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1981 Sov. J. Quantum Electron. 11 186

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Calculations of the accuracy of wavefront reversal utilizing pump radiation with one-dimensional transverse modulation

B. Ya. Zel'dovich and T. V. Yakovleva

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted June 6, 1980)

Kvantovaya Elektron. (Moscow) **8**, 314–321 (February 1981)

The problem of wavefront reversal in stimulated Brillouin backscattering in a pump field with one-dimensional transverse modulation for the reversing mode of the scattered field is considered. Calculations are made of corrections corresponding to "serpentine" distortions. The fraction of these distortions is $\sim x\ln(1/x)$, where $x = g/k\theta^2$ is the product of the gain g (in reciprocal centimeters) and of the Fresnel length $(k\theta)^{-1}$ of a pump field with the divergence θ. Specific differences between the one-dimensional and two-dimensional problems are discussed.

PACS numbers: 42.65.Cq

1. INTRODUCTION

Wavefront reversal (WFR) in stimulated backscattering of light has been the subject of intensive experimental and theoretical investigations (see, for example, Refs. 1–3). A fairly detailed theory of WFR has been developed in various investigations[1,4-12] and a detailed study has also been made of the reversal accuracy, i.e., calculations have been made of the distortions of the structure of the backscattered field compared with that of the pump field.[4,8] In particular, this problem has been solved for the case of incomplete spatial modulation of the pump wave.[12] In all these investigations it was assumed that the pump field is spatially modulated along both coordinates of the transverse cross section of the beam, i.e., a two-dimensional WFR problem was analyzed.

However, also of interest may be experimental situations for which a one-dimensional WFR problem is more adequate, i.e., the case when the pump field varies only along one transverse coordinate. This formulation of the WFR problem is examined in the present quantitative theoretical study of the WFR accuracy for a pump beam with one-dimensional spatial modulation. Calculations are made of "serpentine" distortions in a pump-wavefront-reversing field. It has been established that these small-scale distortions are caused by pulling of the scattered field into active serpentine microwaveguides formed by the gain profile in an inhomogeneous pump field.

2. BASIC EQUATIONS

In the approximation of scalar monochromatic fields of the pump $E_L(x,z)$ and scattered waves $E_s(x,z)$, the parabolic equations for the slow amplitudes [i.e., after isolating rapidly oscillating factors $\exp(\pm ikz)$] in the one-dimensional case take the following form:

$$\frac{\partial E_L}{\partial z}+\frac{i}{2k}\frac{\partial^2}{\partial x^2}E_L=0; \tag{1}$$

$$\frac{\partial E_s}{\partial z}-\frac{i}{2k}\frac{\partial^2}{\partial x^2}E_s=\frac{1}{2}G\,|E_L(x,z)|^2E_s(x,z). \tag{2}$$

We shall assume that the scattered wave propagates in the $+Z$ direction and the pump wave propagates in the $-Z$ direction. The X axis is taken to be the direction of variation of the transverse structure of the two fields whilst these fields are homogeneous along the Y axis. This implies in particular that the wave vectors $\mathbf{k}_L$ and $\mathbf{k}_s$ are situated in the plane XZ. We shall neglect the small frequency shift $G|E_L|^2\sim(k_L-k)_s/k_L\sim10^{-5}$ in scattering of the stimulated Brillouin type. The quantity $G|E_L|^2$ characterizes the local intensity gain (in reciprocal centimeters). In Eq. (1) we shall neglect the influence of the scattered field on the pump field.

We shall express the solution of Eq. (1) as a Fourier series (which corresponds to the problem of a one-dimensional waveguide, i.e., bounded only with respect to X, with periodic boundary conditions of the Born–von Karman type):

$$E_L(x,z)=\sum_q C(q)\,e^{iqx+iq^2z/2k}. \tag{3}$$

Here, q is the modulus of the transverse component, with respect to the Z axis, of the wave vector of the corresponding angular component of the pump field: $\mathbf{k}_\perp=\mathbf{q}=q\mathbf{e}_x$. We shall assume that the amplitudes $C(q)$ of the various angular components of the pump field are independent Gaussian random quantities:

$$\langle C^*(q_1)C(q_2)\rangle=T(q_1)\,\delta(q_1-q_2), \tag{4}$$

where $\delta(q_1-q_2)$ is the discrete Kronecker δ function, the angular brackets denote averaging over the ensemble of fluctuations of the amplitudes $C(q)$. The average pump field intensity with respect to x is given by

$$I_L=\sum_q T(q)=\frac{1}{l}\int|E_L(x,z)|^2dx. \tag{5}$$

Here, l is the transverse dimension of the beam (optical waveguide) along the X axis.

We shall now analyze the solution of Eq. (2) for the scattered wave field $E_s(x,z)$. Expanding the amplitude $E_s(x,z)$ as a Fourier series

$$E_s(x,z)=\sum_q S(q,z)\,e^{ixq-iq^2z/2k}, \tag{6}$$

we obtain the following equations for the Fourier components $S(q,z)$:

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$$\frac{\partial S(q,z)}{\partial z}=\frac{1}{2}G\sum_{q_1,q_2,q_3}C(q_1)C^*(q_2)S(q_3,z)\,\delta(q_1-q_2+q_3-q)\,e^{i\Gamma z}, \quad (7a)$$

$$\Gamma=\frac{1}{2k}(q_1^2-q_2^2-q_3^2+q^2)=(q_1-q_2)(q_1+q_3)/k. \quad (7b)$$

It can be seen from Eq. (7a) that the scattered field component $S(q,z)$ is formed as a result of scattering of the component $S(q_3,z)$ of this field by a permittivity perturbation $\delta\varepsilon(x,z)\sim\exp[i(q_1-q_2)x+i(q_1^2-q_2^2)z/2k]$ produced in the scattering medium as a result of interference of the angular components of the pump field $C(q_1)$ and $C^*(q_2)$. By virtue of the Bragg condition for the x components of the wave vectors, the equality $q=q_3+q_1-q_2$ is found [the second expression in Eq. (7b) was obtained using this condition]. The quantity Γ characterizes the deviation from the Bragg condition for the z components of the wave vectors.

3. MODE APPROXIMATION

It has been established (see Refs. 4 and 5) that the mode approximation to solve Eq. (7a) involves allowing only for "automatic Bragg" or, in other words, coherent processes in all the processes involving scattering of the amplitude $S(q_3,z)$ by the gratings $\delta\varepsilon\to C(q_1)C^*(q_2)$. This implies that in the triple sum over q_1, q_2, q_3 only the following two groups of terms are retained: 1) $q_1=q_2$, $q_3=q$; 2) $q_1=-q_3$, $q_2=-q_2$ (for further details see Ref. 12).

Equation (7a) in the mode approximation takes the form

$$\frac{\partial S(q,z)}{\partial z}=\frac{1}{2}GI_LS(q,z)+\frac{1}{2}GC^*(-q)\sum_{q_1}S(q_1,z)C(-q_1). \quad (8)$$

In deriving Eq. (8) allowance was made for the condition $|C(q)|^2\ll I_L$, i.e., for the smallness of the intensity of each individual angular component compared with the total intensity I_L. The solutions of Eq. (8), described as modes, take the form

$$S^{(m)}(q,z)=e^{\mu z}m(q);\quad E_s^{(m)}(x,z)=e^{\mu z}\sum_q m(q)\,e^{iqx-iq^2z/2k},$$

i.e., in the mode approximation the existence of an inhomogeneous gain distribution in the medium is only observed as the exponential factor $\exp(\mu,z)$ and the spatial structure of the scattered field satisfies the free wave equation.

For modes uncorrelated with the pump field we have $\sum_{q_1}S^{(\mathrm{unc})}(q_1,z)C(-q_1)=0$, and the increment is $\mu_0=\frac{1}{2}GI_L$. As far as the mode correlated with the pump field is concerned, expressing this in the form $S^{\mathrm{cor}}(q,z)=f(q,z)C^*(-q)$, it is readily established that the function $f(q,z)$ is independent of q, i.e., the mode correlated with the pump field accurately reverses the pump wavefront:

$$f(z)=f(0)e^{\mu_1 z};\ \mu_1=GI_L;\quad S^{\mathrm{cor}}(q,z)=C^*(-q)f(0)e^{\mu_1 z}. \quad (9)$$

Thus, in the mode approximation, the scattered field comprises the sum of the solution which accurately reverses the pump wavefront and many solutions corresponding to uncorrelated waves. These are amplified with respect to z with an increment half that for the generating solution. Thus, we shall neglect these uncorrelated modes formed from the spontaneous noise in the cross section $z=0$ as compared with the strong pump-wavefront-reversing mode.

4. CALCULATIONS OF SERPENTINE DISTORTIONS IN THE FIRST ORDER OF PERTURBATION THEORY

In order to determine the structure of the scattered field more accurately, not only the automatic Bragg scattering processes but all these processes should be taken into account. Thus, we shall express Eq. (7a) in the form

$$\frac{\partial S(q,z)}{\partial z}-\frac{1}{2}GI_LS(q,z)-\frac{1}{2}GC^*(-q)\sum_{q_1}C(-q_1)S(q_1,z)$$
$$=\frac{1}{2}G\sum_{q_1,q_2,q_3}{}'\,C(q_1)C^*(q_2)S(q_3,z)\,\delta(q_1-q_2+q_3-q)\,e^{i\Gamma z}. \quad (10)$$

The prime in the sum on the right-hand side of Eq. (10) implies that all automatic Bragg terms are eliminated, i.e., Σ' corresponds to random incoherent scattering processes. These processes may distort the wavefront of the scattered field compared with the pump field (serpentine distortions) and may also change the z dependence of the reversing solution. Thus, we shall express the solution of Eq. (10) in the form

$$S(q,z)=e^{\mu_1 z}A(z)\,C^*(-q)+F_1(q,z), \quad (11)$$

where $F_1(q,z)$ is the amplitude of the serpentine distortions. We shall subject the field of these distortions to the condition of orthogonality with respect to the conjugate pump field:

$$\sum_q F_1(q,z)\,C(-q)=0.$$

Possible corrections to the z dependence of the reversing mode, i.e., to the coefficient at $C^*(-q)$, as a result of incoherent processes will be included in the factor $A(z)$. We shall solve Eq. (10) considering its right-hand side to be a perturbation. Substituting Eq. (11) into Eq. (10) and allowing for the condition $|C(q)|^2\ll I_L$, after various transformations we obtain the following equation for $F_1(q,z)$:

$$\frac{\partial F_1}{\partial z}-\mu_0F_1=e^{\mu_1 z}\frac{G}{2}A(z)\sum{}'\,C(q_1)C^*(q_2)C^*(-q_3)$$
$$\times\,\delta(q_1-q_2+q_3-q)\,e^{i\Gamma z}. \quad (12)$$

Equation (12) is formulated in the first order of perturbation theory. In deriving this equation, the unperturbed zeroth-order solution is substituted for $S(q)$ on the right-hand side of Eq. (10). When integrating Eq. (12), we shall assume that the dependence $A(z)$ is slow and in this case, for fairly large z (i.e., for $GI_Lz\gtrsim 3$) we obtain

$$\int_0^z A(z')\exp\left[\left(\frac{1}{2}GI_L+i\Gamma\right)z'\right]dz'$$
$$\approx A(z)\exp\left[\left(\frac{1}{2}GI_L+i\Gamma\right)z\right]\Big/\left(\frac{1}{2}GI_L+i\Gamma\right).$$

From this it follows that the amplitude of the distortions $F_1(q,z)$ is

$$F_1(q,z)=\frac{G}{2}A\sum_{q_1,q_2,q_3}{}'\,C(q_1)C^*(q_2)C^*(-q_3)$$
$$\times\,\delta(q_1-q_2+q_3-q)\,e^{\mu_1 z+i\Gamma z}\Big/\left(\frac{1}{2}GI_L+i\Gamma\right). \quad (13)$$

Squaring Eq. (13) with respect to the modulus and averaging over the ensemble of fluctuations of the pump amplitudes $C(q)$ allowing for Eq. (4), assuming statistical independence of components with different q, we obtain the intensity of an individual angular component of the noise field:

$$\langle |F_1(q,z)|^2\rangle = \frac{G^2}{2} A^2 e^{2\mu_1 z} \sum'_{q_1,q_2,q_3} \langle |C(q_1)|^2\rangle \langle |C(q_2)|^2\rangle \times \langle |C(q_3)|^2\rangle \delta(q_1-q_2-q_3-q) \Big/ \left[\left(\frac{1}{2}GI_L\right)^2+\Gamma^2\right]. \qquad (14)$$

The factor $1/[(\frac{1}{2}GI_L)^2+\Gamma^2(q_1,q_2,q_3,q)]$ characterizes the efficiency of excitation of the distortions $F_1(q,z)$ as a result of a particular scattering process. Using Eqs. (13) and (14), it is readily established that the main contribution to the serpentine distortions is made by those scattering processes for which the Bragg condition is approximately satisfied: $|\Gamma(q_1,q_2,q_3,q)| \lesssim \frac{1}{2}GI_L$. These processes may be termed "random Bragg" scattering processes unlike the automatic Bragg processes allowed for in the mode approximation.

Calculating the total intensity of the serpentine distortions averaged over the ensemble of fluctuations of the pump amplitudes $C(q)$, we obtain

$$I_n = \Sigma\langle |F_1(q,z)|^2\rangle = G^2 I_L^3 \; A^2 e^{2\mu_1 z} f_2/2k^2; \qquad (15a)$$

$$f_2 = \iiint d\theta_1 d\theta_2 d\theta_3 j(\theta_1)\, j(\theta_2)\, j(\theta_3)/[(GI_L/2k)^2+(\theta_1-\theta_2)^2(\theta_1-\theta_3)^2]. \qquad (15b)$$

Here, we have $\theta = q/k$; $j(\theta)$ is the normalized angular distribution function of the fluctuation components of the pump field $\int_{-\pi}^{\pi} j(\theta)d\theta = 1$; $j(\theta) = (kl/2\pi I_L)T(k\theta)$. Introducing the normalized angular distribution function of the noise field $j_n(\theta,z) = kl/2\pi I_n(z)\langle|F_1(q,z)|^2\rangle$, we obtain using Eqs. (14) and (15)

$$j_n(\theta) = f_1(\theta)/f_2; \qquad (16a)$$

$$f_1(\theta) = \iint d\theta_1 d\theta_2 j(\theta_1)\; j(\theta_2)\; j(\theta_1-\theta_2-\theta)[(GI_L/2k)^2+(\theta_1-\theta_2)^2(\theta+\theta_2)^2]. \qquad (16b)$$

Substituting the specific form of the function $j(\theta)$ into the expressions for f_2 and $f_1(\theta)$ and calculating the corresponding integrals, using Eqs. (15) and (16), we can find the total intensity and angular distribution of the serpentine noise for various angular distributions $j(\theta)$ of the pump field. The quantitative characteristic of the distortions R is the ratio of their intensity I_n to the intensity of that part of the scattered field which accurately reverses the pump wavefront $I_{rev} = |A(z)|^2 e^{2\mu_1 z}$. Allowing for Eq. (15), we obtain $R = G^2 I_L^2 f_2/2k^2$.

We shall now analyze in greater detail the integrals f_2 and $f_1(\theta)$ [Eqs. (15b) and (16b), respectively]. We shall discuss calculations of the triple integral f_2. It is initially more convenient to integrate with respect to θ_3. In this case, the function in the integrand consists of two cofactors: $j(\theta_3)$ and $[(GI_L/2k)^2+(\theta_1-\theta_2)^2(\theta_1-\theta_3)^2]^{-1}$. The characteristic width of the second cofactor as a function of θ_3 for constant θ_1 and θ_2 is $(GI_L/k)/|\theta_1-\theta_2|$. We shall compare this with the width of the pump angular spectrum $\Delta\theta_L$, i.e., with the width of the function $j(\theta_3)$.

We shall *first* analyze the limits of integration with respect to θ_1 and θ_2, where $\Delta\theta_L \gtrsim GI_L/k|\theta_1-\theta_2|$, i.e., $|\theta_1-\theta_2| \gtrsim GI_L/k\Delta\theta_L$. For these values of θ_1 and θ_2 the factor $[(GI_L/2k)^2+(\theta_1-\theta_2)^2(\theta_1-\theta_3)^2]^{-1}$ in the integral with respect to θ_3 is an extremely narrow function compared with the angular distribution function of the pump field $j(\theta_3)$ and, when integrating, this may be replaced by a delta function with the appropriate normalization:

$$\left[\left(\frac{GI_L}{2k}\right)^2+(\theta_1-\theta_2)^2(\theta_1-\theta_3)^2\right]^{-1} \to \pi\frac{2k}{GI_L}\delta[(\theta_1-\theta_2)(\theta_1-\theta_3)]. \qquad (17)$$

We shall now analyze the range of variation of θ_1 and θ_2 in which $|\theta_1-\theta_2| \lesssim GI_L/k\Delta\theta_L$, when the width of the function $j(\theta_3)$ is small compared with the range of variation with respect to θ_3 of the factor $[(GI_L/2k)^2+(\theta_1-\theta_2)^2(\theta_1-\theta_3)^2]^{-1}$. In this case, $j(\theta_3)$ may be replaced by $\delta(\theta_3)$ with a coefficient of 1 (as a result of the normalizing condition and also assuming that the central direction of propagation of the pump beam is parallel to the Z axis). As a result, we obtain the following expression for f_2:

$$f_2 \approx \frac{2\pi k}{GI_L} \iint_{|\theta_1-\theta_2|\gtrsim GI_L/k\Delta\theta_L} d\theta_1 d\theta_2 j^2(\theta_1)\, j(\theta_2)/|\theta_1-\theta_2| + \iint_{|\theta_1-\theta_2|\lesssim GI_L/k\Delta\theta_L} d\theta_1 d\theta_2 j(\theta_1)\, j(\theta_2)\left[\left(\frac{GI_L}{2k}\right)^2+(\theta_1-\theta_2)^2\theta_1^2\right]^{-1}. \qquad (18)$$

We shall analyze in greater detail the second term in Eq. (18). Comparing the ranges of variation with respect to θ_2 of the functions $j(\theta_2)$ and $[(GI_L/2k)^2+(\theta_1-\theta_2)^2\theta_1^2]^{-1}$ and also allowing for the condition $|\theta_1-\theta_2| \lesssim GI_L/k\Delta\theta_L \ll \Delta\theta_L$ (we shall assume that $GI_L/k\Delta\theta_L^2 \ll 1$, since otherwise, as will become apparent subsequently, perturbation theory is not valid), it is readily established that the second term in Eq. (18) may be replaced by $4k/GI_L\Delta\theta_L \int d\theta_1 j^2(\theta_1)$, i.e.,

$$f_2 = \frac{2k}{GI_L}\left\{\pi \iint_{|\theta_1-\theta_2|\gtrsim GI_L/k\Delta\theta_L} d\theta_1 d\theta_2 j^2(\theta_1)\, j(\theta_2)\frac{1}{|\theta_1-\theta_2|} + \frac{2}{\Delta\theta_L}\int d\theta_1 j^2(\theta_1)\right\}. \qquad (19)$$

We note some characteristic features in the one-dimensional case under study compared with the two-dimensional case (see Refs. 7 and 12). In both cases, the main contribution to the serpentine distortions is made by *random Bragg scattering processes* with $\boldsymbol{\Gamma} \sim 0$. When integrating with respect to $\boldsymbol{\theta}_3$ in the two-dimensional problem, the substitution

$$\left[\left(\frac{GI_L}{2k}\right)^2+\frac{\Gamma^2}{k^2}\right]^{-1} \to \pi\frac{2k}{GI_L}\delta\left(\frac{\Gamma}{k}\right), \qquad (20)$$

where

$$\Gamma/k = (\boldsymbol{\theta}_1-\boldsymbol{\theta}_2)(\boldsymbol{\theta}_1-\boldsymbol{\theta}_3),$$

is only valid for moderately high values $|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|$. Nevertheless, it may be assumed that, in the two-dimensional case, the substitution (20) is valid over the whole range of variation of the two-dimensional vectors $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ since the condition $\boldsymbol{\Gamma}\sim 0$ merely implies that $\boldsymbol{\theta}_1-\boldsymbol{\theta}_2 \perp \boldsymbol{\theta}_1-\boldsymbol{\theta}_3$. The range where $\boldsymbol{\theta}_1-\boldsymbol{\theta}_2 \sim 0$, makes a negligible contribution to the appropriate integral with respect to the two-dimensional vectors $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

In contrast, in the one-dimensional problem studied in the present investigation, the condition $\Gamma \sim 0$ necessarily implies the condition $\theta_1-\theta_2 \sim 0$ or $\theta_1-\theta_3 \sim 0$ (the variables θ_2 and θ_3 are converted from one to the other as a result of the simple permutation of the indices $2 \leftrightarrow 3$ in the integrand). In the one-dimensional case, the contribution made to the integral by the range of variation of θ_1 and θ_2 where $\theta_1-\theta_2 \sim 0$ cannot be neglected and if this range is not analyzed separately, we obtain a divergent integral. Thus, in the one-dimensional case, the substitution (17) using a delta function cannot be considered to be valid throughout the limits of integration. Thus, two terms with different limits of integration appear in Eq. (18).

Similarly, analyzing the integral $f_1(\theta)$ [Eq. (16b)] and comparing the characteristic ranges of variation with respect to θ_1 and θ_2 of the various cofactors in the function in the integrand, it is readily established that the following expression is valid:

$$f_1(\theta) \approx j(-\theta)\frac{2k}{GI_L}\left\{\pi \int\limits_{|\theta_2+\theta| \gtrsim GI_L/k\Delta\theta_L} j^2(\theta_2)\frac{d\theta_2}{|\theta_2+\theta|} + \frac{1}{(\Delta\theta_L)^2}\right\}. \quad (21)$$

A distinguishing feature of Eq. (21) is that the intensity of the distortions $f_1(\theta)$ is only nonzero for those values of θ which (to within the trivial substitution $\theta \to -\theta$ since stimulated backscattering is being analyzed) are found within the pump field. In contrast, for $j(\theta)$ in the form of a "circular stand" of radius θ_0 in the two-dimensional problem, the serpentine distortions are nonzero within a circle of radius $\theta_0\sqrt{2}$ in angular space (although within this circle the distortions are distributed nonuniformly with a higher intensity at the center).

These two conclusions, applicable to one- and two-dimensional problems respectively, are readily obtained by analyzing the position, on a Ewald sphere, of the wave vectors which approximately satisfy the Bragg condition for scattering by the gratings $\delta\varepsilon \sim C(q_1)C^*(q_2)$.

We shall give values of the relative intensity R and angular distribution $j_n(\theta)$ of the serpentine noise calculated using Eqs. (15)–(17), (19), and (21) for various forms of the angular distribution of the pump field $j(\theta)$. The expressions given below are valid assuming that $|\ln(GI_L/k\Delta\theta_L^2)| \gtrsim 3$.

For a Gaussian angular distribution $j(\theta) = e^{-\theta^2/\theta_0^2}/\sqrt{\pi}\theta_0$, we obtain

$$R \approx \frac{GI_L}{k\theta_0^2}\frac{2}{\sqrt{3}}\ln\frac{k\theta_0^2}{GI_L}; \quad (22a)$$

$$j_n(\theta) \approx e^{-3\theta^2/\theta_0^2}\sqrt{3}\frac{1}{\theta_0\sqrt{\pi}}. \quad (22b)$$

It can be seen from Eq. (22b) that the angular distribution function of the noise is narrower than the angular distribution function of the pump field.

For a parabolic distribution

$$j(\theta) = \begin{cases} \frac{3}{4\theta_0}\left(1 - \frac{\theta^2}{\theta_0^2}\right) & \text{for } |\theta| \leqslant \theta_0; \\ 0 & \text{for } |\theta| > \theta_0 \end{cases}$$

calculations give

$$R = \frac{27}{35}\pi\frac{GI_L}{k\theta_0^2}\ln\left(\frac{2k\theta_0^2}{GI_L}\right); \quad (23a)$$

$$j_n(\theta) = \text{const}\frac{1}{\theta_0}\left(1 - \frac{\theta^2}{\theta_0^2}\right)^3\left[1 + \frac{\ln(1-\theta^2/\theta_0^2)}{\ln(2k\theta_0^2/6I_L)}\right] \text{ for } \theta \leqslant \theta_0. \quad (23b)$$

For a rectangular angular distribution of the pump field $j(\theta)$

$$j(\theta) = \begin{cases} 1/\theta_0 & \text{for } |\theta| \leqslant \theta_0/2; \\ 0 & \text{for } |\theta| > \theta_0/2 \end{cases}$$

we have

$$R = 2\pi\frac{GI_L}{k\theta_0^2}\ln\frac{k\theta_0^2}{GI_L}; \quad (24a)$$

$$j_n(\theta) = j(-\theta)\left[1 + \frac{\ln(1/4 - \theta^2/\theta_0^2)}{2\ln(k\theta_0^2/GI_L)}\right]. \quad (24b)$$

For $\theta/\theta_0 \ll 1$, we have $j_n(\theta) \approx j(-\theta)$; for $\theta/\theta_0 \approx \frac{1}{2}$, we obtain $j_n(\theta) \approx \frac{1}{2}j(-\theta)$ [we note that all the results given above, Eqs. (22)–(24), are valid for $\ln(k\theta_0^2/GI_L) \gtrsim 3$].

If we have $k\theta_0^2/GI_L \lesssim 10$, the integrals (15b) and (16b) should be calculated numerically. Nevertheless, perturbation theory is still valid for calculating the serpentine noise.

5. CONCLUSIONS

Thus, calculations have been made of serpentine distortions in the problem of WFR in stimulated scattering of a pump beam with one-dimensional transverse inhomogeneity. The results differ significantly from the two-dimensional case and are of interest for WFR waves propagating over the surface (such as surface plasmons and polaritons) and also for WFR waves in two-dimensional waveguiding films.

The authors are grateful to V. V. Shkunov for useful discussions.

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Translated by R. M. Durham