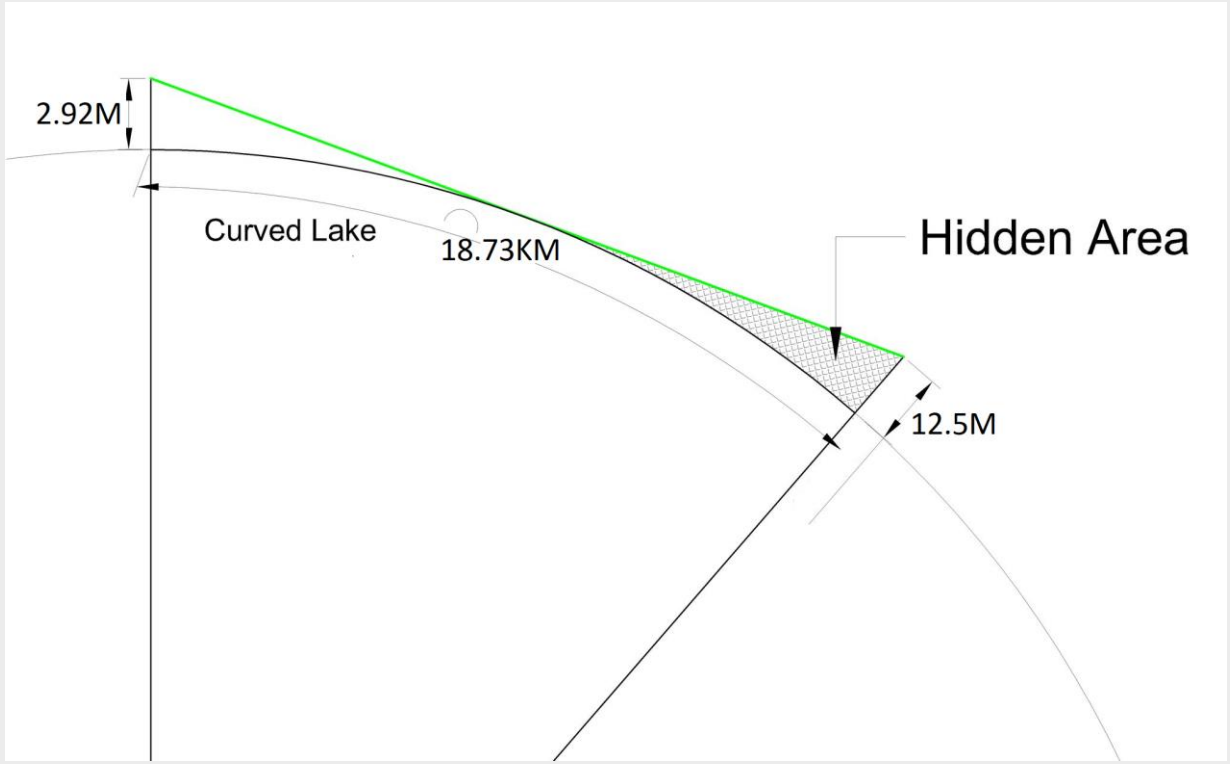
MEASUREMENT 5

Lake Ijssel 22nd April 2018 at 0:00 AM



LASER BEAM HIDDEN HEIGHT CALCULATION MEASUREMENT 5 TARGET 3

The calculations based on a spherical model results in a target hidden height of the Measurement 5 Target 3 position of 12.504 meters (41.024 feet)



CURVATURE CORRECTION WGS84 and MSL

The radius of Earth is 6364.585 km (3954.770 miles) at the 52.872386° latitude (Target 3) on the WGS84 ellipsoid model, and 6364.643 km (3954.806 miles) at the 52.710094° latitude (laser).
The measurement direction heading is 15.25°

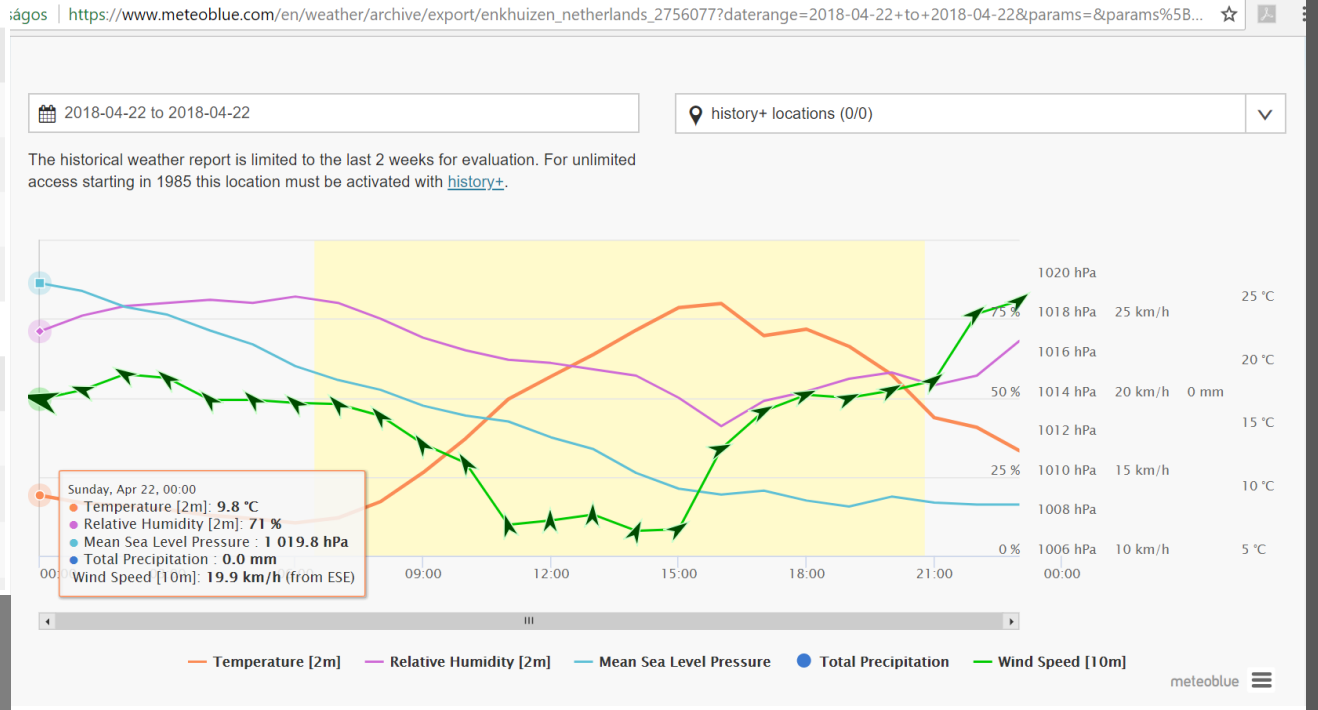
$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

The calculated difference of curvature drop on WGS84 to the spherical model from Laser to Target 3: **+0.237 mm** (+0.0093 inches)

The height of Lake Ijssel above sea level is 0 meters.

Input	Amount
Vacuum Wavelength:	445 Nanometers [nm]
Air Temperature:	9.8 Degrees Celsius
Atmospheric Pressure:	1.0198 Bars
Air Humidity:	71 Relative Humidity, Percent

Output	Result
Wavelength in Ambient Air:	444.872132 Nanometers [nm]
Refractive Index of Air[1]:	1.000287426
Uncertainty of Calculated Index[2]:	0.000000032

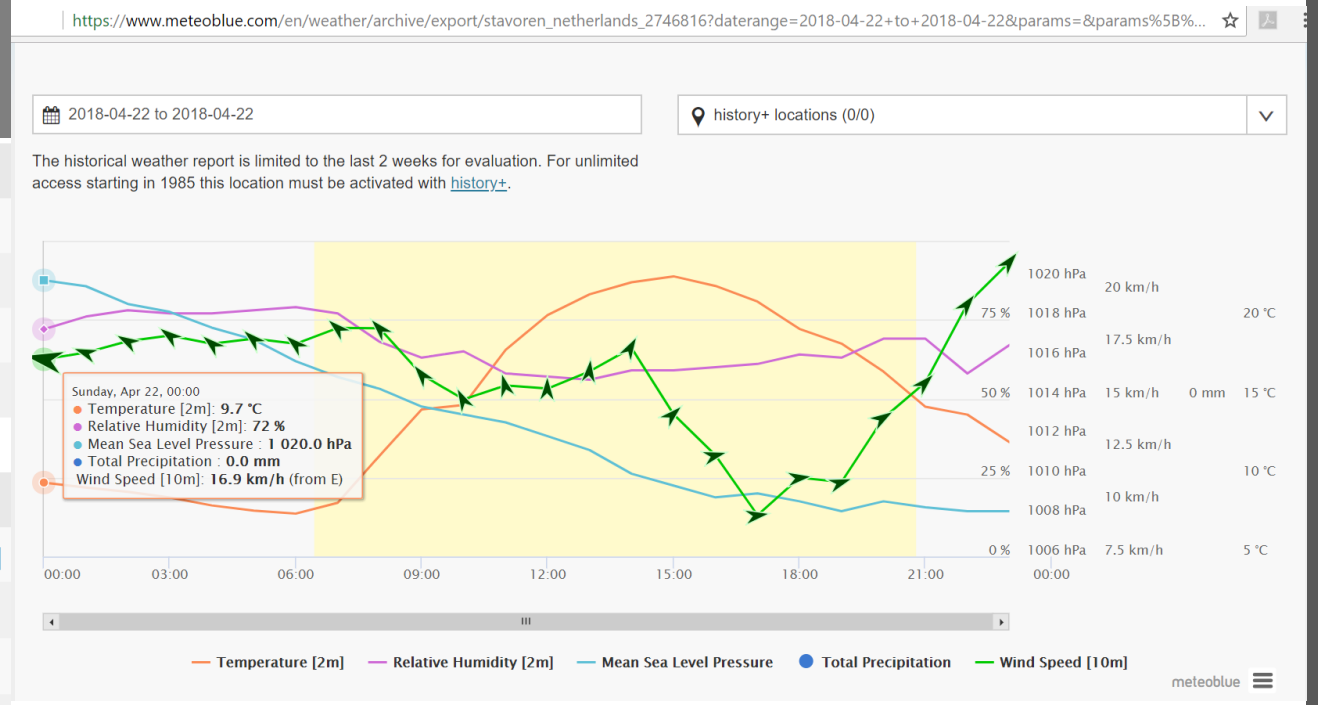


MEASUREMENT 5

Lake Ijssel 22nd April 2018 at 0:00 AM

Input	Amount
Vacuum Wavelength:	445 Nanometers [nm]
Air Temperature:	9.7 Degrees Celsius
Atmospheric Pressure:	1.02 Bars
Air Humidity:	72 Relative Humidity, Percent

Output	Result
Wavelength in Ambient Air:	444.872063 Nanometers [nm]
Refractive Index of Air[1]:	1.000287582
Uncertainty of Calculated Index[2]:	0.000000032



REFRACTION CALCULATION OF 5TH MEASUREMENT

Angle of incidence (θ1): 0.0040°

Refractive index calculation
(based on Modified Edlén Equation)

n1 = 1.000287426 (445nm, 9.8°C, 1.0198Bars, 71%)

n2 = 1.000287582 (445nm, 9.7°C, 1.0200Bars, 72%)

Angle of refraction is calculated with Snell's law:
sin θ2 = (n1 * sinθ1) / n2 = 0.003999999 degrees
Angle of deviation = 0.000000001°

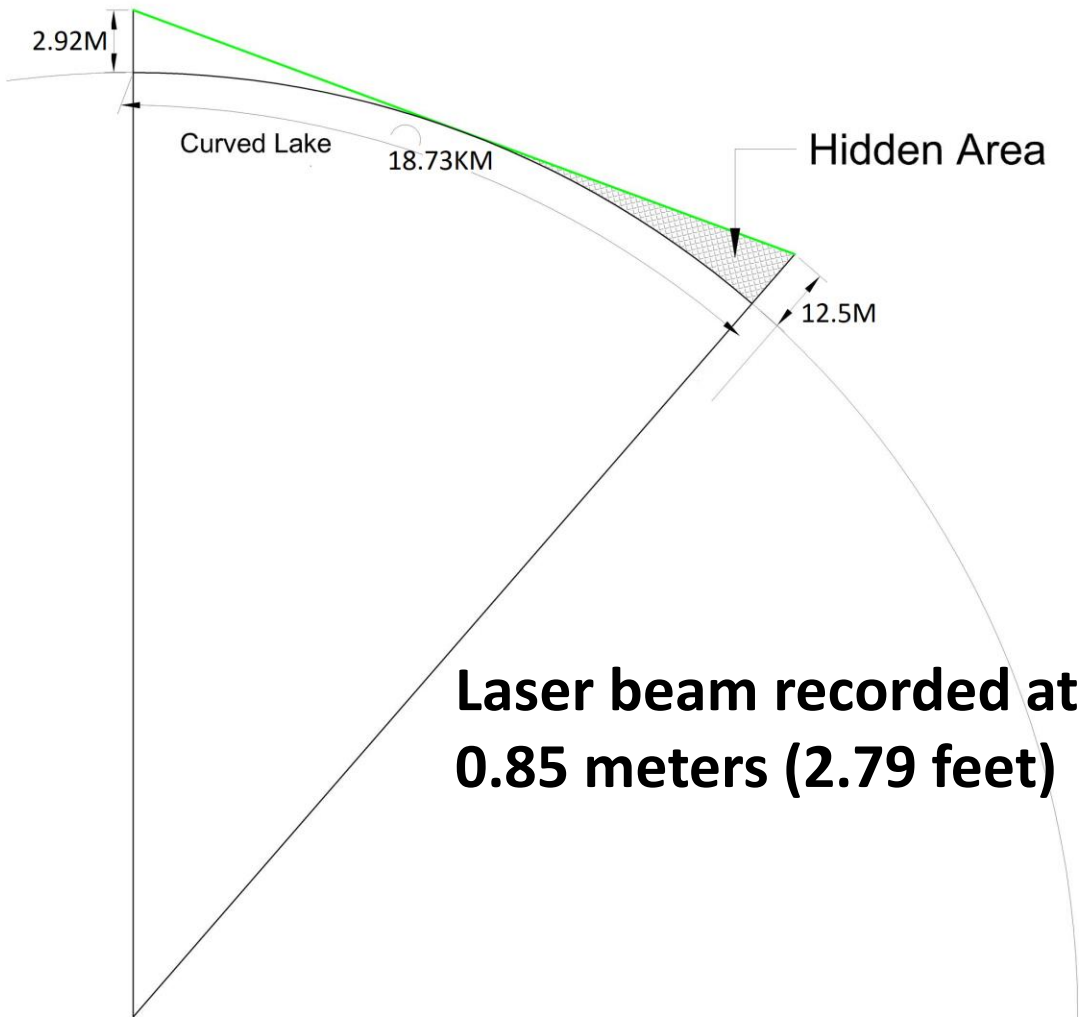
We concluded the ambient conditions refracted the laser beam downward by a maximum of **0.0117 mm** (0.00046 inches)

POSITIONS AND HEIGHT DATA AT TARGET 3 POSITION LAKE IJSSEL

Enkhuizen (laser position):
Latitude = 52.7100944444445° N = 52° 42' 36.34" N
Longitude = 5.2959722222222° E = 5° 17' 45.5" E
GPS ellipsoidal height = 0 meters
Geoid height = 42.409 meters (139.14 feet)

Target 3 (measurement position):
Latitude = 52.8724444444444° N = 52° 52' 20.8" N
Longitude = 5.36933055555556° E = 5° 22' 9.59" E
GPS ellipsoidal height = 0 meters
Geoid height = 42.149 meters (138.28 feet)

Difference geoid height: **260 mm** (10.24 inches)



Laser beam recorded at 0.85 meters (2.79 feet)

Measurement 5
Target hidden height spherical model: 12.504 meters (41.0231 feet)

WGS84 laser to target correction: **+0.16 mm** (0.0063 inches)

MSL correction is **0mm**

Refraction correction (max) **-0.0117 mm** (-0.00046 inch)

EXPECTED target hidden height 12.50415 meters (41.0236 feet)

Difference of geoid height: +260 mm (+12.05 inches)

TARGET 4 POSITION
Lake Ijssel
(Measurement 6)

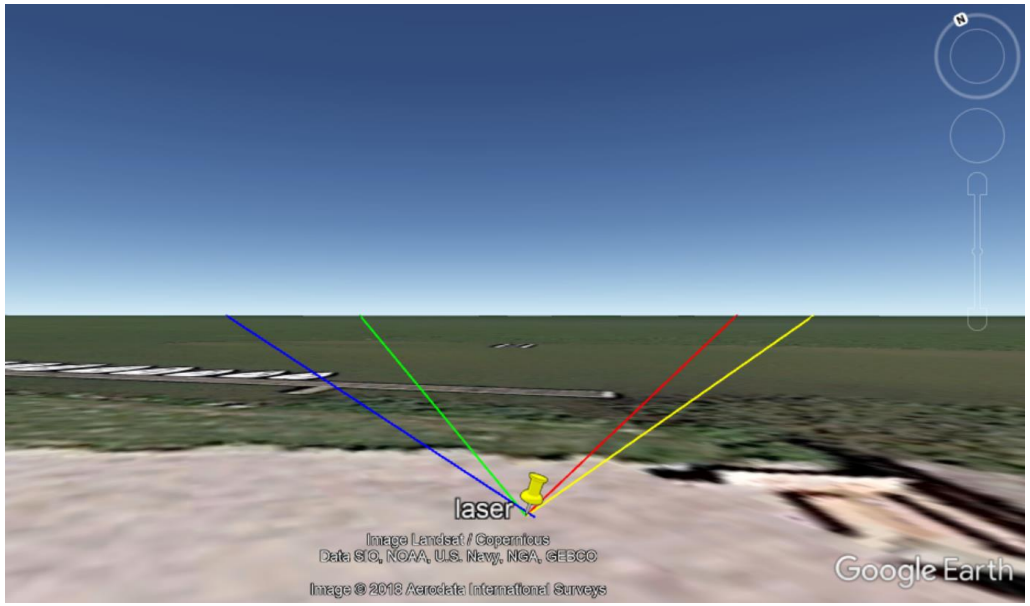
MEASUREMENT 6
LAKE IJSSEL TARGET 4

On the 22nd of April at 2:19 AM, the blue laser pointer was placed on the SALAD at 2.92 meters (9.58 feet) above the lake surface heading to 3.66°.

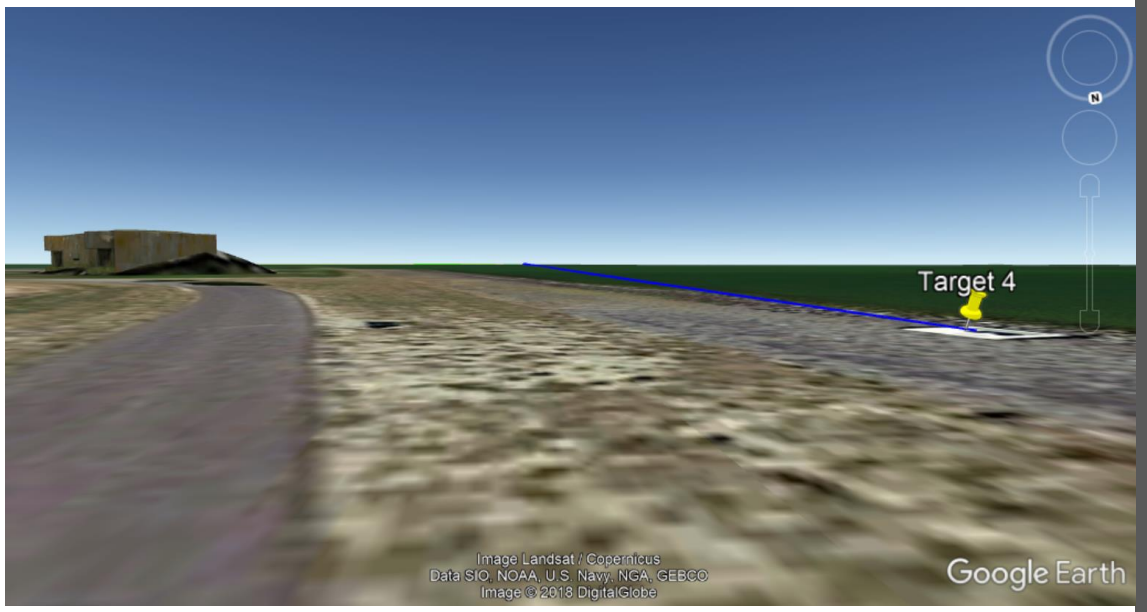
It was seen on the opposite shore at 40.14km (24.94 miles) to Target 4 at 1.5 meters (4.92 feet) above the lake surface.

MEASUREMENT 6

Lake Ijssel 22nd April 2018 at 2:19 AM



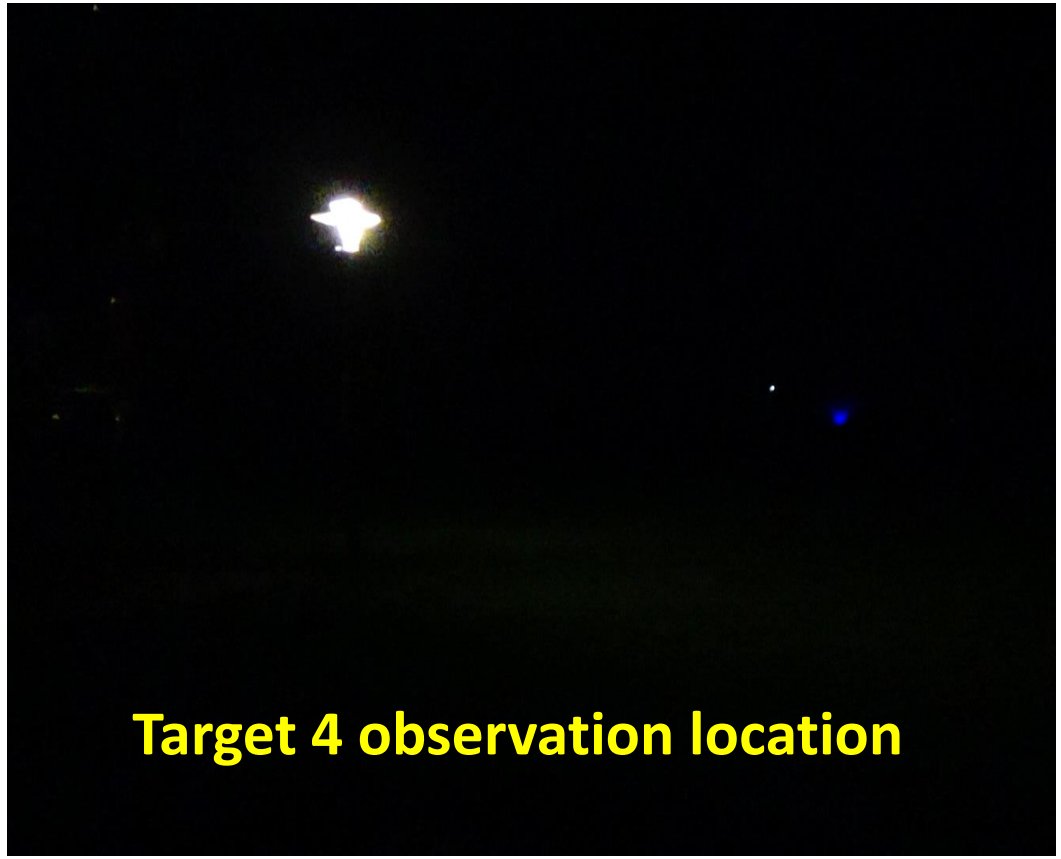
Laser location

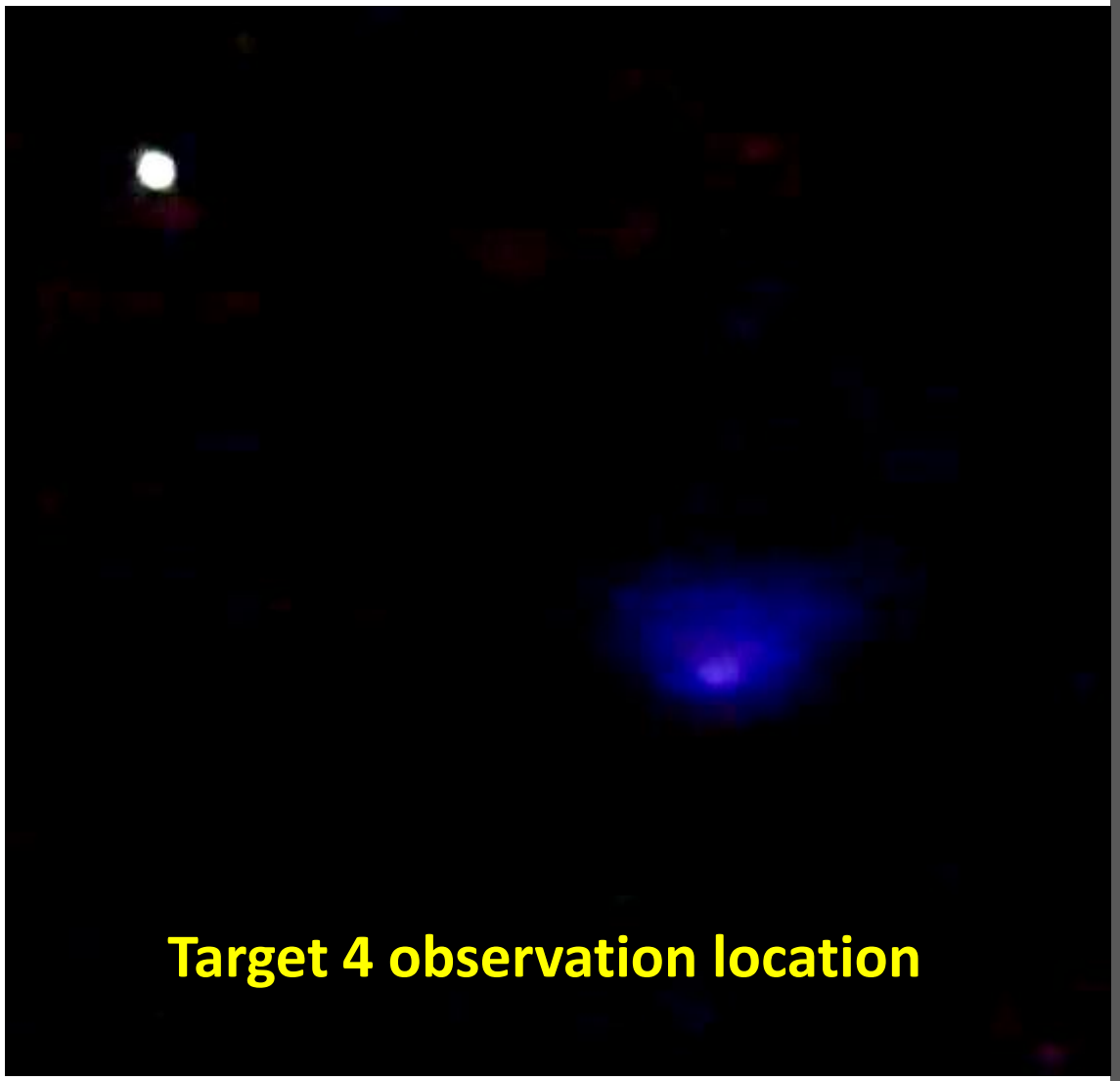


Target 4 observation location

MEASUREMENT 6

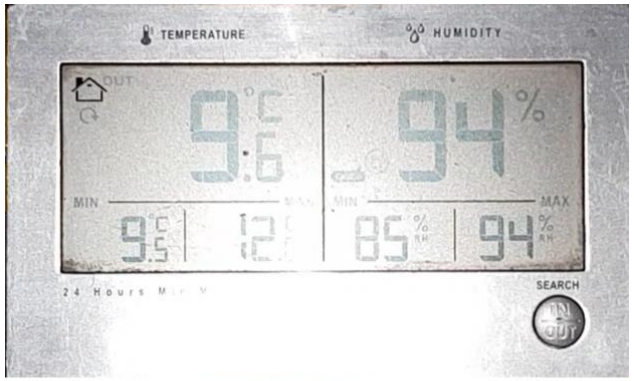
Lake Ijssel 22nd April 2018 at 2:19 AM





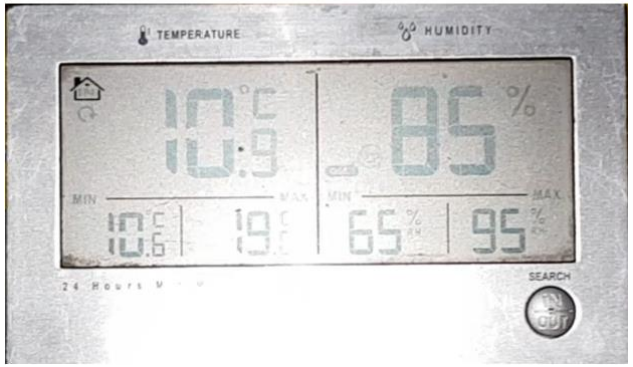
MEASUREMENT 6

Lake Ijssel 22nd April 2018 at 2:19 AM

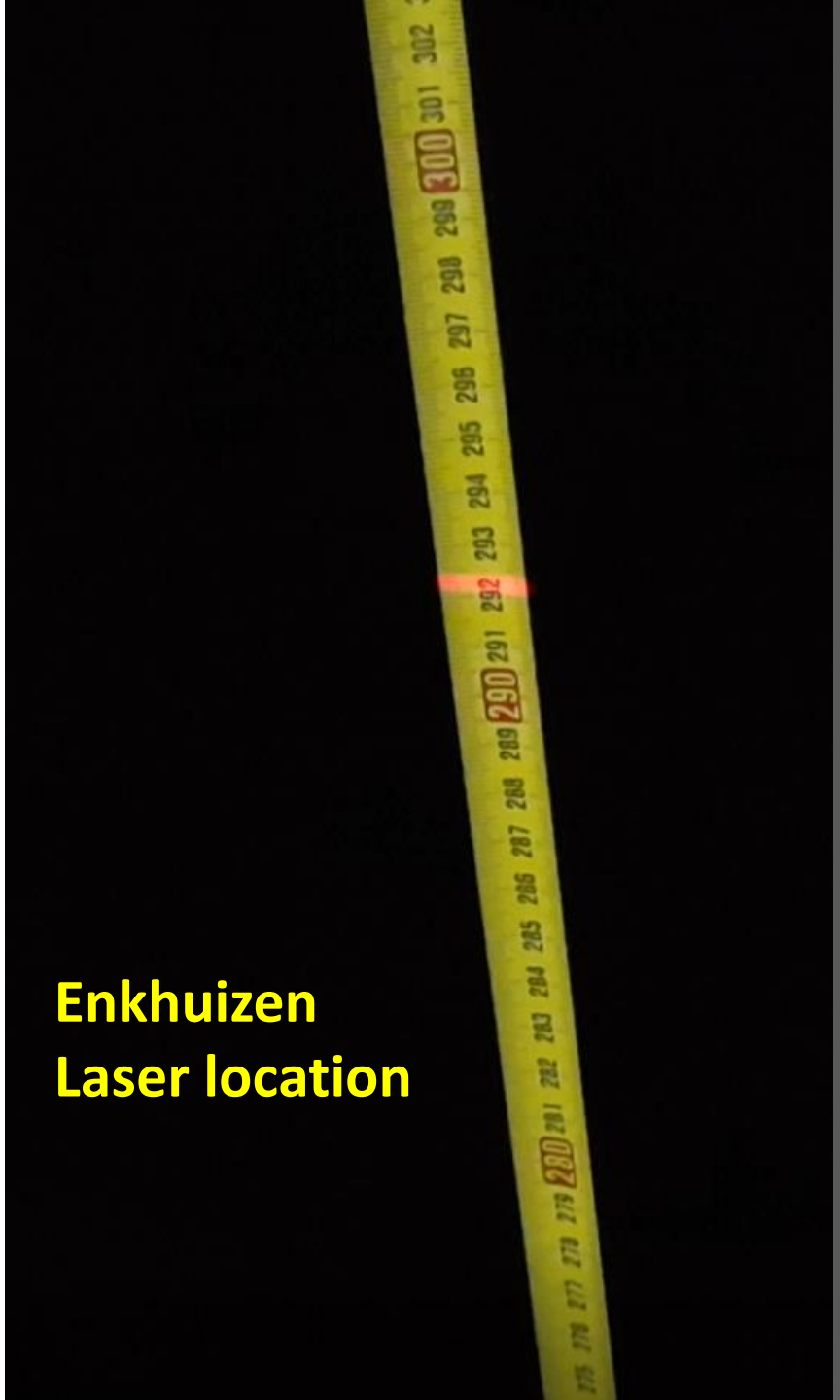


Temperature & humidity

Water level
9.6°C (49.3°F)
94%

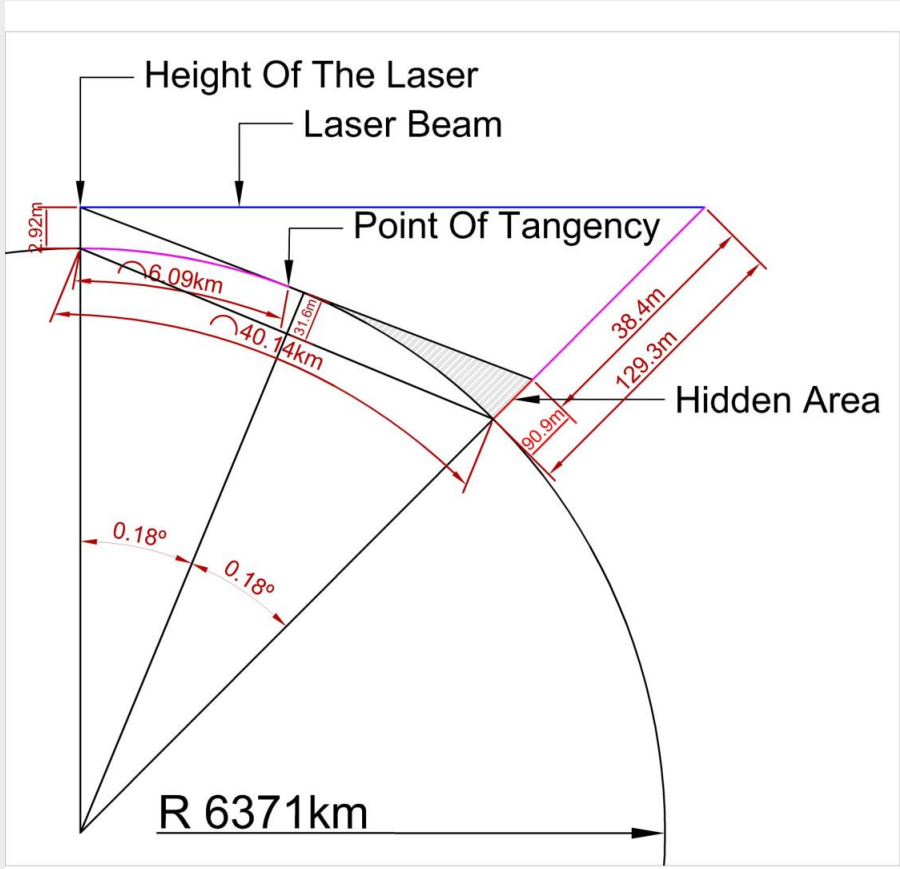


At 2 meters (6.56 feet)
11.6°C (52.9°F)
85%



LASER BEAM HIDDEN HEIGHT CALCULATION MEASUREMENT 6 TARGET 4

The calculations based on a spherical model results in a target hidden height of the Measurement 6 Target 4 position of **90.81** meters (297.33 feet)



CURVATURE CORRECTION WGS84 AND MSL

The radius of Earth is 6364.514 km (3954.726 miles) at the 53.070055° N latitude (Target 4) on the WGS84 ellipsoid model, and 6364.643 km (3954.806 miles) at the 52.710094° N latitude (laser).
The measurement direction heading is 3.66°

$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

The calculated difference of curvature drop on WGS84 to the spherical model from Laser to Target 4:
+2.56 mm (+0.101 inches)

The height of Lake Ijssel above sea level is 0 meters.

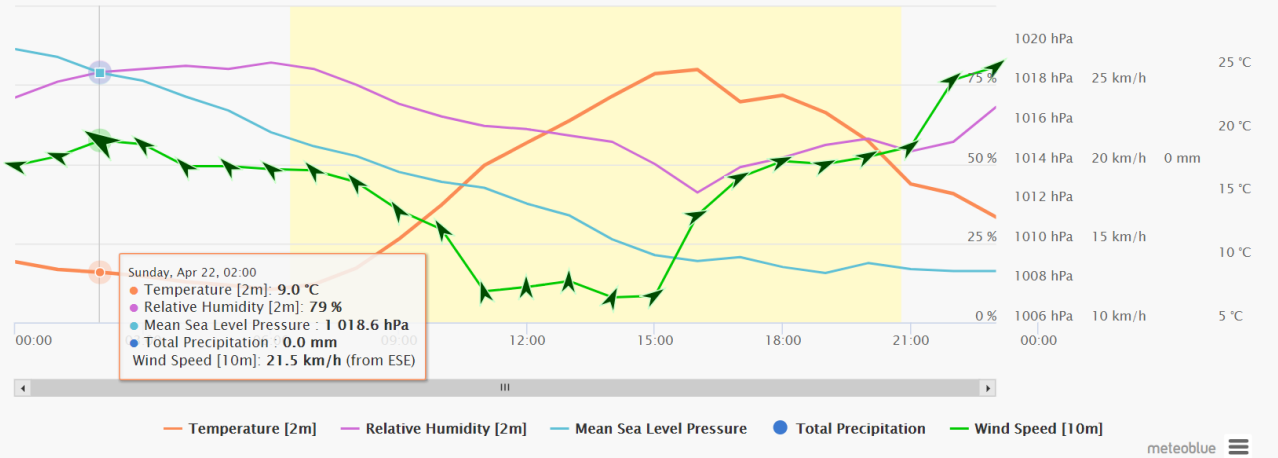
Input	Amount
Vacuum Wavelength:	445 Nanometers [nm]
Air Temperature:	9 Degrees Celsius
Atmospheric Pressure:	1.0186 Bars
Air Humidity:	79 Relative Humidity, Percent

Output	Result
Wavelength in Ambient Air:	444.871927 Nanometers [nm]
Refractive Index of Air[1]:	1.000287887
Uncertainty of Calculated Index[2]:	0.000000032

https://www.meteoblue.com/en/weather/archive/export/enkhuizen_netherlands_2756077?daterange=2018-04-22+to+2018-04-22&params=&params%5B...

2018-04-22 to 2018-04-22 | history+ locations (0/0)

The historical weather report is limited to the last 2 weeks for evaluation. For unlimited access starting in 1985 this location must be activated with history+.



MEASUREMENT 6

Lake Ijssel 22nd April 2018 at 2:19 AM

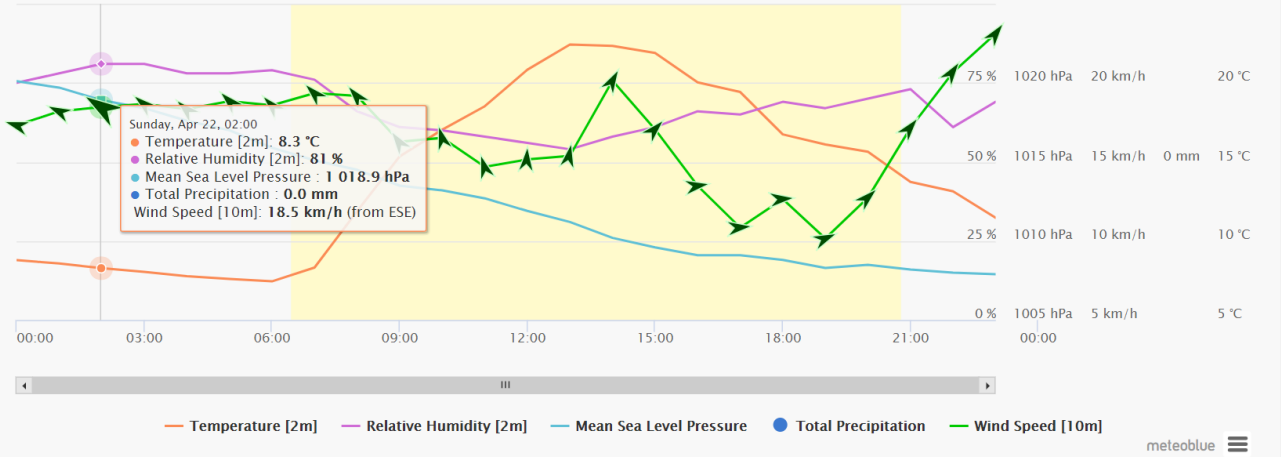
Input	Amount
Vacuum Wavelength:	445 Nanometers [nm]
Air Temperature:	8.3 Degrees Celsius
Atmospheric Pressure:	1.0189 Bars
Air Humidity:	81 Relative Humidity, Percent

Output	Result
Wavelength in Ambient Air:	444.871567 Nanometers [nm]
Refractive Index of Air[1]:	1.000288697
Uncertainty of Calculated Index[2]:	0.000000032

https://www.meteoblue.com/en/weather/archive/export/makkum_netherlands_2751254?daterange=2018-04-22+to+2018-04-22&params=&params%5B%...

2018-04-22 to 2018-04-22 | history+ locations (0/0)

The historical weather report is limited to the last 2 weeks for evaluation. For unlimited access starting in 1985 this location must be activated with history+.



REFRACTION CALCULATION OF 6TH MEASUREMENT

Angle of incidence (θ1): 0.0020°

Refractive index calculation
(based on Modified Edlén Equation)

n1 = 1.000287887 (445nm, 9°C, 1.0186Bars, 79%)

n2 = 1.000288697 (445nm, 8.3°C, 1.0189Bars, 81%)

Angle of refraction is calculated with Snell's law: sin θ2 = (n1 * sinθ1) / n2 = 0.001999998 degrees
Angle of deviation = 0.000000002°

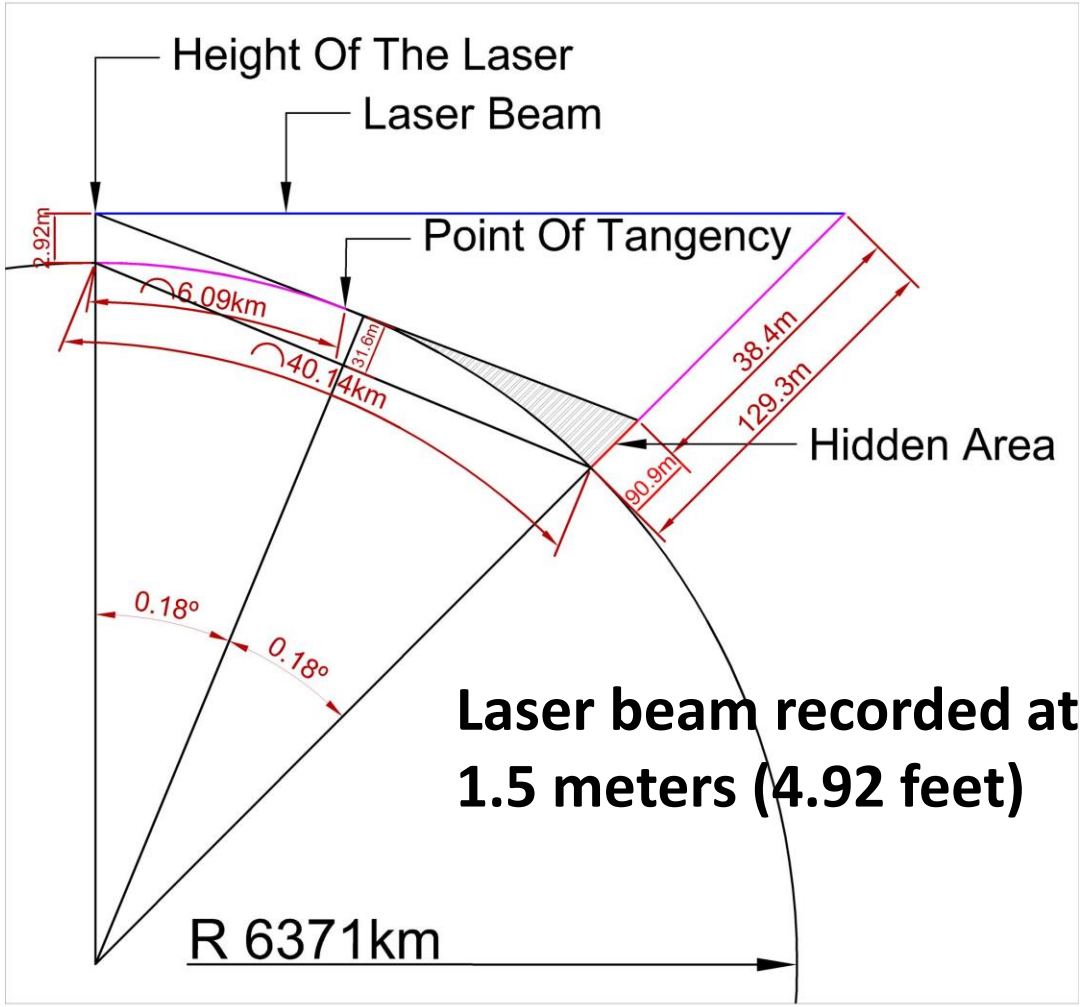
We concluded the ambient conditions refracted the laser beam downward by a maximum of
0.064 mm (0.0025 inches)

POSITIONS AND HEIGHT DATA AT TARGET 4 POSITION LAKE IJSSEL

Enkhuizen (laser position):
Latitude = 52.7100944444445° N = 52° 42' 36.34" N
Longitude = 5.2959722222222° E = 5° 17' 45.5" E
GPS ellipsoidal height = 0 meters
Geoid height = 42.409 meters (139.135 feet)

Target 4 (measurement position):
Latitude = 53.0700555555556° N = 53° 4' 12.2" N
Longitude = 5.33469166666667° E = 5° 20' 4.89" E
GPS ellipsoidal height = 0 meters
Geoid height = 41.847 meters (137.292 feet)

Difference geoid height: 562 mm (22.126 inches)



Measurement 6
Target hidden height spherical model:
90.81 meters (297.92945 feet)

WGS84 laser to target correction:
+2.17 mm (0.086 inch)

MSL correction is **0mm**

Refraction correction (max)
-0.065 mm (-0.00257 inches)

EXPECTED target hidden height
90.81211 meters (297.93637 feet)

Difference of geoid height:
+562 mm (+22.126 inches)

TARGET 4 POSITION
Lake Ijssel
(Measurement 7)

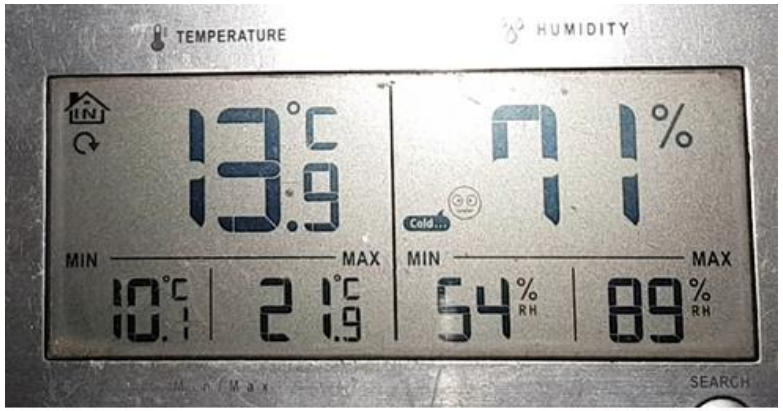
MEASUREMENT 7
LAKE IJSEL TARGET 4

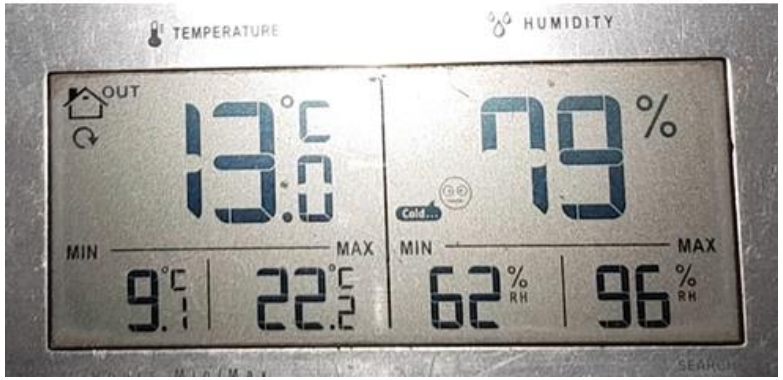
On the 22nd of April at 23:20, the blue laser pointer was placed on the SALAD at 2.92 meters (9.58 feet) above the lake surface heading to 3.66°.

It was seen on the opposite shore at 40.14km (24.94 miles) to Target 4 at 1.5 meters (4.92 feet) above the lake surface.

MEASUREMENT 7

Lake Ijssel 22nd April 2018 at 23:20 PM

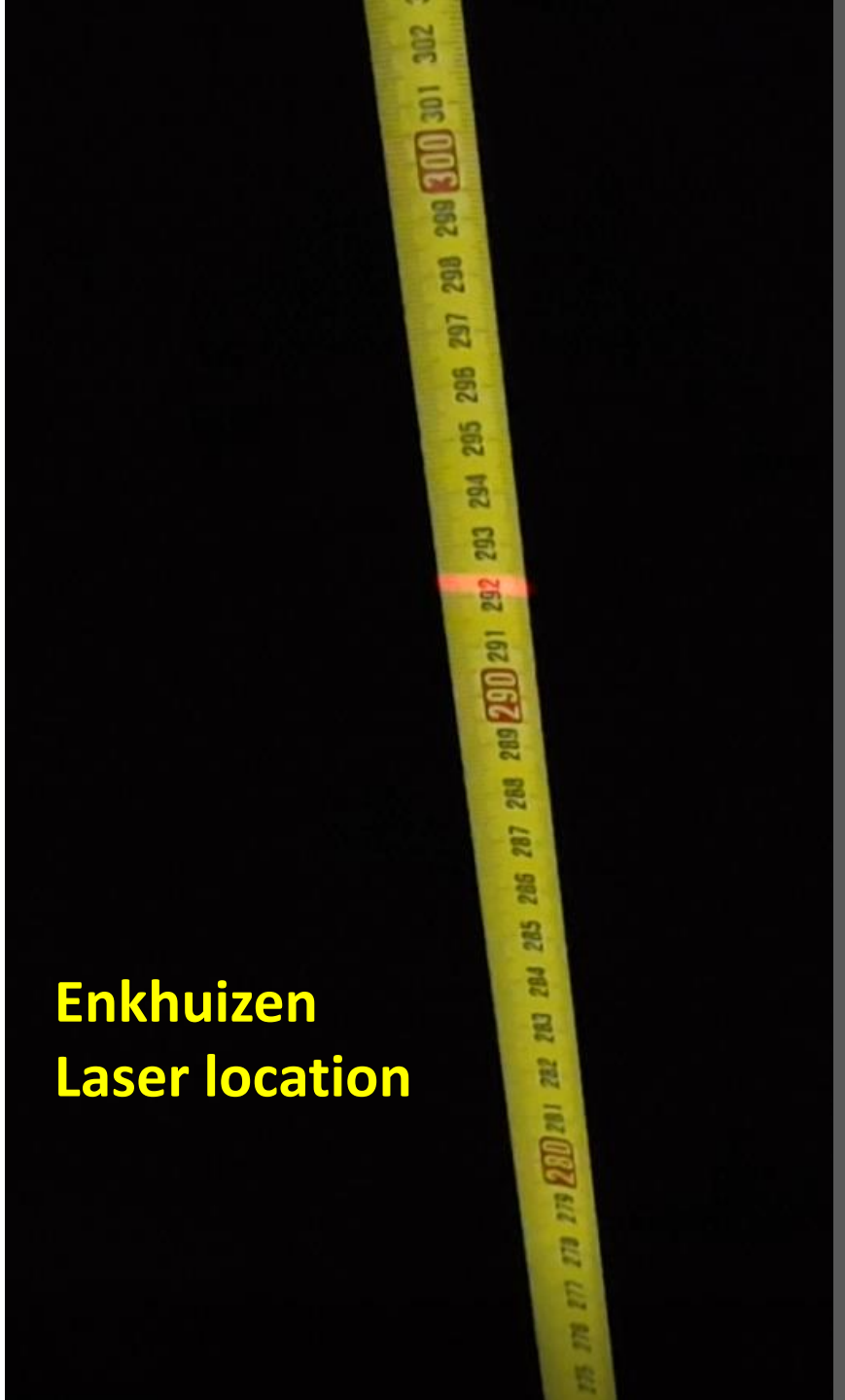




Temperature & humidity

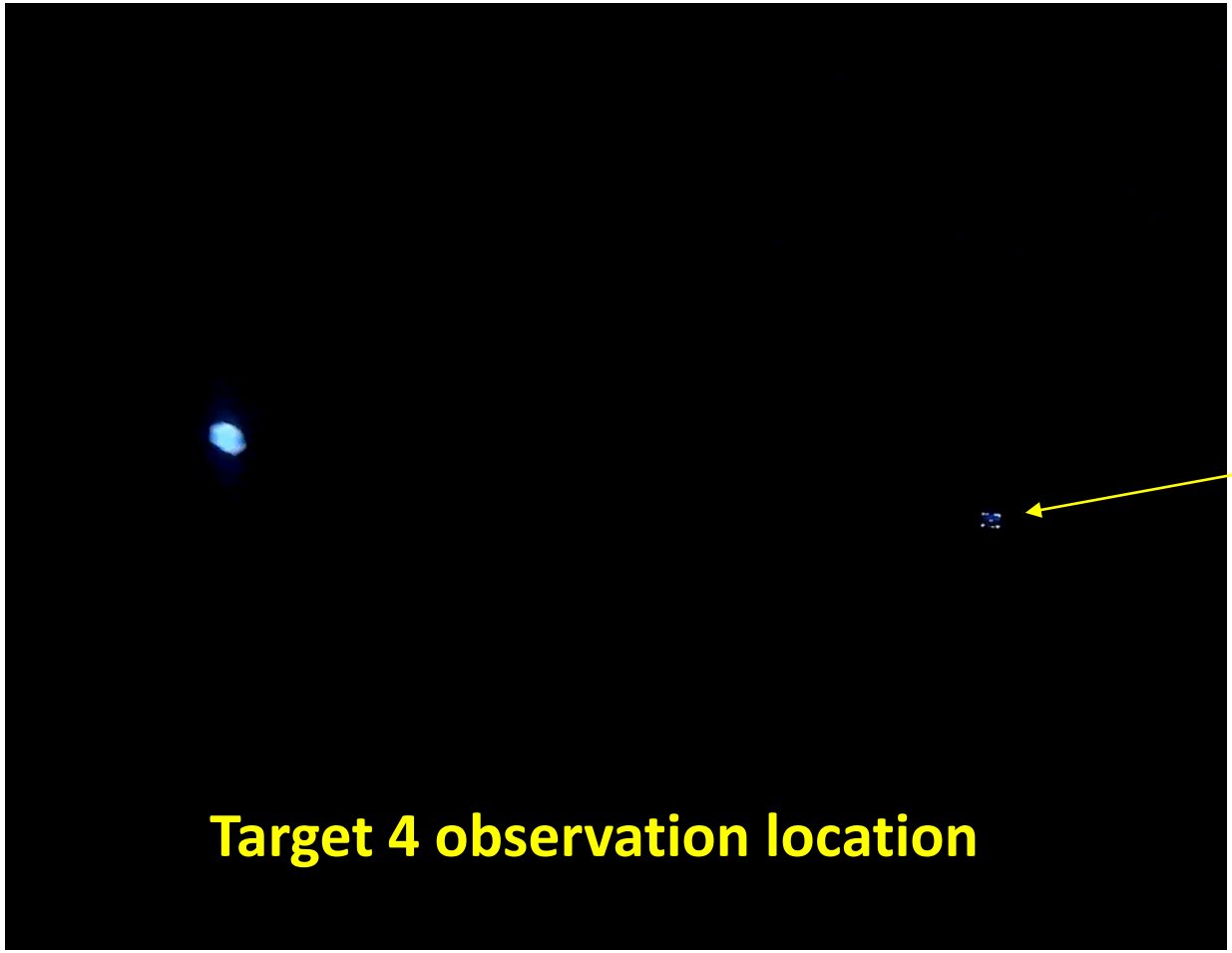
At 2 meters (6.56 feet)
13.9°C (57.0°F)
71%

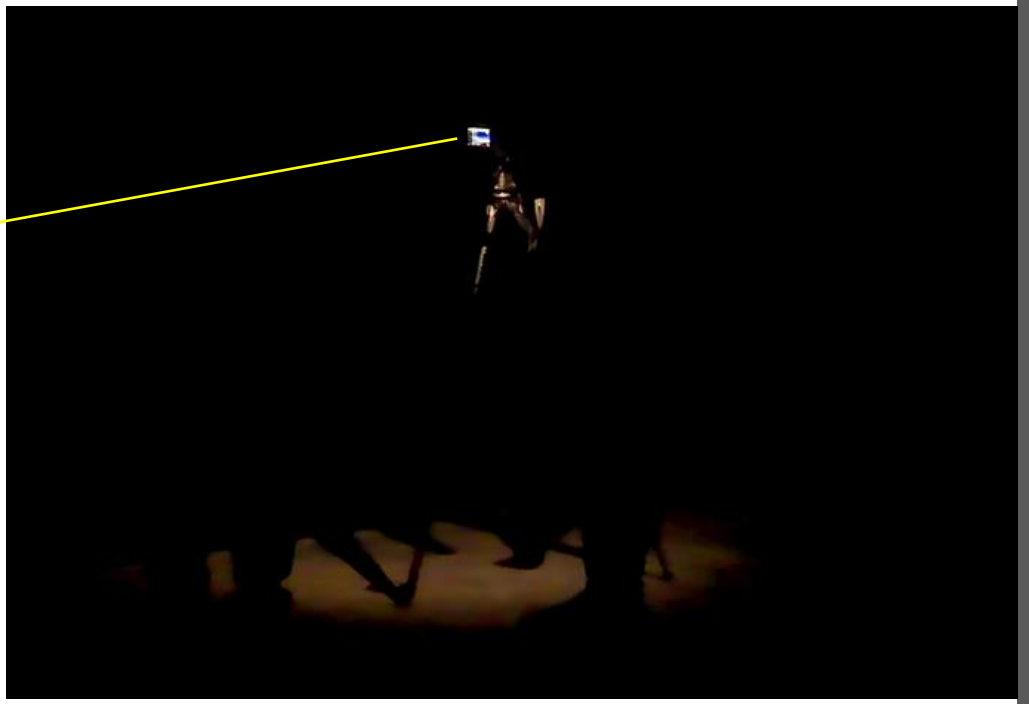
Water level
13.0°C (55,4°F)
79%



MEASUREMENT 7

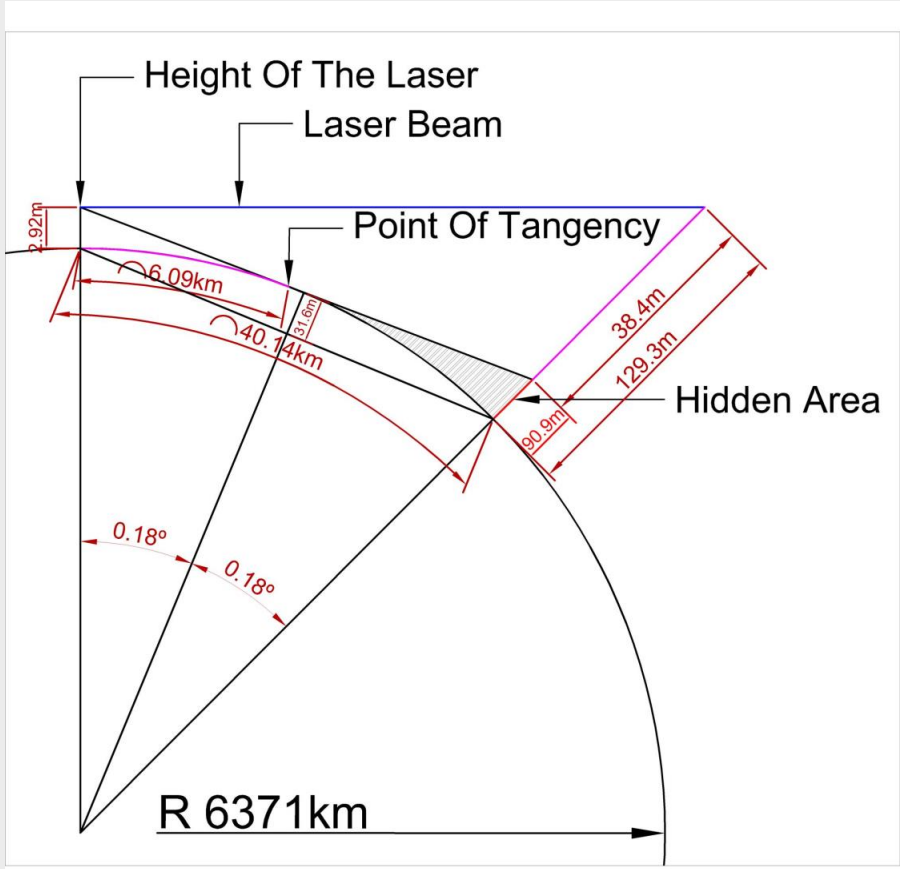
Lake Ijssel 22nd April 2018 at 23:20 PM





LASER BEAM HIDDEN HEIGHT CALCULATION MEASUREMENT 7 TARGET 4

The calculations based on a spherical model results in a target hidden height of the Measurement 7 Target 4 position of **90.81** meters (297.33 feet)



CURVATURE CORRECTION WGS84 and MSL

The radius of Earth is 6364.514 km (3954.726 miles) at the 53.070055° N latitude (Target 4) on the WGS84 ellipsoid model, and 6364.643 km (3954.806 miles) at the 52.710094° N latitude (laser).
The measurement direction heading is 3.66°

$$R = \sqrt{ [(r_1^2 * \cos(B))^2 + (r_2^2 * \sin(B))^2] / [(r_1 * \cos(B))^2 + (r_2 * \sin(B))^2]}$$

The calculated difference of curvature drop on WGS84 to the spherical model from Laser to Target 4: **+2.56 mm** (+0.101 inches)

The height of Lake Ijssel above sea level is 0 meters.

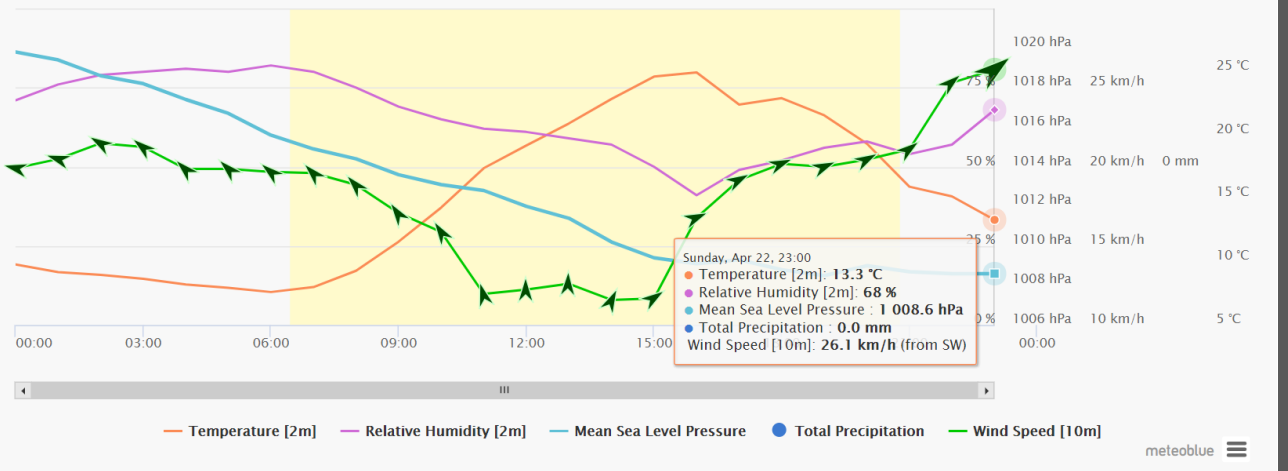
Input	Amount
Vacuum Wavelength:	445 Nanometers [nm]
Air Temperature:	13.3 Degrees Celsius
Atmospheric Pressure:	1.0086 Bars
Air Humidity:	68 Relative Humidity, Percent

Output	Result
Wavelength in Ambient Air:	444.875116 Nanometers [nm]
Refractive Index of Air[1]:	1.000280717
Uncertainty of Calculated Index[2]:	0.000000031

https://www.meteoblue.com/en/weather/archive/export/enkhuizen_netherlands_2756077?daterange=2018-04-22+to+2018-04-22&params=&params%5B...

2018-04-22 to 2018-04-22 | history+ locations (0/0)

The historical weather report is limited to the last 2 weeks for evaluation. For unlimited access starting in 1985 this location must be activated with history+.



MEASUREMENT 7

Lake Ijssel 22nd April 2018 at 23:20

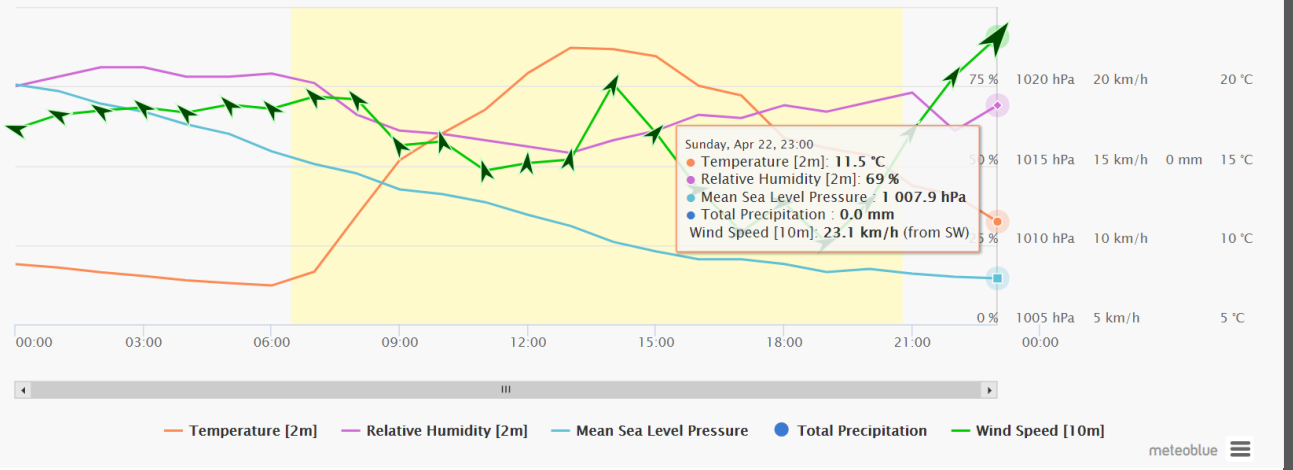
Input	Amount
Vacuum Wavelength:	445 Nanometers [nm]
Air Temperature:	11.5 Degrees Celsius
Atmospheric Pressure:	1.0079 Bars
Air Humidity:	69 Relative Humidity, Percent

Output	Result
Wavelength in Ambient Air:	444.874395 Nanometers [nm]
Refractive Index of Air[1]:	1.000282338
Uncertainty of Calculated Index[2]:	0.000000031

https://www.meteoblue.com/en/weather/archive/export/makkum_netherlands_2751254?daterange=2018-04-22+to+2018-04-22&params=&params%5B%...

2018-04-22 to 2018-04-22 | history+ locations (0/0)

The historical weather report is limited to the last 2 weeks for evaluation. For unlimited access starting in 1985 this location must be activated with history+.



REFRACTION CALCULATION OF 7TH MEASUREMENT

Angle of incidence (θ1): 0.0020°

Refractive index calculation
(based on Modified Edlén Equation)

n1 = 1.000280717 (445nm, 13.3°C, 1.0086Bars, 68%)

n2 = 1.000282338 (445nm, 11.5°C, 1.0079Bars, 69%)

Angle of refraction is calculated with Snell's law: sin θ2 = (n1 * sinθ1) / n2 = 0.001999997 degrees
Angle of deviation = 0.000000003°

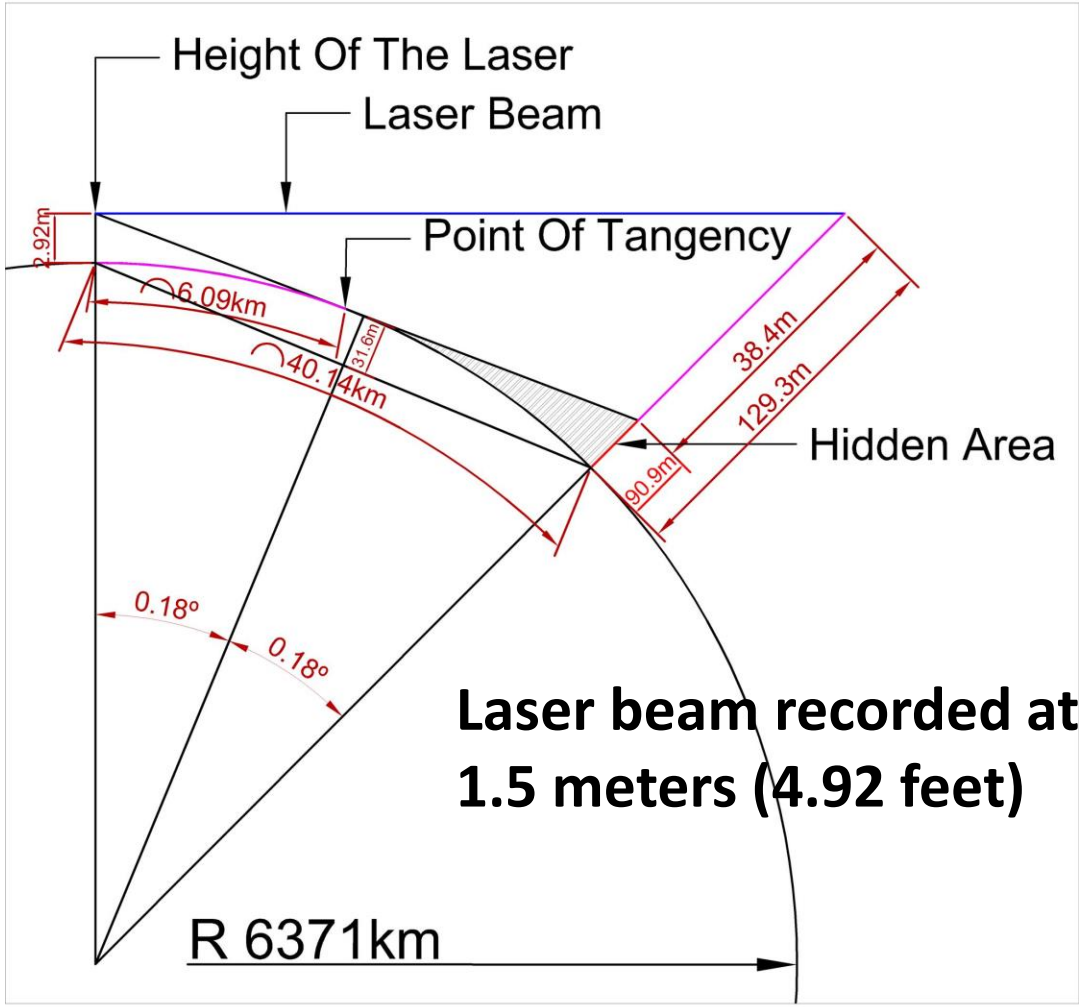
We concluded the ambient conditions refracted the laser beam downward by a maximum of
0.131 mm (0.00512 inches)

POSITIONS AND HEIGHT DATA AT TARGET 4 POSITION LAKE IJSSEL

Enkhuizen (laser position):
Latitude = 52.7100944444445° N = 52° 42' 36.34" N
Longitude = 5.2959722222222° E = 5° 17' 45.5" E
GPS ellipsoidal height = 0 meters
Geoid height = 42.409 meters (139.135 feet)

Target 4 (measurement position):
Latitude = 53.0700555555556° N = 53° 4' 12.2" N
Longitude = 5.33469166666667° E = 5° 20' 4.89" E
GPS ellipsoidal height = 0 meters
Geoid height = 41.847 meters (137.292 feet)

Difference geoid height: 562 mm (22.126 inches)



Measurement 7
Target hidden height spherical model:
90.81 meters (297.92945 feet)

WGS84 laser to target correction:
+2.17 mm (0.086 inches)

MSL correction is **0mm**

Refraction correction (max)
-0.133 mm (-0.00525 inches)

EXPECTED target hidden height
90.81204 meters (297.93614 feet)

Difference of geoid height:
+562 mm (+22.126 inches)

EXPERIMENT RESULTS

Experiment Results of the seven tests indicates there is no curvature on the lake surfaces.

The testing results agree with the analysis of error sources. Feasibility of optimizing optical configuration was verified.

In summary, the testing results of the lake surface were ± 0.2 m, relative accuracy was within 1% (confidence level 98%).

Our experiment results verify the possibility of a point-to-point long-range wireless communication system.

CONCLUSION

The TLT measurements meet the accuracy of the experiment design requirements and provided a definite deliverable output:
The surface of Lake Balaton and Lake Ijssel is not convex and curvature is not detectable.

Through analyzing the error-source models of TLT measurements and corrections for the geoid surfaces, we determined the results of the experiments **definitively prove** our hypothesis of non-convex water surfaces.

CONCLUSION

As all large bodies of water should follow the geopotential surface, we proved these lakes are NON-UNIFORM with the WGS84 model.

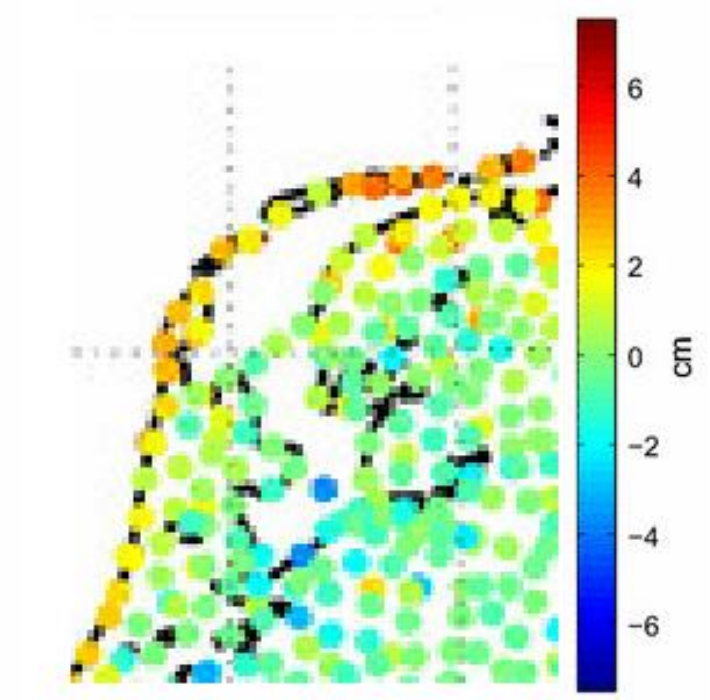
This has a very important impact on the WGS84 model it self:
The **WGS84 MODEL IS INCORRECT** as these large scale height differences are not detectable! The geoid surface heights do NOT conform with the land measurements.

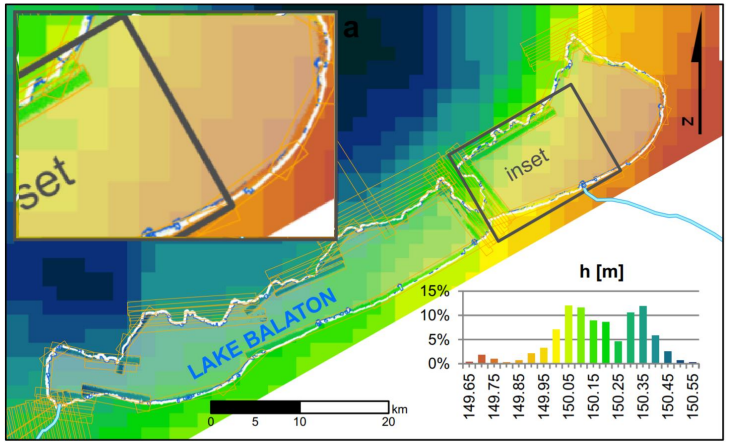
CONCLUSION

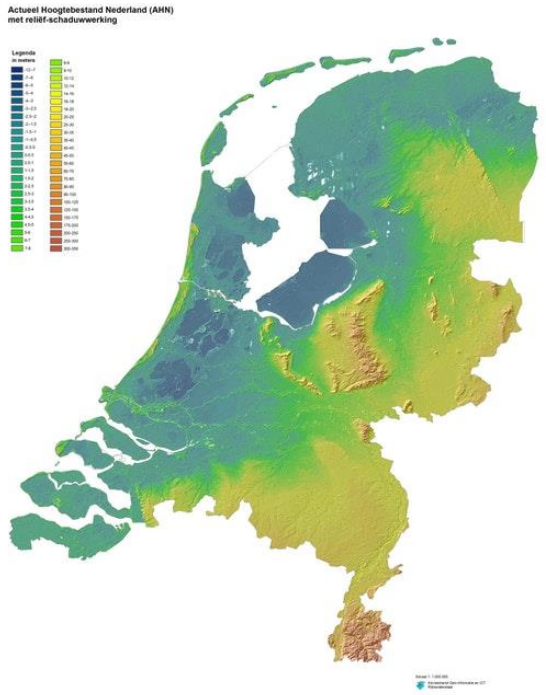
Water surfaces are uniform over the whole area without bending points as they are gravipotential surfaces. Therefore, a measured section is representative for the entire surface area.

Lake Balaton has a total stretch of 77 kilometers that is supposed to have a curvature of 465 meters. Lake Ijssel has a total stretch of 70 kilometers that is supposed to have a curvature of 385 meters.

In the case of a local gravitational anomaly, the geoid surface should indicate that difference in the WGS84 as geoid undulation in height differences.







Where is the missing curvature height difference?

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TABLE 1 - Metrical
Testing results of the lake curvature measurements

Measurement #	1 (Balaton)	2 (Balaton)	3 (Ijssel)	4 (Ijssel)	5 (Ijssel)	6 (Ijssel)	7 (Ijssel)
SPHERICAL Target Hidden Height	4570	3520	18180	40210	12504	90810	90810
Correction WGS84 Laser to Target	0.029	-0.026	0.18	0.439	0.16	2.17	2.17
MSL	-0.119	-0.119	0	0	0	0	0
Refraction	0	0.235	-0.329	-0.494	-0.0117	-0.064	-0.131
EXPECTED Target Hidden Height	4569.91	3520.09	18179.85	40209.95	12504.15	90812.11	90812.04
Variance to Geoid Height	129	-129	233	306	260	562	562
All units in millimeters							

TABLE 1 - Imperial
Testing results of the lake curvature measurements

Measurement #	1 (Balaton)	2 (Balaton)	3 (Ijssel)	4 (Ijssel)	5 (Ijssel)	6 (Ijssel)	7 (Ijssel)
SPHERICAL Target Hidden Height	179.92	138.58	715.75	1583.07	492.28	3575.2	3575.2
Correction WGS84 Laser to Target	0.00114	-0.001	0.00709	0.01728	0.0063	0.08543	0.08543
MSL	-0.00469	-0.004469	0	0	0	0	0
Refraction	0	0.00925	-0.01295	-0.01945	-0.00046	-0.0025	-0.00512
EXPECTED Target Hidden Height	179.918	138.586	715.742	1583.069	492.289	3575.28	3575.277
Variance to Geoid Height	5.79	-5.79	9.17	12.05	10.24	22.13	22.13
All units in inches							

TECHNICAL DATA

Several types, power and wavelength lasers were used in the measurements.

1. Precision green laser device
   output power 2W
   0.08mRad collimation
   555 nm green
2. High power green laser device
   output power 5W
   555 nm green
3. RGB laser device
   output power 9W
   450/555/660nm
4. Green laser device
   output power 3W
   555 nm green
5. Blue laser pointer
   output power 1 to 2W
   445 nm blue mRad

TECHNICAL DATA

Super Accurate Laser Aiming Device (SALAD)

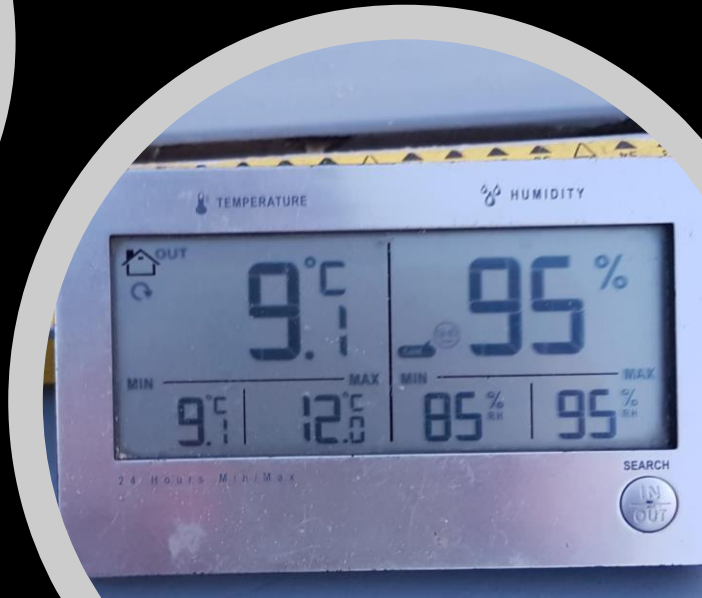
The SALAD was designed specially for this experiment with an aiming accuracy of 0.000024 degree precision. The projected beam can be moved within an increment of less than a centimeter at a distance of 66 km. The SALAD can be remotely controlled and it is able to store the position parameters of the previously measured positions. Automatic GPS targeting system was recently developed.

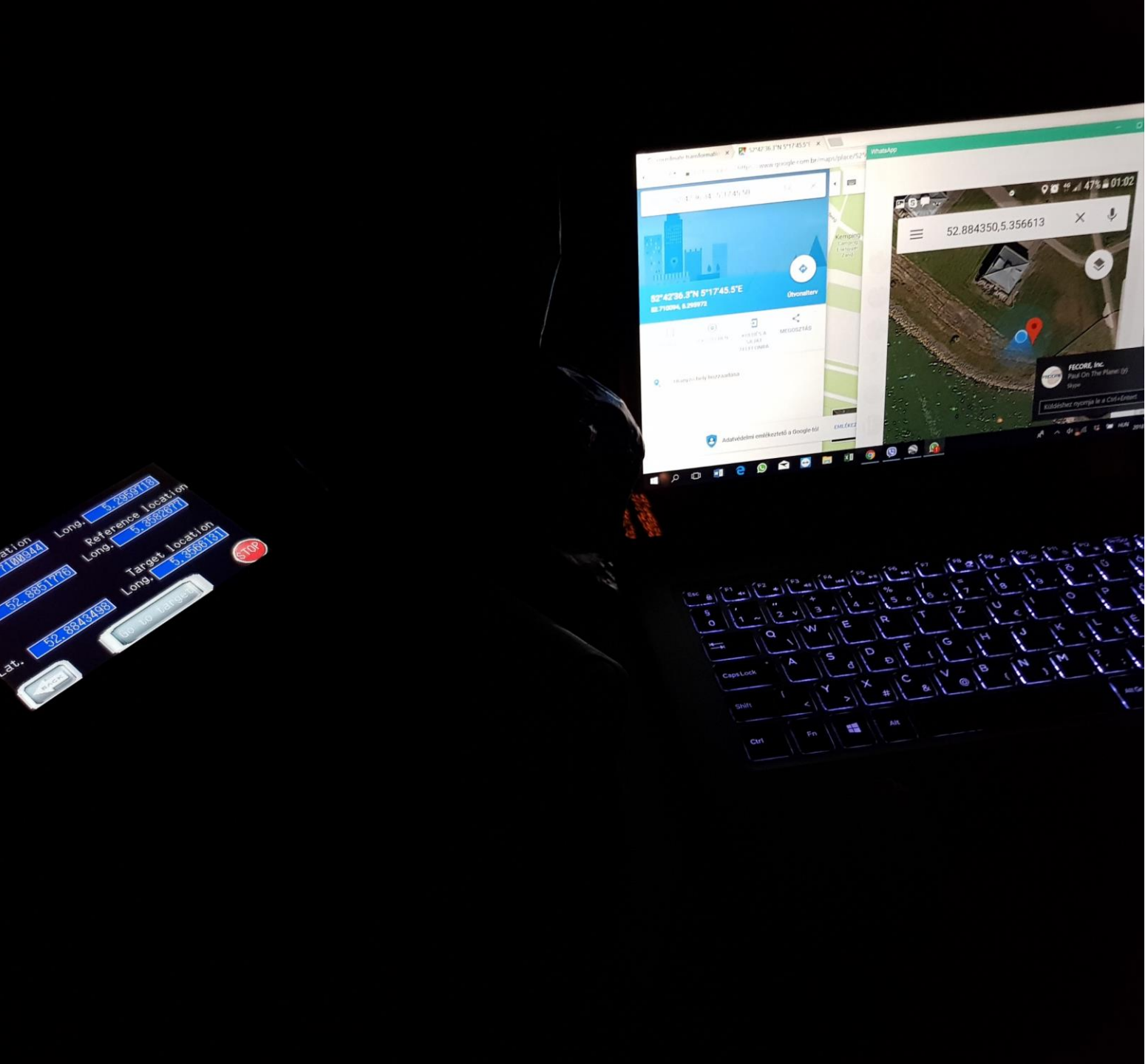
TECHNICAL DATA

Devices used to observe the laser beam:

1. portable canvas with 3x3 meter surface
2. Nikon P900 camera
3. Canon 5d Mark II EFL 17-40 mm f4.0 Lens, at 40mm
4. Samsung S8 and Samsung S6 mobile phones
5. GoPro 5

TEMPERATURE & HUMIDITY READINGS

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Reference location
Long.
Target location
Long.
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Lat.
Go to Target
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FECORE, Inc.


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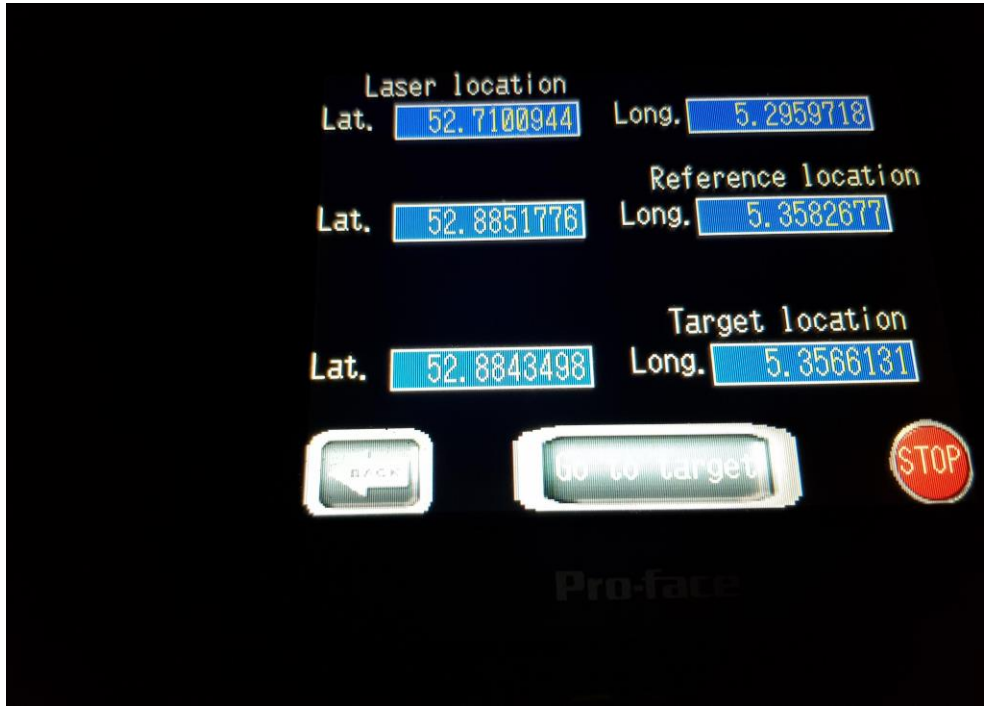
Ver.pos.
87596
Step
1000
STOP
Pro-face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Long.
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Long.
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Target location
Lat.
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Long.
5.3566131
BACK
STOP
Pro-face

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- Cami Knodel
- Jeran Campanella
- Chris Seely
- Paul On The Plane
- Rick Zimmerman
- Rob Skiba
- David Shepherd
- Rick Hummer
- Taboo “Ben” Conspiracy
- David Weiss
- Richard Aubin
- Daisy van Havermaet
- Chris van Matre
- Richard Blades
- And to BILU & Big Foot

SPECIAL THANKS TO OUR FECORE MEMBERS!

Please visit our website to find out more about our projects. We will continue to work on other TLT measurements over sea and land surfaces to prove the real shape of Earth.

Your donations or membership is very much appreciated by our team to continue with the experiments.

FECORE

THANK YOU FOR YOUR ATTENTION!

– to be continued! ☺