

# ON THE CONTINUOUS ABSORPTION COEFFICIENT OF THE NEGATIVE HYDROGEN ION. III

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## ABSTRACT

In this paper the contribution to the continuous absorption coefficient of the negative hydrogen ion by the free-free transitions is evaluated in terms of the matrix elements of the acceleration in the Hartree field of a hydrogen atom. The new coefficients are larger than the earlier determinations by factors exceeding 10 over the entire range of wave lengths of astrophysical interest.

Tables of the continuous absorption coefficient of  $H^-$ , including both the bound-free and the free-free transitions for various temperatures ( $2520^\circ \leq T \leq 10,080^\circ$ ) and wave lengths ( $\lambda > 4000 \text{ \AA}$ ) are also provided. It is further shown that the new coefficients are sufficient to account for the solar continuous spectrum from  $\lambda 4000 \text{ \AA}$  to  $\lambda 25,000 \text{ \AA}$ .

1. *Introduction.*—In two earlier papers<sup>1</sup> the continuous absorption coefficient of the negative hydrogen ion has been considered, and its cross-sections for the radiative processes leading to its ionization have been determined with some degree of definitiveness. And recent astrophysical discussions<sup>2</sup> relating to the continuous spectrum of the sun have shown that the cross-sections which were derived for these “bound-free” transitions of  $H^-$  are adequate to account for the continuous absorption in the solar atmosphere between  $\lambda 4000 \text{ \AA}$  and  $\lambda 10,000 \text{ \AA}$ . Beyond  $\lambda 10,000 \text{ \AA}$ , however, there appears to be an additional source of absorption, which it would be natural to suppose is due to the radiative transitions of free electrons in the field of neutral hydrogen atoms.<sup>3</sup> But the existing evaluations<sup>4</sup> of these “free-free” transitions make them insufficient to account for the observed amount of absorption beyond  $\lambda 10,000 \text{ \AA}$  by factors exceeding 10. Indeed, on the strength of this discrepancy, the existence of a hitherto “unknown source of absorption” has been concluded.<sup>5</sup> However, as similar conclusions relating to the bound-free transitions have proved premature in the past, we have examined the earlier evaluations of the free-free transitions of  $H^-$  and have found, as we shall presently explain, that there are ample grounds for mistrusting them, even as to giving the correct orders of magnitude. We have, accordingly, made some further calculations to obtain estimates of the free-free transitions, on which at least some reliance could be placed. It is the object of this paper to present the results of such calculations and to show that these newly determined cross-sections for the free-free transitions, together with the cross-sections for the bound-free transitions given in Paper II, are sufficient to account for the continuous absorption in the solar and in the stellar atmospheres of neighboring spectral types in a manner which dispels any remaining belief in an “unknown source of absorption.”

2. *The inadequacy of the Born approximation for the evaluation of the free-free transitions of  $H^-$ .*—The essential approximation which underlies all the earlier evaluations<sup>4</sup> is that of Born’s for describing the motion of an electron in the field of a hydrogen atom.

<sup>1</sup> S. Chandrasekhar, *Ap. J.*, **102**, 223, 395, 1945. These papers will be referred to as Papers I and II, respectively.

<sup>2</sup> G. Münch, *Ap. J.*, **102**, 385, 1945; D. Chalonge and V. Kourganoff, *Ann. d’ap.* (in press).

<sup>3</sup> The possible astrophysical importance of this process was first suggested by A. Pannekoek, *M.N.*, **91**, 162, 1931.

<sup>4</sup> L. Nedelsky, *Phys. Rev.*, **42**, 641, 1932; D. H. Menzel and C. L. Pekeris, *M.N.*, **96**, 77, 1935; J. A. Wheeler and R. Wildt, *Ap. J.*, **95**, 281, 1942.

<sup>5</sup> See particularly the discussion of Chalonge and Kourganoff (*op. cit.*).

This is apparent, for example, from the agreement of the calculations of Menzel and Pekeris with those of Wheeler and Wildt, in which the Born approximation is explicitly made. However, on consideration it appears that for the range of energies which occur in stellar atmospheres the Born approximation for incident  $s$ -electrons must be a very bad one; for, as the mean energy of the electrons in a Maxwellian distribution at temperature  $T$  is

$$\frac{1}{2} \bar{k}^2 = 0.0243 \left( \frac{T}{5040} \right) \quad (1)$$

when expressed in atomic units, it is evident that it is only electrons with  $k^2 < 0.05$  that will principally contribute to the continuous absorption. And for  $s$ -electrons with as small energies as these, the Born approximation must fail. This is indeed well known from the work of P. M. Morse and W. P. Allis.<sup>6</sup> But it may be useful to illustrate this failure in a manner which will emphasize the magnitude of the errors to which the Born approximation may lead in the cross-sections for radiative transitions. For this purpose, we have compared in Figure 1 the radial  $s$ -waves

$$\chi_0 (\text{Born}) = \sin k r \quad (2)$$

on the Born approximation with the properly normalized  $s$ -spherical waves in the Hartree field of a hydrogen atom, which the authors have recently tabulated.<sup>7</sup> It is seen that for  $r < 2$  the Hartree waves have amplitudes which are larger than  $\chi_0$  (Born) by factors which, on the average, exceed 3. However, similar comparisons between the  $p$ -waves

$$\chi_1 (\text{Born}) = \frac{\sin k r}{k r} - \cos k r \quad (3)$$

on the Born approximation and the corresponding Hartree waves show that equation (3) provides a satisfactory approximation for the energies in which we are interested. Remembering that, in the evaluation of the matrix elements of the acceleration, the wave functions for  $r < 2$  are all that matters (cf. Fig. 2, in which we have plotted the acceleration  $\ddot{r}$  in the static field of a hydrogen atom) and, further, that the contribution to the absorption coefficient arising from the  $s \rightarrow p$  and  $p \rightarrow s$  transitions must far outweigh all the others, it is apparent that in the Hartree approximation we shall obtain cross-sections which will be larger by ten or more times the values obtained with the Born approximation. In other words, we may expect a correct evaluation of the free-free transitions of  $H^-$  to bring about an agreement between physical theory and the demands of astrophysical data without the need of postulating a still unknown source of continuous absorption.

3. *Formula for the evaluation of the cross-sections for the free-free transitions of  $H^-$  on the Hartree approximation.*—Our remarks in the preceding section have shown the inadequacy of the Born approximation for the evaluation of the free-free transitions of  $H^-$ . The use of the Hartree approximation suggests itself as the next best, though the effects of exchange and polarization may very well be appreciable for the very slow  $s$ -electrons in which we are primarily interested. But it may be hoped that these latter effects will not, at any rate, affect the orders of magnitude of the derived quantities! In any case, to improve on the Hartree approximation would require an amount of numerical work which will be several fold; and the task is immense even as it is.<sup>8</sup> These considerations, to-

<sup>6</sup> *Phys. Rev.*, **44**, 269, 1933.

<sup>7</sup> S. Chandrasekhar and F. H. Breen, *Ap. J.*, **103**, 41; 1946.

<sup>8</sup> For example, the present work has required the numerical integration of 63 radial functions and the evaluation of 523 infinite integrals, not to mention the computation of numerous auxiliary functions and tables. (All this work was done with a Marchant.)

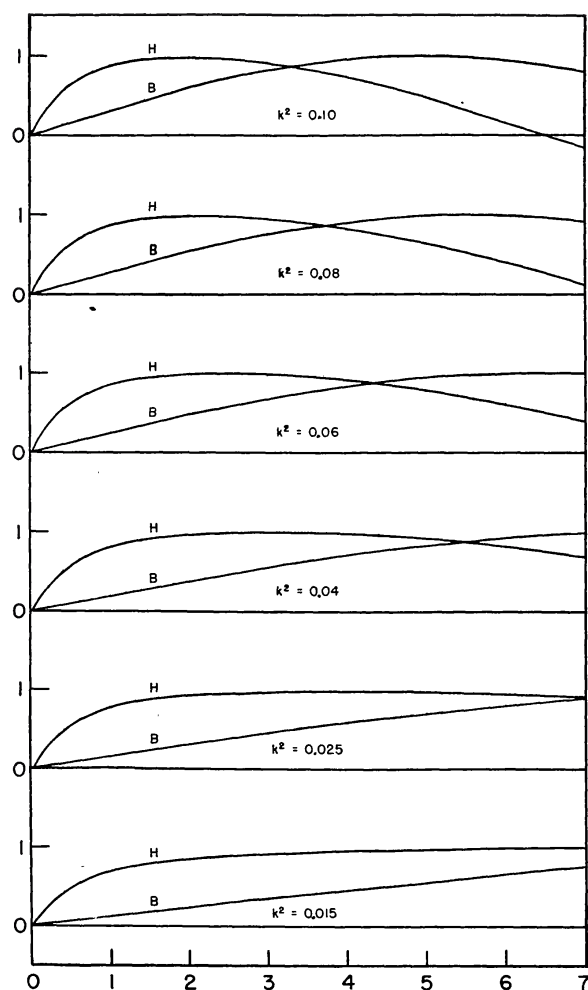


FIG. 1.—A comparison of the radial wave functions of  $s$ -electrons in the field of a hydrogen atom on the Born (B) and the Hartree (H) approximations for various energies of astrophysical interest. (The abscissa measures the distance from the center in units of the Bohr radius).

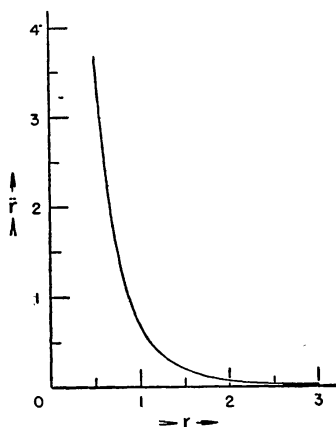


FIG. 2.—The acceleration  $\ddot{r} = [r^{-2} + 2(1 + r^{-1})]e^{-2r}$  of an electron in the Hartree field of a hydrogen atom.

gether with the urgency of the astrophysical needs, have prompted us to undertake in some detail the evaluation of the cross-sections for the free-free transitions of  $H^-$ , using the  $s$ - and the  $p$ -spherical waves in the Hartree field of the hydrogen atom which we have tabulated in an earlier paper.<sup>7</sup>

Now, in the Hartree approximation a wave function representing a plane wave at infinity is given by

$$\Psi = \sum_{l=0}^{\infty} \frac{i^l (2l+1)}{k r} P_l(\mu) \chi_l(r; k^2), \quad (4)$$

where the radial functions  $\chi_l(r; k)$  are solutions of the equation

$$\frac{d^2 \chi_l}{d r^2} + \left\{ k^2 - \frac{l(l+1)}{r^2} + 2 \left( 1 + \frac{1}{r} \right) e^{-2r} \right\} \chi_l = 0, \quad (5)$$

which have unit amplitude at infinity. (In the foregoing equations we have adopted the atomic system of units). For wave functions of the form (4) the standard formula in the quantum theory, which gives the cross-section for a transition in which a free electron with an initial momentum  $k_0$  (in atomic units) becomes an electron with a momentum  $k_1$  by the absorption of a quantum of appropriate energy from an incident beam of unit specific intensity, can be written in the form<sup>9</sup>

$$a(k_0^2; \Delta k^2) = \frac{256\pi^2}{3} \left( \frac{2\pi e^2}{hc} \right) \left( \frac{\hbar^2}{4\pi^2 m e^2} \right)^5 \frac{1}{k_0^2 k_1 (\Delta k^2)^3} \times \sum_{l=1}^{\infty} l \left\{ |(l, k_0^2 | \dot{r} | l-1, k_1^2)|^2 + |(l-1, k_0^2 | \dot{r} | l, k_1^2)|^2 \right\} \text{cm}^5, \quad (6)$$

where the various matrix elements  $(l, k_1^2 | \dot{r} | l-1, k_2^2)$  have to be evaluated in atomic units and

$$\Delta k^2 = k_1^2 - k_0^2. \quad (7)$$

Equation (6) gives the cross-section for a single hydrogen atom in the ground state for the transition in question when there is one electron with momentum  $k_0$  in unit volume. Moreover, the wave length,  $\lambda$ , of the radiation for which formula (6) gives the atomic absorption coefficient is

$$\lambda(\Delta k^2) = \frac{911.3}{\Delta k^2} A. \quad (8)$$

In the Hartree field of a hydrogen atom the acceleration is given by

$$\dot{r} = \left[ \frac{1}{r^2} + 2 \left( 1 + \frac{1}{r} \right) \right] e^{-2r}, \quad (9)$$

and the matrix elements which have to be evaluated are of the type

$$(l, k_0^2 | \dot{r} | l-1, k_1^2) = \int_0^{\infty} \chi_l(r; k_0^2) \left[ \frac{1}{r^2} + 2 \left( 1 + \frac{1}{r} \right) \right] e^{-2r} \chi_{l-1}(r; k_1^2) dr. \quad (10)$$

To obtain the absorption coefficients appropriate for an assembly in which we have a Maxwellian distribution of electrons corresponding to a temperature  $T$ , we must average  $a(k_0^2; \Delta k^2)$  over all initial  $k_0^2$  (and for a fixed  $\Delta k^2$ ) with the weight function

$$197.8 \theta^{3/2} k_0 e^{-31.32 \theta k_0^2} \quad (11)$$

<sup>9</sup> Cf. J. A. Gaunt, *Phil. Trans. R. Soc., London*, A, 229, 163, 1930; see particularly formula (5.24) on p. 194; also Wheeler and Wildt, *op. cit.*, eq. (6) on p. 284.

where

$$\theta = \frac{5040}{T}. \quad (12)$$

In astrophysical applications it is convenient to express the free-free atomic absorption coefficients as per unit electron pressure. If we denote by  $\kappa(\Delta k^2)$  the corresponding atomic absorption coefficient after averaging over a Maxwell distribution of initial velocities, we find that we can write our basic formula in the form

$$\kappa(\Delta k^2) = \frac{7.251 \times 10^{-29} \theta^{5/2}}{(\Delta k^2)^3} \int_0^\infty d(k_0^2) \frac{f(k_0^2)}{k_0^2 k_1} \left. \begin{aligned} & \times \sum_{l=1}^{\infty} l \{ |(l, k_0^2 | \check{r} | l-1, k_1^2)|^2 + |(l-1, k_0^2 | \check{r} | l, k_1^2)|^2 \} \frac{\text{cm}^4}{\text{dyne}}, \end{aligned} \right\} (13)$$

where

$$f(k_0^2) = 100 k_0 e^{-31.32 \theta k_0^2}. \quad (14)$$

4. *Details of the evaluation: tables of the necessary matrix elements.*—The problem of numerically evaluating absorption coefficients for free-free transitions is a specially troublesome one, since for each wave length the coefficients must be computed for a range of initial velocities sufficient to allow for the averaging over Maxwell distributions for various temperatures. If the matrix elements for all the necessary values of  $k_0^2$  and  $\Delta k^2$  must be individually evaluated, then the number of wave functions which would be needed will be so many as to make the problem an impracticable one on this score alone. It may, therefore, be useful to place on record the details of our procedure in this instance.

First, we may observe that the infinite series in equation (13) is so rapidly convergent that we may ignore all terms except the first one. This may be verified in the following manner:

We have already indicated in § 2 that for  $l \geq 1$  we can use the Born approximation without any serious error. We may therefore write

$$\chi_l(r; k^2) = \left( \frac{\pi k r}{2} \right)^{1/2} J_{l+\frac{1}{2}}(k r). \quad (15)$$

Accordingly, for  $l \geq 2$ ,

$$(l, k_1^2 | \check{r} | l-1, k_2^2) = \frac{1}{2} \pi (k_1 k_2)^{1/2} \int_0^\infty r J_{l+\frac{1}{2}}(k_1 r) J_{l-\frac{1}{2}}(k_2 r) \times \left[ \frac{1}{r^2} + 2 \left( 1 + \frac{1}{r} \right) \right] e^{-2r} dr. \quad (16)$$

Writing equation (16) in the form

$$(l, k_1^2 | \check{r} | l-1, k_2^2) = -\frac{1}{2} \pi k_1 k_2 \int_0^\infty r J_{l+\frac{1}{2}}(k_1 r) J_{l-\frac{1}{2}}(k_2 r) \times \frac{d}{dr} \left[ \left( 1 + \frac{1}{r} \right) e^{-2r} \right] dr \quad (17)$$

and integrating by parts, we find after some further reductions that

$$(l, k_1^2 | \check{r} | l-1, k_2^2) = \frac{1}{2} \pi (k_1 k_2)^{1/2} \int_0^\infty (1+r) e^{-2r} [k_1 J_{l-\frac{1}{2}}(k_1 r) J_{l-\frac{1}{2}}(k_2 r) - k_2 J_{l+\frac{1}{2}}(k_1 r) J_{l+\frac{1}{2}}(k_2 r)] dr. \quad (18)$$

The integrals over the Bessel functions which occur in the foregoing equation can be expressed in terms of the Legendre functions of the second kind,  $Q_l(x)$ , and their derivatives,  $Q'_l(x)$ .<sup>10</sup> We find

$$(l, k_1^2 | \check{r} | l-1, k_2^2) = \frac{1}{2} k_1 Q_{l-1}(x) - \frac{1}{2} k_2 Q_l(x) - \frac{1}{k_2} Q'_{l-1}(x) + \frac{1}{k_1} Q'_l(x), \quad (19)$$

where the argument for the Legendre functions is

$$x = \frac{k_1^2 + k_2^2 + 4}{2k_1k_2}. \quad (20)$$

Using equation (19), we can readily verify the fact that all transitions in which the incident electron is characterized by an  $l$  greater than 1 contribute less than a fraction of 1 per cent to the sum in equation (13). It is therefore sufficient to consider only the matrix elements  $(0, k_0^2 | \check{r} | 1, k_1^2)$  and  $(1, k_0^2 | \check{r} | 0, k_1^2)$ . In evaluating these matrix elements it was found convenient to distinguish three cases, discussed below.

i)  $k_0^2 < 0.015$  and  $k_1^2 > 0.015$ .—It has been shown<sup>7</sup> that, when  $k^2 < 0.015$ , the radial functions  $\chi_0(r; k^2)$  and  $\chi_1(r; k^2)$  can be expressed with sufficient accuracy in the form

$$\chi_0(r; k^2) = \frac{1}{A_0(k)} [X_0(r) - k^2 Y_0(r)] \quad (21)$$

and

$$\chi_1(r; k^2) = \frac{1}{A_1(k)} [X_1(r) - k^2 Y_1(r)], \quad (22)$$

where  $A_0(k)$  and  $A_1(k)$  are the factors which normalize the functions in brackets to unit amplitude at infinity and  $X_0, Y_0, X_1,$  and  $Y_1$  are certain functions which have been tabulated. With this representation of the wave functions, the matrix element  $(0, k_0^2 | \check{r} | 1, k_1^2)$  can be expressed in the form

$$(0, k_0^2 | \check{r} | 1, k_1^2) = \frac{1}{A_0(k_0) A_1(k_1)} \left[ \int_0^\infty X_0 X_1 \check{r} dr - k_0^2 \int_0^\infty Y_0 X_1 \check{r} dr - k_1^2 \int_0^\infty X_0 Y_1 \check{r} dr + k_0^2 k_1^2 \int_0^\infty Y_0 Y_1 \check{r} dr \right]. \quad (23)$$

The corresponding expression for  $(1, k_0^2 | \check{r} | 0, k_1^2)$  can be obtained from equation (23) by simply interchanging  $k_0$  and  $k_1$ .

Thus, when  $k_0^2$  and  $k_1^2$  are both less than 0.015, the necessary matrix elements can all be evaluated in terms of four definite integrals which are listed in Table 1.

TABLE 1  
INTEGRALS FOR COMPUTING THE MATRIX ELEMENTS WHEN  $k_0^2$  AND  $k_1^2$  ARE BOTH LESS THAN 0.015

$\int_0^\infty X_0 X_1 \check{r} dr \dots \dots \dots$	0.40044	$\int_0^\infty Y_0 X_1 \check{r} dr \dots \dots \dots$	0.22641
$\int_0^\infty Y_0 Y_1 \check{r} dr \dots \dots \dots$	0.16669	$\int_0^\infty X_0 Y_1 \check{r} dr \dots \dots \dots$	0.09076

ii)  $k_0^2 < 0.015$  and  $k_1^2 \geq 0.015$ .—In this case we can use the representation (20) or (21) only for the radial function describing the incident electron, and the required matrix elements can be expressed in the forms

$$(0, k_0^2 | \check{r} | 1, k_1^2) = \frac{1}{A_0(k_0)} \left[ \int_0^\infty X_0 \chi_1(r; k_1) \check{r} dr - k_0^2 \int_0^\infty Y_0 \chi_1(r; k_1) \check{r} dr \right] \quad (24)$$

<sup>10</sup> Cf. G. N. Watson, *Theory of Bessel Functions*, p. 389, Cambridge University Press, 1944.

and

$$(1, k_0^2 | \bar{r} | 0, k_1^2) = \frac{1}{A_1(k_0)} \left[ \int_0^\infty X_{1\chi_0}(r; k_1) \bar{r} dr - k_0^2 \int_0^\infty Y_{1\chi_0}(r; k_1) \bar{r} dr \right]. \quad (25)$$

The integrals which occur on the right-hand side of equations (24) and (25) have been evaluated for various values of  $k_1^2 \geq 0.015$  and are listed in Table 2; for these values of  $k_1^2$  and for all  $k_0^2 < 0.015$ , the required matrix elements can be found without further numerical integrations according to equations (24) and (25).

iii)  $k_0^2 \geq 0.015$  and  $k_1^2 \geq 0.015$ .—In this case there is no short cut to the evaluation of the matrix elements, and the integrals must all be evaluated individually. However, for this purpose we have only the radial functions tabulated in the paper by Chandrasekhar and Breen.

Our procedure, then, for determining the various matrix elements with assigned  $k_0^2$  and  $\Delta k^2$  was as follows:

TABLE 2  
INTEGRALS FOR COMPUTING THE MATRIX ELEMENTS WHEN  $k_0^2 < 0.015$

$k_1^2$	$\int_0^\infty X_{1\chi_0}(r; k_1^2) \bar{r} dr$	$\int_0^\infty Y_{1\chi_0}(r; k_1^2) \bar{r} dr$	$\int_0^\infty X_{0\chi_1}(r; k_1^2) \bar{r} dr$	$\int_0^\infty Y_{0\chi_1}(r; k_1^2) \bar{r} dr$
0.015	0.64185	0.14271	0.0033763	0.0018942
.030	.71329	.15557	.0067421	.0037565
.045	.74163	.15863	.0101066	.0055872
.060	.75618	.15858	.013458	.0073854
.080	.76622	.15650	.017920	.0097336
.100	.77079	.15328	.022366	.012025
.125	.77307	.14861	.027895	.014809
.150	.77310	.14360	.033392	.017504
.175	.77164	.13840	.038851	.020113
.200	.76954	.13170	.044272	.022636
0.250	0.76357	0.12293	0.054972	0.027421

First, the matrix elements listed in Table 3 were computed with the known radial functions. (This table also includes the matrix elements computed according to eqs. [23], [24], or [25]). The values given in Table 3 show sufficiently smooth differences with both arguments to enable satisfactory interpolation for any intermediate value. Using these directly computed values, we next found, by interpolation, all the matrix elements which were needed for the evaluation of  $\kappa(\Delta k^2)$  according to equation (13); the matrix elements found in this manner are given in Table 4, which was then the basis for our further calculations.

5. *The continuous absorption coefficient of  $H^-$ .*—Even with as complete a table of matrix elements as Table 4, the evaluation of the continuous absorption coefficient for free-free transitions is a tiresome matter, since, for each wave length and temperature for which the coefficient is desired, a numerical integration must be performed. The coefficients tabulated in Table 5 were determined in this manner according to equation (13).

In Table 5 we have also entered the absorption coefficients due to the bound-free transitions for  $\lambda < 16,500$  Å. These latter coefficients were obtained by multiplying the cross-sections derived from those given in Paper II (Table 3) by the factor

$$\phi(\theta) = 4.158 \times 10^{-10} \theta^{5/2} e^{1.726\theta}, \quad (26)$$

which gives the number of  $H^-$  ions present per neutral hydrogen atom and unit electron pressure. The contributions to the continuous absorption coefficient of  $H^-$  by the

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TABLE 3  
 THE MATRIX ELEMENTS  $(0, k_0^2 | \bar{r} | 1, k^2)$  AND  $(1, k_0^2 | \bar{r} | 0, k_1^2)$   
 EVALUATED BY EXACT NUMERICAL INTEGRATIONS

$k^2$	$(0, 0.0025   \bar{r}   1, k^2)$	$(1, 0.0025   \bar{r}   0, k^2)$	$(0, 0.005   \bar{r}   1, k^2)$	$(1, 0.005   \bar{r}   0, k^2)$	$(0, 0.0075   \bar{r}   1, k^2)$	$(1, 0.0075   \bar{r}   0, k^2)$
0.0025	0.0005256	0.0005257				
.0050	.0010513	.0006778	0.001356	0.001356		
.0075	.001577	.0007681	.002033	.001536	0.002304	0.002304
.0100	.002103	.0008293	.002711	.001659	.003073	.002488
.0125	.002629	.0008738	.003390	.001747	.003842	.002621
.0150	.003153	.0009022	.004066	.001804	.004608	.002707
.0300	.006297	.001003	.008119	.002005	.009201	.003008
.0450	.009439	.001043	.01217	.002085	.01379	.003128
.060	.01257	.001063	.01621	.002126	.01837	.003189
.080	.01674	.001077	.02158	.002154	.02446	.003232
.100	.02089	.001084	.02694	.002167	.03053	.003251
.125	.02605	.001087	.03360	.002174	.03808	.003261
.150	.03119	.001087	.04022	.002174	.04558	.003261
.175	.03629	.001085	.04680	.002170	.05304	.003255
.200	.04135	.001082	.05333	.002164	.06044	.003246
0.250	0.05135	0.001074	0.06622	0.002147	0.07506	0.003221

$k^2$	$(0, 0.01   \bar{r}   1, k^2)$	$(1, 0.01   \bar{r}   0, k^2)$	$(0, 0.0125   \bar{r}   1, k^2)$	$(1, 0.0125   \bar{r}   0, k^2)$	$(0, 0.015   \bar{r}   1, k^2)$	$(1, 0.015   \bar{r}   0, k^2)$
0.010	0.003318	0.003318				
.0125	.004148	.003495				
.015	.004975	.003609	0.005241	0.004513	0.005412	0.005412
.030	.009935	.004011	.010467	.005015	.010808	.006015
.045	.01489	.004171	.01569	.005215	.01620	.006254
.060	.01983	.004253	.02090	.005317	.02158	.006377
.080	.02641	.004309	.02783	.005388	.02873	.006463
.100	.03296	.004335	.03473	.005421	.03586	.006502
.125	.04112	.004348	.04332	.005437	.04473	.006522
.150	.04922	.004349	.05186	.005438	.05355	.006523
.175	.05727	.004341	.06035	.005428	.06232	.006516
.200	.06527	.004329	.06877	.005414	.07102	.006494
0.250	0.08105	0.004296	0.08541	0.005372	0.08820	0.006445

$k^2$	$(0, 0.02   \bar{r}   1, k^2)$	$(1, 0.02   \bar{r}   0, k^2)$	$(0, 0.025   \bar{r}   1, k^2)$	$(1, 0.025   \bar{r}   0, k^2)$	$(0, 0.03   \bar{r}   1, k^2)$	$(1, 0.03   \bar{r}   0, k^2)$
0.02	0.007594	0.007593	0.007842	0.009471	0.008021	0.01135
.04	.01517	.008260	.01567	.010303	.01603	.01237
.06	.02270	.008506	.02345	.010609	.02398	.01274
.08	.03023	.008619	.03123	.01075	.03194	.01291
.10	.03773	.008672	.03898	.01082	.03987	.01299
.125	.04707	.008699	.04862	.01085	.04974	.01303
.150	.05635	.008701	.05821	.01085	.05955	.01303
.175	.06557	.008685	.06774	.01083	.06930	.01301
.200	.07473	.008663	.07721	.01081	.07899	.01298
0.250	0.09277	0.008597	0.09590	0.01072	0.09811	0.01288



TABLE 3—Continued

$k^2$	(0, 0.035   $\bar{r}$   1, $k^2$ )	(1, 0.035   $\bar{r}$   0, $k^2$ )	(0, 0.04   $\bar{r}$   1, $k^2$ )	(1, 0.04   $\bar{r}$   0, $k^2$ )	(0, 0.06   $\bar{r}$   1, $k^2$ )	(1, 0.06   $\bar{r}$   0, $k^2$ )
0.04	0.01630	0.01444	0.01650	0.01651		
.06	.02440	.01487	.02470	.01700	0.02544	0.02544
.08	.03249	.01507	.03290	.01723	.03388	.02578
.100	.04056	.01516	.04107	.01733	.04230	.02595
.125	.05060	.01521	.05123	.01739	.05279	.02603
.150	.06058	.01521	.06135	.01740	.06321	.02605
.175	.07050	.01519	.07139	.01737	.07357	.02600
.200	.08036	.01515	.08137	.01732	.08387	.02594
.250	0.09982	0.01504	0.10109	0.01720	.10422	.02576
0.350					0.1440	0.02528

$k^2$	(0, 0.08   $\bar{r}$   1, $k^2$ )	(1, 0.08   $\bar{r}$   0, $k^2$ )	(0, 0.1   $\bar{r}$   1, $k^2$ )	(1, 0.1   $\bar{r}$   0, $k^2$ )	(0, 0.125   $\bar{r}$   1, $k^2$ )	(1, 0.125   $\bar{r}$   0, $k^2$ )
0.080	0.03435	0.03435				
.100	.04289	.03457	0.04317	0.04317		
.125	.05352	.03469	.05388	.04332	0.05408	0.05408
.150	.06410	.03471	.06453	.04339	.06478	.05417
.175	.07462	.03466	.07513	.04330	.07543	.05406
.200	.08507	.03458	.08566	.04321	.08602	.05395
.250	.1057	.03434	.1065	.04292	.1070	.05361
.350	0.1461	0.03372	.1472	.04216	.1480	.05270
.450			.1866	.04130	0.1876	0.05165
.50			.2057	.04086		
.60			.2427	.03996		
.70			.2783	.03906		
0.80			0.3123	0.03818		

$k^2$	(0, 0.15   $\bar{r}$   1, $k^2$ )	(1, 0.15   $\bar{r}$   0, $k^2$ )	(0, 0.175   $\bar{r}$   1, $k^2$ )	(1, 0.175   $\bar{r}$   0, $k^2$ )	(0, 0.2   $\bar{r}$   1, $k^2$ )	(1, 0.2   $\bar{r}$   0, $k^2$ )
0.150	0.06485	0.06485	0.06478	0.07552	0.06467	0.08614
.175	.07552	.06478				
.200	.08614	.06467	.08607	.07533		
.250	.1071	.06428	.1071	.07490	.1070	.08547
.350	.1483	.06321	.1483	.07369	.1482	.08414
.450	.1881	.06198	0.1882	0.07230	.1882	.08259
.50	.2074	.06134			.2076	.08177
.60	.2451	.06005			.2455	.08010
.70	0.2812	0.05874			.2819	.07835
0.80					0.3169	0.07676

$k^2$	(0, 0.25   $\bar{r}$   1, $k^2$ )	(1, 0.25   $\bar{r}$   0, $k^2$ )	(0, 0.35   $\bar{r}$   1, $k^2$ )	(1, 0.35   $\bar{r}$   0, $k^2$ )	(0, 0.45   $\bar{r}$   1, $k^2$ )	(1, 0.45   $\bar{r}$   0, $k^2$ )
0.25	0.1064	0.1064				
.35	.1476	.1049	0.1457	0.1457		
.40	.1678	.1040	.1658	.1446		
.45	.1876	.1030	.1856	.1434	0.1829	0.1829
.50	.2071	.1021	.2050	.1422	.2023	.1815
.60	.2451	.1001	.2430	.1396	.2402	.1785
.70	.2817	.09809	.2798	.1370	.2769	.1754
0.80	0.3169	0.09603	0.3152	0.1344	0.3124	0.1722

TABLE 3—Continued

$k^2$	$(0, 0.5 \bar{\nu} 1, k^2)$	$(1, 0.5 \bar{\nu} 0, k^2)$	$(0, 0.6 \bar{\nu} 1, k^2)$	$(1, 0.6 \bar{\nu} 0, k^2)$
0.50.....	0.2008	0.2008	.....	.....
.60.....	.2385	.1976	0.2351	0.2351
.70.....	.2752	.1942	.2716	.2314
0.80.....	0.3107	0.1909	0.3070	0.2277

bound-free and free-free transitions can thus be brought together in the same system ( $\text{cm}^4/\text{dyne}$ ).

In Figure 3 we have illustrated the results of Table 5 for  $T = 6300^\circ$  ( $\theta = 0.8$ ). For comparison we have also included the earlier evaluations.

In Table 6 the coefficients given in Table 5 have been reduced by the stimulated emission factor

$$(1 - e^{-31.32\theta \Delta k^2}), \quad (27)$$

which is quite appreciable in the infrared. The values given in Table 6 are such as to enable satisfactory interpolation for most astrophysical applications.

Finally, in Table 7 we give the net absorption coefficient of  $H^-$ , including both the bound-free and the free-free transitions.

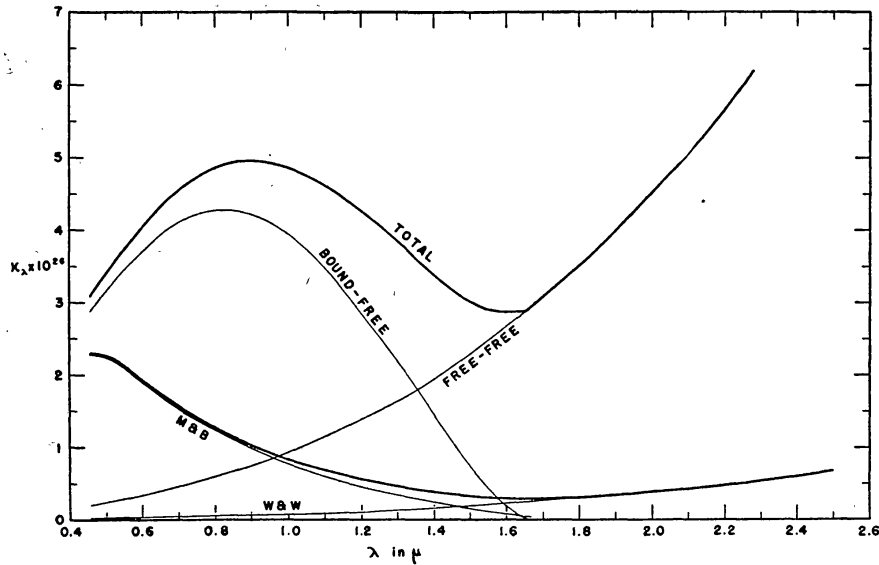


FIG. 3.—The continuous absorption coefficient of  $H^-$  per neutral hydrogen atom and per unit electron pressure for a temperature of  $6300^\circ$  K. The results of earlier determinations of the same quantities are included for comparison. The curve *M & B* is derived from the cross-sections of Massey and Bates for the bound-free transitions, and the curve *W & W* corresponds to the results of Wheeler and Wildt.

TABLE 4

THE MATRIX ELEMENTS  $(0, k_0^z | \pi | 1, k_0^z + \Delta k^z = k_1^z)$  AND  $(1, k_0^z | \pi | 0, k_0^z + \Delta k^z = k_1^z)$  FOR VARIOUS VALUES OF  $k_0^z$  AND  $\Delta k^z$  DERIVED BY INTERPOLATION FROM TABLE 3

$\Delta k^z$	$k_0^z = 0.0025$		$k_0^z = 0.0050$		$k_0^z = 0.0075$		$k_0^z = 0.0100$		$k_0^z = 0.0125$		$k_0^z = 0.0150$		$k_0^z = 0.020$	
	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$
0	0.000526	0.001356	0.001356	0.001356	0.002304	0.003318	0.004370	0.005412	0.006473	0.007594	0.008649	0.009720	0.010808	0.011900
0.005	0.000577	0.002711	0.001659	0.002711	0.003842	0.004975	0.006112	0.007209	0.008306	0.009406	0.010506	0.011606	0.012706	0.013806
0.010	0.000768	0.004069	0.001815	0.004069	0.005274	0.006428	0.007582	0.008736	0.009890	0.011044	0.012198	0.013352	0.014506	0.015660
0.015	0.000924	0.005417	0.001903	0.005417	0.006604	0.007881	0.009159	0.010437	0.011715	0.012993	0.014271	0.015549	0.016827	0.018105
0.020	0.004725	0.009961	0.001958	0.009961	0.008436	0.009935	0.011599	0.013263	0.014927	0.016591	0.018255	0.019919	0.021583	0.023247
0.025	0.005773	0.008119	0.002005	0.008119	0.009967	0.011559	0.013221	0.014875	0.016489	0.018103	0.019717	0.021331	0.022945	0.024559
0.030	0.006821	0.010112	0.002041	0.010112	0.011550	0.013244	0.014908	0.016562	0.018216	0.019870	0.021524	0.023178	0.024832	0.026486
0.040	0.008916	0.012117	0.002085	0.012117	0.013140	0.014830	0.016520	0.018200	0.019880	0.021560	0.023240	0.024920	0.026600	0.028280
0.050	0.011006	0.014875	0.01054	0.014875	0.01761	0.01983	0.02206	0.02432	0.02658	0.02884	0.03110	0.03336	0.03562	0.03788
0.060	0.01309	0.01065	0.02114	0.01065	0.03180	0.03208	0.03236	0.03264	0.03292	0.03320	0.03348	0.03376	0.03404	0.03432
0.070	0.01518	0.01073	0.02148	0.01073	0.03228	0.03241	0.03254	0.03267	0.03280	0.03293	0.03306	0.03319	0.03332	0.03345
0.080	0.01726	0.01078	0.02159	0.01078	0.03241	0.03254	0.03267	0.03280	0.03293	0.03306	0.03319	0.03332	0.03345	0.03358
0.090	0.01934	0.01082	0.02166	0.01082	0.03249	0.03262	0.03275	0.03288	0.03301	0.03314	0.03327	0.03340	0.03353	0.03366
0.100	0.02141	0.01084	0.02169	0.01084	0.03280	0.03280	0.03280	0.03280	0.03280	0.03280	0.03280	0.03280	0.03280	0.03280
0.120	0.02554	0.01087	0.03360	0.01087	0.03261	0.03261	0.03261	0.03261	0.03261	0.03261	0.03261	0.03261	0.03261	0.03261
0.140	0.02965	0.01087	0.03890	0.01087	0.04483	0.04483	0.04483	0.04483	0.04483	0.04483	0.04483	0.04483	0.04483	0.04483
0.160	0.03374	0.01086	0.02174	0.01086	0.05081	0.05081	0.05081	0.05081	0.05081	0.05081	0.05081	0.05081	0.05081	0.05081
0.180	0.03781	0.01084	0.02169	0.01084	0.05675	0.05675	0.05675	0.05675	0.05675	0.05675	0.05675	0.05675	0.05675	0.05675
0.200	0.04186	0.01082	0.02162	0.01082	0.06269	0.06269	0.06269	0.06269	0.06269	0.06269	0.06269	0.06269	0.06269	0.06269
0.225	0.04687	0.01078	0.02154	0.01078	0.06997	0.06997	0.06997	0.06997	0.06997	0.06997	0.06997	0.06997	0.06997	0.06997

$\Delta k^z$	$k_0^z = 0.025$		$k_0^z = 0.030$		$k_0^z = 0.035$		$k_0^z = 0.040$		$k_0^z = 0.045$		$k_0^z = 0.050$		$k_0^z = 0.055$	
	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$	$(0, k_0^z   \pi   1, k_1^z)$	$(1, k_0^z   \pi   0, k_1^z)$
0	0.009805	0.009805	0.01204	0.01204	0.01428	0.01651	0.01874	0.02097	0.02321	0.02545	0.02769	0.02993	0.03217	0.03441
0.005	0.01176	0.009989	0.01404	0.01221	0.01630	0.01856	0.02081	0.02306	0.02531	0.02756	0.02981	0.03206	0.03431	0.03656
0.010	0.01372	0.01017	0.01603	0.01237	0.01833	0.02060	0.02288	0.02515	0.02742	0.02969	0.03196	0.03423	0.03650	0.03877
0.015	0.01567	0.01030	0.01802	0.01251	0.02035	0.02265	0.02495	0.02724	0.02954	0.03184	0.03413	0.03643	0.03872	0.04102
0.020	0.01762	0.01042	0.02000	0.01268	0.02237	0.02470	0.02702	0.02932	0.03161	0.03391	0.03621	0.03851	0.04081	0.04311
0.025	0.01956	0.01050	0.02199	0.01268	0.02440	0.02675	0.02909	0.03141	0.03371	0.03601	0.03831	0.04061	0.04291	0.04521
0.030	0.02151	0.01050	0.02398	0.01274	0.02642	0.02880	0.03117	0.03350	0.03581	0.03811	0.04041	0.04271	0.04501	0.04731
0.040	0.02540	0.01066	0.02796	0.01284	0.03047	0.03284	0.03521	0.03758	0.03995	0.04232	0.04469	0.04706	0.04943	0.05180
0.050	0.02929	0.01073	0.03194	0.01291	0.03451	0.03689	0.03926	0.04163	0.04400	0.04637	0.04874	0.05111	0.05348	0.05585
0.060	0.03317	0.01077	0.03591	0.01295	0.03854	0.04092	0.04329	0.04566	0.04803	0.05040	0.05277	0.05514	0.05751	0.05988
0.070	0.03704	0.01080	0.03987	0.01299	0.04257	0.04500	0.04743	0.04986	0.05229	0.05472	0.05715	0.05958	0.06201	0.06444
0.080	0.04091	0.01083	0.04382	0.01301	0.04659	0.04914	0.05168	0.05423	0.05677	0.05932	0.06187	0.06442	0.06697	0.06952
0.090	0.04477	0.01084	0.04777	0.01302	0.05060	0.05326	0.05591	0.05856	0.06121	0.06386	0.06651	0.06916	0.07181	0.07446
0.100	0.04862	0.01085	0.05171	0.01303	0.05460	0.05731	0.06001	0.06266	0.06531	0.06796	0.07061	0.07326	0.07591	0.07856
0.120	0.05630	0.01085	0.05955	0.01303	0.06245	0.06521	0.06796	0.07071	0.07346	0.07621	0.07896	0.08171	0.08446	0.08721
0.140	0.06394	0.01084	0.06736	0.01302	0.07030	0.07313	0.07596	0.07879	0.08162	0.08445	0.08728	0.09011	0.09294	0.09577
0.160	0.07153	0.01082	0.07512	0.01299	0.07823	0.08113	0.08403	0.08693	0.08983	0.09273	0.09563	0.09853	0.10143	0.10433
0.180	0.07909	0.01080	0.08284	0.01296	0.08623	0.08923	0.09223	0.09523	0.09823	0.10123	0.10423	0.10723	0.11023	0.11323
0.200	0.08659	0.01077	0.09050	0.01292	0.09402	0.09754	0.10106	0.10458	0.10810	0.11162	0.11514	0.11866	0.12218	0.12570
0.225	0.09390	0.01073	0.10001	0.01287	0.10367	0.10733	0.11099	0.11465	0.11831	0.12197	0.12563	0.12929	0.13295	0.13661



TABLE 5\*  
THE CONTINUOUS ABSORPTION COEFFICIENT OF THE NEGATIVE HYDROGEN ION PER NEUTRAL HYDROGEN ATOM AND PER UNIT ELECTRON PRESSURE FOR VARIOUS TEMPERATURES ( $\theta = 5040/T$ ) AND WAVE LENGTHS

$\Delta k^2$	$\lambda A$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1.0$	$\theta = 1.2$	$\theta = 1.4$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2.0$
0.005	182300	1.86 (-30)	2.03 (-30)	2.17 (-30)	2.29 (-30)	2.40 (-30)	2.49 (-30)	2.66 (-30)	2.79 (-30)	2.91 (-30)	3.01 (-30)	3.09 (-30)
0.010	91130	1.55 (-23)	1.70 (-23)	1.84 (-23)	1.95 (-23)	2.06 (-23)	2.16 (-23)	2.34 (-23)	2.50 (-23)	2.64 (-23)	2.77 (-23)	2.89 (-23)
0.015	60750	2.02 (-24)	2.24 (-24)	2.44 (-24)	2.61 (-24)	2.78 (-24)	2.94 (-24)	3.23 (-24)	3.50 (-24)	3.76 (-24)	4.00 (-24)	4.23 (-24)
0.020	45560	6.27 (-25)	7.00 (-25)	7.67 (-25)	8.30 (-25)	8.89 (-25)	9.48 (-25)	1.06 (-24)	1.16 (-24)	1.27 (-24)	1.37 (-24)	1.46 (-24)
0.025	36450	2.77 (-25)	3.12 (-25)	3.45 (-25)	3.76 (-25)	4.06 (-25)	4.36 (-25)	4.93 (-25)	5.49 (-25)	6.05 (-25)	6.60 (-25)	7.14 (-25)
0.030	30380	1.49 (-25)	1.69 (-25)	1.88 (-25)	2.06 (-25)	2.25 (-25)	2.43 (-25)	2.78 (-25)	3.14 (-25)	3.49 (-25)	3.84 (-25)	4.20 (-25)
0.040	22780	9.00 (-26)	1.03 (-25)	1.16 (-25)	1.28 (-25)	1.40 (-25)	1.52 (-25)	1.77 (-25)	2.01 (-25)	2.26 (-25)	2.51 (-25)	2.77 (-25)
0.050	18230	4.17 (-26)	4.84 (-26)	5.51 (-26)	6.18 (-26)	6.86 (-26)	7.54 (-26)	8.92 (-26)	1.03 (-25)	1.18 (-25)	1.33 (-25)	1.48 (-25)
0.060	15190	2.34 (-26)	2.76 (-26)	3.18 (-26)	3.60 (-26)	4.03 (-26)	4.48 (-26)	5.38 (-26)	6.32 (-26)	7.29 (-26)	8.29 (-26)	9.31 (-26)
0.070	13020	1.48 (-26)	1.77 (-26)	2.06 (-26)	2.36 (-26)	2.66 (-26)	2.98 (-26)	3.63 (-26)	4.30 (-26)	5.01 (-26)	5.74 (-26)	6.48 (-26)
0.080	11390	0.117 (-26)	0.219 (-26)	0.382 (-26)	0.634 (-26)	1.01 (-26)	1.56 (-26)	3.48 (-26)	0.723 (-25)	1.43 (-25)	2.70 (-25)	4.97 (-25)
0.090	10130	1.02 (-26)	1.23 (-26)	1.44 (-26)	1.66 (-26)	1.89 (-26)	2.13 (-26)	2.63 (-26)	3.14 (-26)	3.69 (-26)	4.25 (-26)	4.82 (-26)
0.100	9113	0.398 (-26)	0.746 (-26)	1.30 (-26)	2.16 (-26)	3.45 (-26)	5.33 (-26)	11.9 (-26)	2.47 (-25)	4.87 (-25)	9.22 (-25)	17.0 (-25)
0.120	7594	7.41 (-27)	9.02 (-27)	1.07 (-26)	1.24 (-26)	1.42 (-26)	1.61 (-26)	2.00 (-26)	2.41 (-26)	2.85 (-26)	3.29 (-26)	3.76 (-26)
0.140	6509	0.603 (-26)	1.13 (-26)	1.97 (-26)	3.28 (-26)	5.23 (-26)	8.08 (-26)	18.0 (-26)	3.74 (-25)	7.37 (-25)	14.0 (-25)	25.7 (-25)
0.160	5695	5.64 (-27)	6.93 (-27)	8.27 (-27)	9.68 (-27)	1.11 (-26)	1.27 (-26)	1.59 (-26)	1.92 (-26)	2.28 (-26)	2.65 (-26)	3.03 (-26)
0.180	5063	0.719 (-26)	1.35 (-26)	2.36 (-26)	3.91 (-26)	6.24 (-26)	9.65 (-26)	21.5 (-26)	4.46 (-25)	8.80 (-25)	16.7 (-25)	30.7 (-25)
0.200	4556	4.45 (-27)	5.50 (-27)	6.61 (-27)	7.78 (-27)	8.99 (-27)	1.03 (-26)	1.29 (-26)	1.58 (-26)	1.87 (-26)	2.18 (-26)	2.51 (-26)
0.225	4050	0.774 (-26)	1.45 (-26)	2.53 (-26)	4.20 (-26)	6.71 (-26)	10.4 (-26)	23.1 (-26)	4.80 (-25)	9.46 (-25)	17.9 (-25)	33.0 (-25)
		2.98 (-27)	3.73 (-27)	4.53 (-27)	5.38 (-27)	6.27 (-27)	7.20 (-27)	9.16 (-27)	1.13 (-26)	1.35 (-26)	1.58 (-26)	1.82 (-26)
		0.778 (-26)	1.46 (-26)	2.55 (-26)	4.23 (-26)	6.75 (-26)	10.4 (-26)	23.3 (-26)	4.83 (-25)	9.52 (-25)	18.1 (-25)	33.2 (-25)
		2.15 (-27)	2.72 (-27)	3.33 (-27)	3.98 (-27)	4.66 (-27)	5.38 (-27)	6.89 (-27)	8.52 (-27)	1.02 (-26)	1.20 (-26)	1.39 (-26)
		0.725 (-26)	1.36 (-26)	2.37 (-26)	3.94 (-26)	6.28 (-26)	9.72 (-26)	21.6 (-26)	4.49 (-25)	8.86 (-25)	16.8 (-25)	30.9 (-25)
		1.63 (-27)	2.08 (-27)	2.57 (-27)	3.08 (-27)	3.63 (-27)	4.20 (-27)	5.42 (-27)	6.72 (-27)	8.10 (-27)	9.55 (-27)	1.11 (-26)
		0.657 (-26)	1.23 (-26)	2.15 (-26)	3.57 (-26)	5.70 (-26)	8.81 (-26)	19.6 (-26)	4.08 (-25)	8.04 (-25)	15.2 (-25)	28.0 (-25)
		1.29 (-27)	1.65 (-27)	2.05 (-27)	2.47 (-27)	2.92 (-27)	3.39 (-27)	4.39 (-27)	5.47 (-27)	6.61 (-27)	7.81 (-27)	9.06 (-27)
		0.591 (-26)	1.11 (-26)	1.94 (-26)	3.21 (-26)	5.13 (-26)	7.93 (-26)	17.7 (-26)	3.67 (-25)	7.23 (-25)	13.7 (-25)	25.2 (-25)
		1.04 (-27)	1.35 (-27)	1.68 (-27)	2.03 (-27)	2.41 (-27)	2.81 (-27)	3.65 (-27)	4.56 (-27)	5.52 (-27)	6.53 (-27)	7.59 (-27)
		0.532 (-26)	0.997 (-26)	1.74 (-26)	2.89 (-26)	4.61 (-26)	7.13 (-26)	15.9 (-26)	3.30 (-25)	6.51 (-25)	12.3 (-25)	22.7 (-25)
		8.30 (-28)	1.08 (-27)	1.35 (-27)	1.64 (-27)	1.95 (-27)	2.28 (-27)	2.97 (-27)	3.72 (-27)	4.52 (-27)	5.60 (-27)	6.23 (-27)
		0.468 (-26)	0.877 (-26)	1.53 (-26)	2.54 (-26)	4.06 (-26)	6.28 (-26)	14.0 (-26)	2.90 (-25)	5.73 (-25)	10.9 (-25)	20.0 (-25)

\* The upper entries give the contributions to the absorption coefficients by the free-free transitions. The lower entries (for  $\lambda > 16,500 \text{ \AA}$ ) give the corresponding contributions due to the bound-free transitions derived from the cross-sections given in *A. P. J., 102, 395, 1945*. The numbers in parentheses are the powers of 10, by which the corresponding entries should be multiplied to get the absorption coefficients in the unit  $\text{CM}^2/\text{dyn}$ .

TABLE 6\*

THE CONTINUOUS ABSORPTION COEFFICIENT OF THE NEGATIVE HYDROGEN ION PER NEUTRAL HYDROGEN ATOM AND PER UNIT ELECTRON PRESSURE FOR VARIOUS TEMPERATURES AND WAVE LENGTHS AFTER ALLOWING FOR THE STIMULATED EMISSION FACTOR  $(1 - e^{-h\nu/kT})$

$\Delta k^{\dagger}$	$\lambda \text{ \AA}$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1.0$	$\theta = 1.2$	$\theta = 1.4$	$\theta = 1.6$	$\theta = 1.8$	$\theta = 2.0$
†	...	...	...	...	...	...	...	...	...	...	...	...
0.005	182300	2.92 (-29)	3.81 (-29)	4.76 (-29)	5.74 (-29)	6.76 (-29)	7.81 (-29)	9.98 (-29)	1.22 (-28)	1.46 (-28)	1.70 (-28)	1.93 (-28)
	...	1.17 (-24)	1.53 (-24)	1.91 (-24)	2.30 (-24)	2.71 (-24)	3.13 (-24)	4.01 (-24)	4.91 (-24)	5.85 (-24)	6.80 (-24)	7.77 (-24)
	91130	2.93 (-25)	3.84 (-25)	4.80 (-25)	5.79 (-25)	6.83 (-25)	7.90 (-25)	1.01 (-24)	1.24 (-24)	1.48 (-24)	1.72 (-24)	1.97 (-24)
	60750	1.31 (-25)	1.72 (-25)	2.15 (-25)	2.60 (-25)	3.07 (-25)	3.55 (-25)	4.56 (-25)	5.61 (-25)	6.69 (-25)	7.79 (-25)	8.91 (-25)
	45560	7.45 (-26)	9.77 (-26)	1.22 (-25)	1.48 (-25)	1.75 (-25)	2.03 (-25)	2.61 (-25)	3.21 (-25)	3.83 (-25)	4.46 (-25)	5.10 (-25)
	36450	4.81 (-26)	6.32 (-26)	7.93 (-26)	9.61 (-26)	1.14 (-25)	1.32 (-25)	1.69 (-25)	2.09 (-25)	2.49 (-25)	2.90 (-25)	3.32 (-25)
	30380	3.38 (-26)	4.44 (-26)	5.57 (-26)	6.77 (-26)	8.01 (-26)	9.29 (-26)	1.20 (-25)	1.47 (-25)	1.76 (-25)	2.05 (-25)	2.34 (-25)
	22780	1.94 (-26)	2.56 (-26)	3.22 (-26)	3.91 (-26)	4.64 (-26)	5.38 (-26)	6.94 (-26)	8.54 (-26)	1.02 (-25)	1.19 (-25)	1.36 (-25)
	18230	1.27 (-27)	1.68 (-26)	2.11 (-26)	2.57 (-26)	3.05 (-26)	3.54 (-26)	4.56 (-26)	5.62 (-26)	6.70 (-26)	7.79 (-26)	8.91 (-26)
	15190	0.0710 (-26)	0.148 (-26)	0.279 (-26)	0.493 (-26)	0.825 (-26)	1.32 (-26)	3.12 (-26)	0.671 (-25)	1.36 (-25)	2.61 (-25)	4.86 (-25)
	13020	6.77 (-27)	8.97 (-27)	1.13 (-26)	1.38 (-26)	1.63 (-26)	1.89 (-26)	2.44 (-26)	3.00 (-26)	3.58 (-26)	4.16 (-26)	4.76 (-26)
	...	0.265 (-26)	0.545 (-26)	1.02 (-26)	1.79 (-26)	2.97 (-26)	4.74 (-26)	11.0 (-26)	2.35 (-25)	4.72 (-25)	9.05 (-25)	16.7 (-25)
	11390	5.29 (-27)	7.02 (-27)	8.84 (-27)	1.08 (-26)	1.27 (-26)	1.48 (-26)	1.90 (-26)	2.34 (-26)	2.79 (-26)	3.26 (-26)	3.73 (-26)
	...	0.431 (-26)	0.879 (-26)	1.63 (-26)	2.83 (-26)	4.68 (-26)	7.42 (-26)	17.1 (-26)	3.63 (-25)	7.24 (-25)	13.8 (-25)	25.5 (-25)
	10130	4.27 (-27)	5.65 (-27)	7.12 (-27)	8.66 (-27)	1.03 (-26)	1.19 (-26)	1.53 (-26)	1.89 (-26)	2.25 (-26)	2.63 (-26)	3.02 (-26)
	...	0.544 (-26)	1.10 (-26)	2.03 (-26)	3.50 (-26)	5.74 (-26)	9.07 (-26)	20.8 (-26)	4.38 (-25)	8.70 (-25)	16.6 (-25)	30.6 (-25)
	9113	3.52 (-27)	4.66 (-27)	5.87 (-27)	7.14 (-27)	8.46 (-27)	9.81 (-27)	1.26 (-26)	1.56 (-26)	1.86 (-26)	2.18 (-26)	2.50 (-26)
	...	0.612 (-26)	1.23 (-26)	2.25 (-26)	3.86 (-26)	6.31 (-26)	9.92 (-26)	22.6 (-26)	4.74 (-25)	9.40 (-25)	17.9 (-25)	32.9 (-25)
	7594	2.52 (-27)	3.34 (-27)	4.21 (-27)	5.11 (-27)	6.06 (-27)	7.03 (-27)	9.06 (-27)	1.12 (-26)	1.34 (-26)	1.57 (-26)	1.81 (-26)
	...	0.659 (-26)	1.31 (-26)	2.37 (-26)	4.02 (-26)	6.52 (-26)	10.2 (-26)	23.0 (-26)	4.80 (-25)	9.50 (-25)	18.0 (-25)	33.2 (-25)
	6509	1.91 (-27)	2.52 (-27)	3.18 (-27)	3.86 (-27)	4.57 (-27)	5.31 (-27)	6.86 (-27)	8.50 (-27)	1.02 (-26)	1.20 (-26)	1.39 (-26)
	...	0.644 (-26)	1.26 (-26)	2.26 (-26)	3.82 (-26)	6.16 (-26)	9.60 (-26)	21.5 (-26)	4.49 (-25)	8.86 (-25)	16.8 (-25)	30.9 (-25)
	5695	1.50 (-27)	1.98 (-27)	2.49 (-27)	3.03 (-27)	3.59 (-27)	4.17 (-27)	5.40 (-27)	6.72 (-27)	8.10 (-27)	9.55 (-27)	1.11 (-26)
	...	0.603 (-26)	1.17 (-26)	2.09 (-26)	3.51 (-26)	5.63 (-26)	8.75 (-26)	19.6 (-26)	4.07 (-25)	8.04 (-25)	15.2 (-25)	28.0 (-25)
	5063	1.21 (-27)	1.60 (-27)	2.01 (-27)	2.44 (-27)	2.90 (-27)	3.38 (-27)	4.39 (-27)	5.47 (-27)	6.61 (-27)	7.81 (-27)	9.06 (-27)
	...	0.556 (-26)	1.07 (-26)	1.90 (-26)	3.18 (-26)	5.09 (-26)	7.90 (-26)	17.6 (-26)	3.67 (-25)	7.23 (-25)	13.7 (-25)	25.2 (-25)
	4556	9.98 (-28)	1.32 (-27)	1.66 (-27)	2.02 (-27)	2.40 (-27)	2.80 (-27)	3.65 (-27)	4.56 (-27)	5.52 (-27)	6.53 (-27)	7.59 (-27)
	...	0.509 (-26)	0.974 (-26)	1.72 (-26)	2.87 (-26)	4.60 (-26)	7.12 (-26)	15.9 (-26)	3.30 (-25)	6.51 (-25)	12.3 (-25)	22.7 (-25)
	4050	8.05 (-28)	1.06 (-27)	1.34 (-27)	1.64 (-27)	1.95 (-27)	2.27 (-27)	2.97 (-27)	3.72 (-27)	4.52 (-27)	5.60 (-27)	6.23 (-27)
	...	0.454 (-26)	0.865 (-26)	1.52 (-26)	2.53 (-26)	4.05 (-26)	6.27 (-26)	14.0 (-26)	2.90 (-25)	5.73 (-25)	10.9 (-25)	20.0 (-25)

\* The arrangement of this table is the same as Table 5.

† The entries in this line when divided by  $(\Delta k^{\dagger})^2$  will give the corresponding absorption coefficients to a sufficient accuracy for all  $\Delta k^{\dagger} < 0.005$ .

TABLE 7\*  
 THE CONTINUOUS ABSORPTION COEFFICIENT OF THE NEGATIVE HYDROGEN ION DUE TO THE FREE-FREE AND THE BOUND-FREE  
 TRANSITIONS FOR VARIOUS TEMPERATURES AND WAVE LENGTHS AFTER ALLOWING  
 FOR THE STIMULATED EMISSION FACTOR  $(1 - e^{-h\nu/kT})$

$\Delta\lambda^{\dagger}$	$\lambda\lambda$	$\theta=0.5$	$\theta=0.6$	$\theta=0.7$	$\theta=0.8$	$\theta=0.9$	$\theta=1.0$	$\theta=1.2$	$\theta=1.4$	$\theta=1.6$	$\theta=1.8$	$\theta=2.0$
†												
0.005	182300	3.513 (-29)	4.593 (-29)	5.730 (-29)	6.915 (-29)	8.140 (-29)	9.404 (-29)	1.202 (-28)	1.474 (-28)	1.754 (-28)	2.042 (-28)	2.330 (-28)
0.010	91130	1.168 (-24)	1.528 (-24)	1.906 (-24)	2.301 (-24)	2.710 (-24)	3.132 (-24)	4.006 (-24)	4.913 (-24)	5.846 (-24)	6.795 (-24)	7.765 (-24)
0.015	60750	2.934 (-25)	3.840 (-25)	4.796 (-25)	5.794 (-25)	6.829 (-25)	7.897 (-25)	1.012 (-24)	1.243 (-24)	1.481 (-24)	1.724 (-24)	1.971 (-24)
0.020	45560	1.313 (-25)	1.720 (-25)	2.151 (-25)	2.602 (-25)	3.066 (-25)	3.554 (-25)	4.560 (-25)	5.610 (-25)	6.688 (-25)	7.793 (-25)	8.907 (-25)
0.025	36450	7.447 (-26)	9.771 (-26)	1.223 (-25)	1.482 (-25)	1.750 (-25)	2.028 (-25)	2.606 (-25)	3.209 (-25)	3.829 (-25)	4.463 (-25)	5.102 (-25)
0.030	30380	4.811 (-26)	6.370 (-26)	7.925 (-26)	9.607 (-26)	1.136 (-25)	1.317 (-25)	1.695 (-25)	2.088 (-25)	2.493 (-25)	2.904 (-25)	3.321 (-25)
0.040	22780	3.375 (-26)	4.440 (-26)	5.575 (-26)	6.765 (-26)	8.006 (-26)	9.289 (-26)	1.196 (-25)	1.474 (-25)	1.759 (-25)	2.049 (-25)	2.343 (-25)
0.050	18230	1.941 (-26)	2.560 (-26)	3.220 (-26)	3.913 (-26)	4.636 (-26)	5.383 (-26)	6.935 (-26)	0.854 (-25)	1.019 (-25)	1.186 (-25)	1.356 (-25)
0.060	15190	1.270 (-26)	1.679 (-26)	2.115 (-26)	2.572 (-26)	3.049 (-26)	3.541 (-26)	4.561 (-26)	0.561 (-25)	0.669 (-25)	0.779 (-25)	0.890 (-25)
0.070	13020	0.973 (-26)	1.342 (-26)	1.784 (-26)	2.324 (-26)	2.996 (-26)	3.846 (-26)	6.363 (-26)	1.071 (-25)	1.832 (-25)	3.167 (-25)	5.488 (-25)
0.080	11390	0.942 (-26)	1.443 (-26)	2.153 (-26)	3.164 (-26)	4.600 (-26)	6.631 (-26)	13.46 (-26)	2.652 (-25)	5.077 (-25)	9.462 (-25)	17.22 (-25)
0.090	10130	0.959 (-26)	1.580 (-26)	2.517 (-26)	3.910 (-26)	5.953 (-26)	8.902 (-26)	19.02 (-26)	3.861 (-25)	7.519 (-25)	14.15 (-25)	25.90 (-25)
0.100	9113	0.970 (-26)	1.665 (-26)	2.741 (-26)	4.366 (-26)	6.770 (-26)	10.261 (-26)	22.29 (-26)	4.565 (-25)	8.928 (-25)	16.84 (-25)	30.86 (-25)
0.120	7594	0.963 (-26)	1.695 (-26)	2.838 (-26)	4.575 (-26)	7.152 (-26)	10.901 (-26)	23.83 (-26)	4.894 (-25)	9.586 (-25)	18.10 (-25)	33.16 (-25)
0.140	6509	0.911 (-26)	1.640 (-26)	2.786 (-26)	4.532 (-26)	7.124 (-26)	10.895 (-26)	23.90 (-26)	4.915 (-25)	9.632 (-25)	18.19 (-25)	33.34 (-25)
0.160	5695	0.834 (-26)	1.513 (-26)	2.581 (-26)	4.206 (-26)	6.618 (-26)	10.126 (-26)	22.22 (-26)	4.570 (-25)	8.959 (-25)	16.92 (-25)	31.02 (-25)
0.180	5063	0.753 (-26)	1.369 (-26)	2.337 (-26)	3.809 (-26)	5.993 (-26)	9.169 (-26)	20.12 (-26)	4.140 (-25)	8.117 (-25)	15.33 (-25)	28.12 (-25)
0.200	4556	0.676 (-26)	1.230 (-26)	2.100 (-26)	3.423 (-26)	5.385 (-26)	8.238 (-26)	18.08 (-26)	3.721 (-25)	7.298 (-25)	13.79 (-25)	25.30 (-25)
0.225	4050	0.608 (-26)	1.105 (-26)	1.887 (-26)	3.074 (-26)	4.837 (-26)	7.401 (-26)	16.25 (-26)	3.346 (-25)	6.564 (-25)	12.41 (-25)	22.76 (-25)
		0.534 (-26)	0.971 (-26)	1.656 (-26)	2.698 (-26)	4.246 (-26)	6.499 (-26)	14.28 (-26)	2.941 (-25)	5.772 (-25)	10.91 (-25)	20.02 (-25)

\* The coefficients are per neutral hydrogen atom and per unit electron pressure. The numbers in parentheses give the powers of 10 by which the corresponding entries should be multiplied to get the coefficients in the unit  $\text{CM}^2/\text{dyne}$ .  
 † The entries in this line when multiplied by  $(\lambda A/1000)^2$  will give the absorption coefficients for the various  $\theta$ -values for all wave lengths  $\lambda > 180,000 \text{ \AA}$ .

6. *Concluding remarks.*—It is not our intention here to make detailed applications of our new absorption coefficients to various astrophysical problems. But one comparison is of interest. In Figure 4 we have compared the theoretical variation of the continuous absorption coefficient of  $H^-$  with wave length for  $T = 6300^\circ$  with that derived by Chalonge and Kourganoff in their recent discussion of the continuous spectrum of the sun for the same temperature. It is seen that the agreement is very satisfactory. In any event, there is no basis for the conclusion that the free-free transitions of  $H^-$  are not

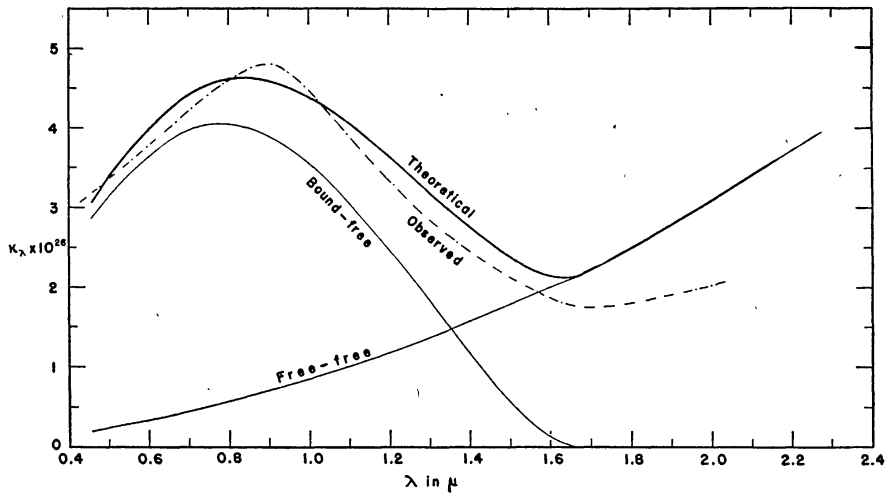


FIG. 4.—A comparison of the theoretical variation of the continuous absorption coefficient with wave length of  $H^-$  for  $T = 6300^\circ$ , with the corresponding variation derived empirically from the solar continuous spectrum by Chalonge and Kourganoff.

adequate to account for the observed amount of absorption in the red beyond  $\lambda 10,000 \text{ \AA}$ . Indeed, it would appear that the negative ion of hydrogen by itself is able to account quantitatively for the entire continuous spectrum of the sun over the range of wave lengths  $\lambda 4000 \text{ \AA}$  to  $25,000 \text{ \AA}$ .

On the physical side we should, however, emphasize that, while our new coefficients for the free-free transitions of  $H^-$  are probably to be trusted generally, the importance of exchange and polarization for the slow  $s$ -electrons may lead to further changes. It is likely that these effects will have a tendency to reduce our coefficients somewhat. We hope to return to these questions in the near future.