*Proceedings of the Institute of Radio Engineers*
*Volume 18, Number 1* *January, 1930*

# MULTIPLE SIGNALS IN SHORT-WAVE TRANSMISSION*

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***Summary***—*This paper presents an analysis of the facsimile records obtained recently in the transmissions between New York, U. S. A., and Somerton, England. Since the speed of the scanning spot in the facsimile apparatus is accurately known, these records permit the measurement of the time intervals between the various signals which produce the distortion in the received record. Thus the facsimile apparatus can be used as an oscillograph for Kennelly-Heaviside layer measurements after the method employed by Breit and Tuve and others.*

*The results of the analysis confirm, in general, the results of other experimenters and extend them in the direction of giving information as to the angle within which the useful radiation is confined at the transmitter. A knowledge of this angle is then shown to yield important information on the distortion to be expected on different wavelengths. A detailed summary of the results is included at the end of the paper.*

## Introduction

THEORIES of short-wave transmission depend in the main on pre-knowledge of the Heaviside layer; and on the other hand this pre-knowledge must be based on some theory of short-wave ray transmission, for the Heaviside layer can only be probed by means of short wireless waves, and the interpretation of the results involves the transmission theory. The methods used for probing involve the reception of more than one ray, generally a direct ray over the surface of the earth and an indirect ray which has travelled up to the Heaviside layer and back. Either the difference in time of arrival of short groups is measured or a sustained signal is sent with a varying frequency, and the variations of interference are made to give the requisite information. Information may also be obtained by observing the directions of the incoming waves. The first method has been extensively used in America, and the second by Appleton in England. The first method has been thoroughly discussed, on the assumption that the wave follows a ray path determined by the ionic density in the Heaviside layer.

On the assumption that the gradient of ionic density is everywhere vertical, the difference in time of arrival of the direct signal and the reflected signal gives at once the angle of transmission and the height of the apex $B$ of the ray, i.e., the equivalent height of the layer. (See Fig. 7.)

*Dewey decimal classification: R113.6.

The method, however, fails for short-distance transmission on sufficiently short waves where the density of the layer is not sufficient to bend the rays down. In fact, the method fails where the receiver is within the skip distance.

In recent tests by Kenrick and Jen[1] the experiments were successful on a 67-m wave, but nothing was received on a 33-m wave, suggesting that the latter was too short a wave. In order to get results on shorter waves than this, experiments must be made outside the skip where the time between the direct and echo signals can be made to yield very definite information. An opportunity for examining such a case on a 22-m wave was afforded by the facsimile records obtained recently in the transmissions between New York and Somerton.

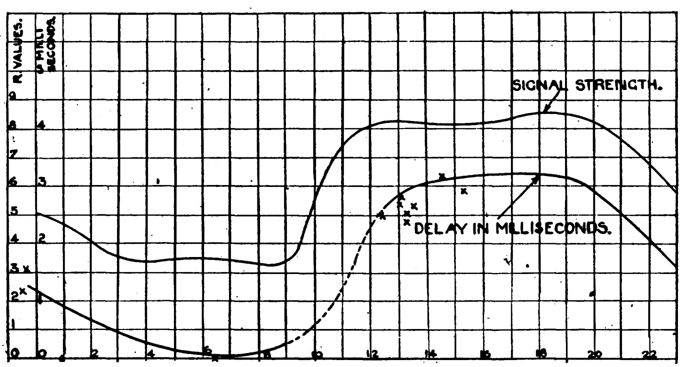



Fig. 1—Echo duration (October-November, 1928) and signal strength. (Station WAJ, 13480 kc.)

The interpretation of the results is fundamentally the same as the above with the proviso that the rays are more nearly horizontal than vertical in such conditions. For the purpose of the analysis of the results, the facsimile apparatus is considered in the nature of an oscillograph. The signals made by the transmitter are in general of sufficiently short duration not to overlap with the echoes. In fact the arrangement is essentially similar to the type of apparatus used in the experiments of Breit and Tuve and other experimenters in America; a faithful record of each signal made at the transmitter and modified in transmission is made at the receiver. For the present purpose it is hardly necessary to enter into details of the apparatus, and it is sufficient to state that practically perfect reproduction is obtained over short distances where the transmission distortions are not present.

[1]PROC. I. R. E., **17,** 711; April, 1929.

## Short Echoes

A preliminary analysis of short echoes (3 or 4 millisec.) exhibited by the facsimile records taken on the circuit between New York and Somerton on 13480 kc or 22.255-m wave during the period August,

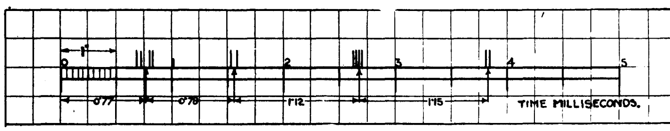



Fig. 2—Echoes.

1928, to January, 1929, has brought to light facts which may be of considerable practical importance in facsimile working. They are also of great significance in constructing a working model of the Heaviside layer and giving a rational explanation of the bewildering variety of results obtained in short-wave interception.

In a preliminary survey something like 20 or 30 cases of the echo signals shown on the facsimile photographs have been measured up, and these are typical selections from the large amount of material accumulated. A fairly close examination of the main bulk of the material suggests that very little will be added but repetitions by measuring it all.

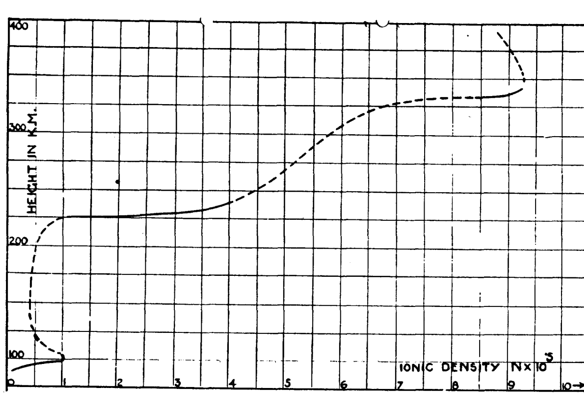



Fig. 3—The Heaviside layer.

Anyone who knows the actual mechanism of the facsimile gear will realize that it gives an excellent record of the mutilation of signals in their passage from the transmitter to receiver, a comparison of the transmitted and received picture immediately showing up the signal mutilation.

Examination of the records shows that if one short signal (<0.5 millisec. duration) is transmitted this may be reproduced as 1, 2, 3, 4, 5, or even 6 separate signals at the receiver. Thus if a single line (drawn perpendicular to the direction of scan of the light spot) is transmitted, it is reproduced as a group of 2, 3, 4, or 5 lines closely spaced.

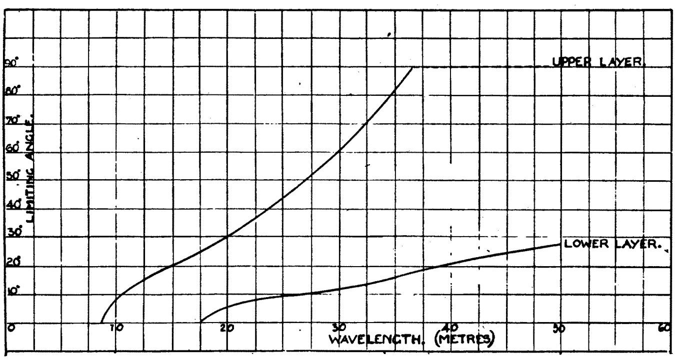



Fig. 4—Limiting angle for transmission.

We may call the first signal that arrives the main signal and the latter short echoes of it. If the distance between the lines on the received photograph is measured it is possible to calculate the time intervals when the speed of the scanning spot is known.

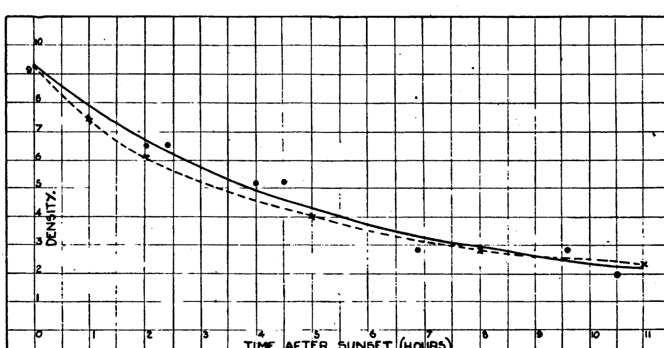



Fig. 5—Ionic recombination.

The speed of the scanning spot is determined by the frequency of the controlling fork and is accurately known.

It is therefore possible to make accurate measurements of the time intervals between the main signal and the various echoes. With a

300-cycle fork the scanning speed is 30 min. per sec. and clearly separates short signals (of the order of 1/5000 sec.) with time intervals less than a millisecond.

Most of the pictures were run off near the middle of the day (when signals were strongest) and the information on night transmission is meager.

It appears that during the hours 1200 to about 1500 echoes up to the number of three or four are prevalent in nearly every case, with delays up to 3.8 millisec. and in one case up to 5.3 millisec.

In the evening echoes are still prevalent, but the extreme delay (between first and last) seems to decrease throughout the night and to be a minimum about sunrise here and to increase rapidly as the sun rises at New York, tending to its maximum value when the sun is on the meridian half way between Somerton and New York. (See samples in Figs. 8, 9, and 10.)

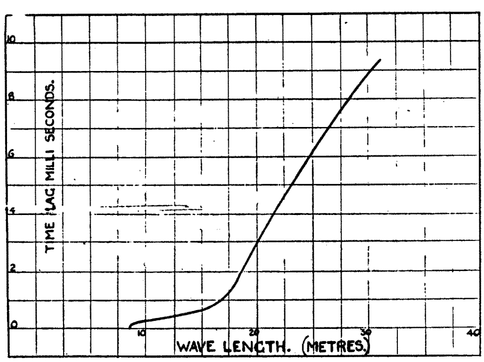



Fig. 6—Observed curve.

The results are plotted on curve 1 showing the total echo duration as a function of the local time for the period between October 15th and November 7th. It shows a diurnal variation very similar to the signal strength curve of WAJ, the station used during this period, showing that when signals are strong multiples are prevalent and disappear as the signal weakens.

The time between each echo appears to be between 0.7 and 1.2 millisec. and the later ones are more widely spaced than the earlier ones (of any given group).

The times of individual echoes are rather irregular, but on particular occasions we have found a very regular arrangement repeated again and again within the period of a minute or so.

These are represented in Fig. 2 and may be taken as a representative set of echoes.

The maximum delay (between the first and last signal) may be used to give very interesting data concerning the Heaviside layer which can be used to determine the behavior as regards facsimile echoes on other wavelengths. This use depends on the following relation derived from the ray theory.

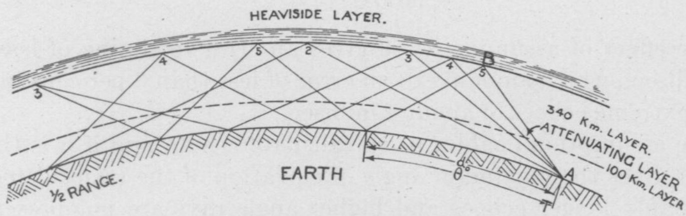



Fig. 7—Theoretical recombination curve.

If the gradient in the layer is purely vertical (as we may assume is the case when conditions are nearly uniform between transmitter and receiver) and if the ray does not depart from the earth's surface more than a small fraction of the earth's radius, then the time of travel of any one of the echo signals, say the $n^{\text{th}}$, is

$$\tau_n = \frac{d}{c} \; \frac{1}{\cos \theta n}$$

the echo time is $\tau_n - \tau_1$

$$\frac{d}{c}\left[\frac{1}{\cos\,\theta_n} - \frac{1}{\cos\,\theta_1}\right]\tau_n - \tau_1$$

since $d$, $c$, and $\tau_n - \tau_1$ are known

$$\frac{1}{\cos\,\theta_n} - \frac{1}{\cos\,\theta}$$

is given.

This gives a relation between $\theta_1$ and $\theta_n$, but not $\theta_1$ and $\theta_n$ separately.

The assumption made here is that $\theta_1$ is very small so that $1/\cos\,\theta_1$ is practically unity. This can be justified in the present case from our knowledge of the Heaviside layer. Thus we know from various experimenters, notably Appleton, that there is a maximum density of $10^5$ electrons per cu. cm at the height of very closely 100 km. See Fig. 3.

On a 22-m wave all rays with initial angles of elevation less than 9 deg. are bent down by this layer, so that if we assume that the first signal is produced by rays which are confined to regions below this first layer

$$\frac{1}{\cos\theta_1} < \frac{1}{\cos 9 \text{ deg.}}$$

i.e.,

$$\frac{1}{0.9877} = 1.0123.$$

The effect of assuming $\theta_1 = 0$ gives an error in timing of less than 0.2 millisec., which amounts to an error of less than 5 per cent in cases of the extreme echoes of about 4 millisec.

The assumption enables us to calculate $\cos\theta_n$, and if $n$ is the last echo, it gives the maximum angle of elevation of the transmitted ray. Presumably longer echoes and higher angle rays are not present because such high-angle rays would penetrate the layer and not be returned to earth. Values of $\theta_n$ determined in this manner are given below.

| Date | G.M.T. | $\theta_n$ | $N$. | |
|---|---|---|---|---|
| | | | $h = 240$ km | $h = 340$ km |
| Oct. 20th | 1517 | 30 deg. 57 min. | $7.80 \times 10^5$ | $8.40 \times 10^5$ |
| " 21st | 2315 | 20 " 16 " | $4.5 \times 10^5$ | $5.2 \times 10^5$ |
| " 21st | 1302 | 29 " 46 " | $7.40 \times 10^5$ | $8.05 \times 10^5$ |
| " | | 30 " 07 " | $7.60 \times 10^5$ | $8.15 \times 10^5$ |
| Nov. 12th | 1321 | 28 " 35 " | $7.05 \times 10^5$ | $7.70 \times 10^5$ |
| " 7th | 1328 | 28 " 35 " | $7.05 \times 10^5$ | $7.70 \times 10^5$ |
| | | 28 " 30 " | $7.0 \times 10^5$ | $7.65 \times 10^5$ |
| | 1536 | 34 " 36 " | $9.1 \times 10^5$ | $9.8 \times 10^5$ |
| | | 34 " 23 " | $9.0 \times 10^5$ | $9.6 \times 10^5$ |
| Nov. 2nd | 1435 | 31 " 47 " | $7.8 \times 10^5$ | $8.8 \times 10^5$ |
| Mean Value | | 30 " 47 " | $7.75 \times 10^5$ | $8.43 \times 10^5$ |

The maximum value o $\theta_n$ is therefore less than 35 deg. and the mean 30 deg. 47 min.

We may therefore state that on 22 m over the Somerton—New York circuit the *ray angles of the transmitted rays are less than 35 deg. and usually less than 31 deg.*

The higher angle rays are usually weaker than the others, and it follows that the *main energy is transmitted along rays* $<20$ deg. elevation.

This is a complete confirmation of our previous results obtained with the cardioid receiver, i.e., that long-distance communication is effected with relatively low-angle rays.

These results determine the ray angles with immensely greater accuracy than any balanced aerial system is likely to do.

These ray angles can now be used to calculate the maximum density (at the apex of the rays) from the relation

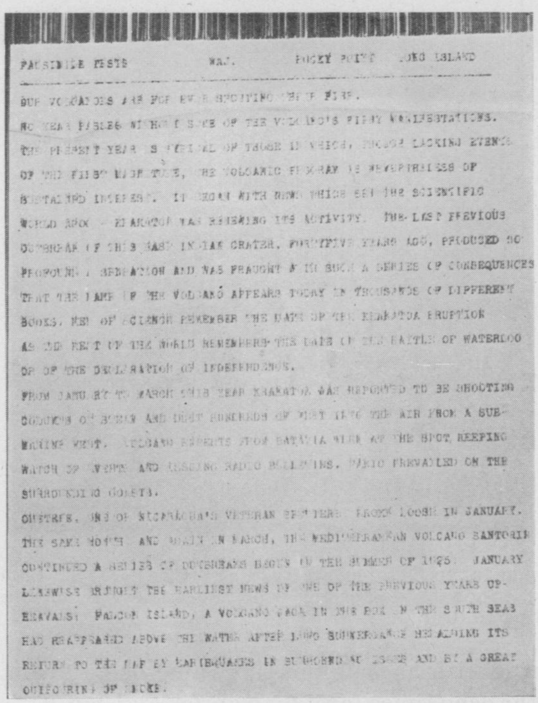
$$\cos\theta = \frac{R+h}{R}\,\frac{\mu}{\mu_0}\cdot\frac{\mu}{\mu_0} = \sqrt{1-\frac{Ne^2c^2}{\pi m n^2}} \quad \text{or} \quad \sqrt{1-\frac{n_0{}^2}{n^2}}$$

where $h$ is the height of the ray above the earth at its apex. when $h/R = x$ is small and $\theta$ is small

$$\frac{n_0{}^2}{n^2} = \frac{Ne^2c^2}{\pi m n^2} = \sin^2\theta + 2x$$

from which $N$ can be determined.

This contains the unknown quantity $x$, the ratio of the height of the layer to the earth's radius, so that $N$ cannot be *accurately* determined unless $x$ is known. If $\theta$ is large, however, so that $\sin\theta \gg 2x$, for even the

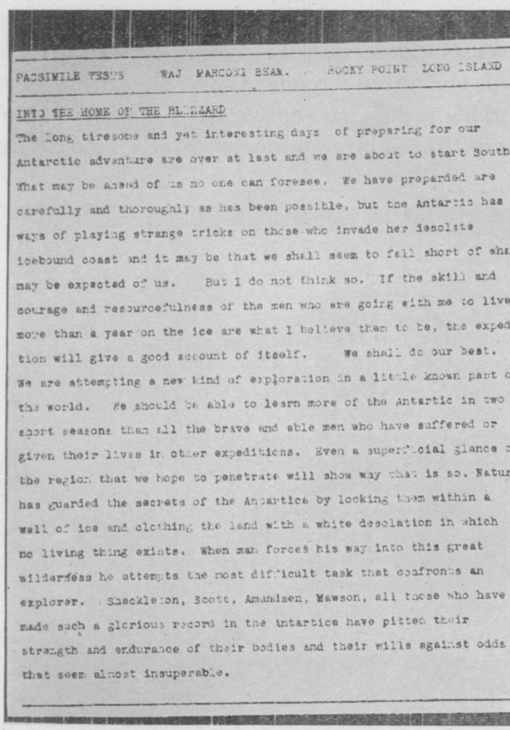

FACSIMILE TESTS [illegible] ROCKY POINT LONG ISLAND

OUR VOLCANOES ARE FOR EVER SPOUTING THEIR FIRE.
NO YEAR PASSES WITHOUT SOME OF THE VOLCANO'S FIERY MANIFESTATIONS.
THE PRESENT YEAR IS TYPICAL OF THOSE IN WHICH, THOUGH LACKING EVENTS OF THE FIRST MAGNITUDE, THE VOLCANIC PROGRAM IS NEVERTHELESS OF SUSTAINED INTEREST. IT BEGAN WITH NEWS WHICH SET THE SCIENTIFIC WORLD AGOG – KRAKATOA WAS RENEWING ITS ACTIVITY. THE LAST PREVIOUS OUTBREAK OF THIS EAST INDIAN CRATER, FORTYFIVE YEARS AGO, PRODUCED SO PROFOUND A SENSATION AND WAS FRAUGHT WITH SUCH A SERIES OF CONSEQUENCES THAT THE NAME OF THE VOLCANO APPEARS TODAY IN THOUSANDS OF DIFFERENT BOOKS. MEN OF SCIENCE REMEMBER THE DATE OF THE KRAKATOA ERUPTION AS THE REST OF THE WORLD REMEMBERS THE DATE OF THE BATTLE OF WATERLOO OR OF THE DECLARATION OF INDEPENDENCE.
FROM JANUARY TO MARCH THIS YEAR KRAKATOA WAS REPORTED TO BE SHOOTING COLUMNS OF [illegible] AND DUST HUNDREDS OF FEET INTO THE AIR FROM A SUBMARINE VENT. VOLCANO EXPERTS FROM BATAVIA WERE AT THE SPOT KEEPING WATCH OF EVENTS AND ISSUING RADIO BULLETINS. PANIC PREVAILED ON THE SURROUNDING COASTS.
OMETEPE, ONE OF NICARAGUA'S VETERAN SPOUTERS, BROKE LOOSE IN JANUARY. THE SAME MONTH AND AGAIN IN MARCH, THE MEDITERRANEAN VOLCANO SANTORIN CONTINUED A SERIES OF OUTBREAKS BEGUN IN THE SUMMER OF 1925. JANUARY LIKEWISE BROUGHT THE EARLIEST NEWS OF ONE OF THE PREVIOUS YEARS UPHEAVALS: FALCON ISLAND, A VOLCANO PEAK IN THE BOX IN THE SOUTH SEAS HAD REAPPEARED ABOVE THE WATER AFTER LONG SUBMERGENCE HERALDING ITS RETURN TO THE MAP BY EARTHQUAKES IN SURROUNDING [illegible] AND BY A GREAT OUTPOURING OF SMOKE.

Fig. 8—Specimen taken October 21, 1928 at 2335 G. M. T., Somerton. Speed, 300 cycles. Lighter limit. Many drop outs. Showing decreasing echo time at midnight. High-angle echo tending to drop out. About 1.48 millisecond delay.

maximum possible values of $h$, the effect of $x$ on $N$ in inappreciable. Unfortunately in this case $\sin^2\theta$ is of the order 0.25 and $2x$ may be 0.08 so that an error of 30 per cent may be made, but the actual error will probably be much less since $x$ can be estimated within fairly narrow limits from the other observations and the error in $2x$ is not likely to be greater than 0.03, giving a percentage error in $N$ of about 12 per cent.

Two sets of $N$ values are given, one with $h$ assumed to be 226 km. (Derived from measurements made by Appleton,[2] and Breit, Tuve, and Dahl[3] on 100-m and 75-m waves, respectively. (Kenrick and Jen)[1].



FACSIMILE TESTS WAJ MARCONI BEAM. ROCKY POINT LONG ISLAND

INTO THE HOME OF THE BLIZZARD

The long tiresome and yet interesting days of preparing for our Antarctic adventure are over at last and we are about to start South. What may be ahead of us no one can foresee. We have preparded are carefully and thoroughly as has been possible, but the Antartic has ways of playing strange tricks on those who invade her desolate icebound coast and it may be that we shall seem to fall short of what may be expected of us. But I do not think so. If the skill and courage and resourcefulness of the men who are going with me to live more than a year on the ice are what I believe them to be, the expedition will give a good account of itself. We shall do our best. We are attempting a new kind of exploration in a little known part of the world. We should be able to learn more of the Antartic in two short seasons than all the brave and able men who have suffered or given their lives in other expeditions. Even a superficial glance of the region that we hope to penetrate will show way that is so. Nature has guarded the secrets of the Antartica by locking them within a wall of ice and clothing the land with a white desolation in which no living thing exists. When man forces his way into this great wilderness he attempts the most difficult task that confronts an explorer. Shackleton, Scott, Amundsen, Mawson, all those who have made such a glorious record in the Antartica have pitted their strength and endurance of their bodies and their wills against odds that seem almost insuperable.

Fig. 9—Specimen taken October 22, 1928 at 0620 G. M. T., Somerton. Speed, 150 cycles. Showing decreasing echo time at early morning. High-angle echo tending to drop out.

The other set is with $h=340$ km, this value being given by the measurement of the echo times (also by Kenrick and Jen.)[1]

$N$ appears to be of the order 8 to $9\times10^5$, i.e., nearly 10 times the maximum value of $N$ in the 100-km layer.

At this point it is necessary to sketch briefly our knowledge of the Heaviside layer as determined by other experiments.

[2] *Nature*, p. 445, March 23, 1929.
[3] Proc. I. R. E. **16**, 1236; September, 1928.

Interference experiments made by Appleton and Hollingworth and signal-strength measurements made by the author on long waves show that the electron density increases rapidly above 75 km and rises to a maximum of $10^5$ at 100 km (in daylight.)

Recent experiments made by Appleton on 100-m wave and described in *Nature*[2] show that the measured effective height of the layer (by interference methods) jumps discontinuously at irregular times from 100 to 226 km, which may be interpreted on the assumption that the lower layer is so nearly only just sufficient to reflect the 100-m wave that a very slight diminution in $N$ from time to time exposes a higher layer at 226 km. The density between $N=10^5$ at 100 km and $N=10^5$ at 226 km is then everywhere less than this value.

Measurements made on 75 m and 67 m show that in the daytime the signal is reflected at an effective height of 226 km, and that the density there is $2.4\times10^5$ or greater. We may therefore sketch in the average day density as in Fig. 3, which is derived from the average of the data. There is a fairly well-defined layer up to $N=10^5$ at 90 to 100 km, another at least $2.4\times10^5$ at 226 km.

Finally at nighttime Kenrick and Jen[1] results seem to show another layer exposed by the recombination of ions below about 344 km height. From these data it seems reasonable to suppose that the 22-m echo rays are reflected between 240- and 340-km heights, and the maximum densities in the layer are calculated for these two heights and give the probable limits. The mean densities corresponding to the two heights are 7.75 and 8.43, the latter probably more nearly correct.

## Maximum Density in the Layer

The fact that there is a certain maximum delay implies a certain maximum angle of transmission, approximately 35 deg. It is well known that corresponding to a certain maximum density in the layer there is a limit to the angle of projection of the ray if this ray is to return to earth again. It seems certain that higher angles are not present in the 22-m transmissions because rays of such angles escape through the layer. The values of $N$ given above represent fairly closely the limiting density in the upper layer.

With this data it is possible to plot a curve giving the limiting angle $\theta$ as a function of the wavelength. It may be derived from the relation $\cos\theta = R+h/R\ \mu_{\min}/\mu_0$

Since knowing $N_{\max}$, $\mu_{\min}/\mu_0$ is known for every wavelength. This curve is shown in Fig. 4.

It will be observed that it meets the $x$ axis at 8.6 m. Implying that for wavelengths less than 8.6 m the relation above cannot be satisfied because even glancing angle rays of higher frequencies are not sufficiently bent to come to earth.

*8.6 m is in fact the short-wave day limit.*

This acts as a check on the values of $N$, for we find, in fact, that this is very close to the day-wave limit. Thus 10-m waves transmitted in England have been received in Australia, New York, and Buenos Aires.

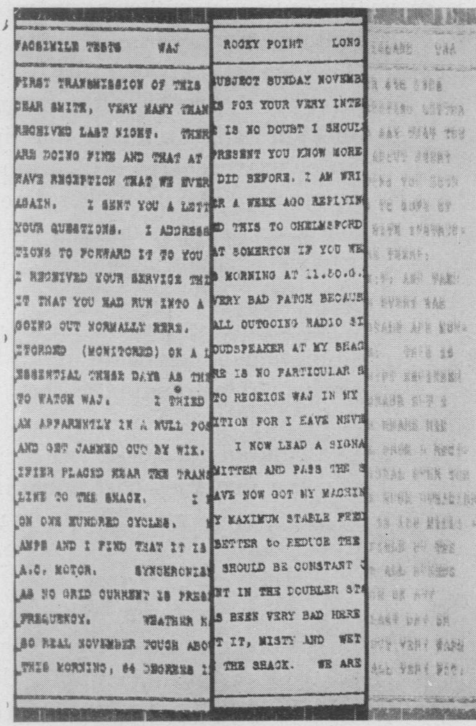



Fig. 10—Specimen taken November 8, 1928 at 1415 G. M. T., Somerton. Speeds: 1st column, 150 cycles; 2nd column, 100 cycles; 3rd column, 300 cycles. Showing increased echo-signal separation with increased scanning speed.

Sporadic and very occasional reception of an 8.67-m beam at Poldhu has been recorded in New York. Reports of occasional long-distance transmissions on waves shorter than this have been received, but definite instances of such transmissions appear to be wanting, and they are probably exceptional.

We may therefore say that there is considerable evidence that the short-wave limit lies between 8 and 10 m in confirmation of the above.

Experiments with a balanced frame and vertical setup to determine the ray angles from the 28 to 32-m Dutch stations, though not of a high order of accuracy on account of the blurring of the balance point by scattered energy, also provide confirmatory evidence.

The angles found lie between 50 deg. and 68 deg. and the corresponding maximum densities (which can be determined independently of the value of $x$) lie between $8.4\times10^5$, and $1.3\times10^6$ with a mean $9.3\times10^5$, a figure slightly greater than that determined from the echo measures, i.e., $8.43\times10^5$. Considering the relative inaccuracy of these experiments the agreement is as good as could be expected.

The mean height of the layer determined from these values of $\theta$ is $250\pm55$, which lies between the limits set out previously.

These results giving the limiting density of the upper atmosphere seem to rest on a very secure basis and to be much more definite than previous deductions of the layer density in the Heaviside layer. This maximum density it will be observed is derived for full daylight conditions.

The curves (Figs. 1 and 5) show, on the same reasoning as previously given, that the maximum echo time decreases throughout the night, and it follows that the density decreases during the night hours and is a minimum just before sunrise.

The absence of rays of higher angle than 35 deg. has been attributed to the insufficiency of electrons.

It is certainly a Heaviside layer effect as the $\theta_{\min}$ depends on the local time and is very much less at night.

The only other effect besides electron limitation which can limit $\theta$ is attenuation which, however, is likely to be reduced the higher the ray angle, so that the electron limitation hypothesis seems almost unassailable.

In Fig. 4 we have plotted the maximum ray angle as a function of $\lambda$, but this maximum ray angle also gives by relation (1) the maximum echo time; we can therefore plot the maximum possible echo time (max. duration between main and echo signal) as a function of $\lambda$. This is given in Fig. 6 for daylight conditions. It will be seen that it decreases very rapidly with $\lambda$ and *is only about 0.8 millisec. at* $\lambda=16m$. This has a very practical significance in facsimile working.

*The maximum echo lag on a 16-m transmission to New York is likely to be only 1/5 of that on 22 m.*

*Four or five times the speed of picture transmission could be used on such a service.*

(This is neglecting scattering echoes which do not seem to be serious in the picture service, at least on 22 m and when signals are relatively strong.)

Summarizing, we may say that this conclusion is logically reached contingent on the two assumptions.

(1) That the main signal (first received) is transmitted at practically glancing incidence.

(2) That the lag of the last echo is limited wholly by insufficiency of electron density in the upper layer, both of which seem to be open to very little criticism or doubt.

## Origin of the Multiple Echoes

So far we have been considering the time interval between the first and last signal giving the maximum transmission angle and the maximum density in the layer.

The individual echo signals, as has already been stated, are rather irregular; there is the possibility that they may overlap, in some cases producing phase opposition which obscures the main position of the echo signal. Cases have been found where, however, the echoes are so definitely repeated that they represent without doubt the true sequence of the echoes. Those are represented in Fig. 2.

They represent a set of five signals each implying a definite ray starting in the five definite directions $\theta_1 < 5$deg.; $\theta_2 = 16$ deg. 45 min.; $\theta_3 = 21$ deg. 34 min.; $\theta_4 = 29$ deg. 32 min.; and $\theta_5 = 34$ deg. 36 min.

The question arises, what is the path of these different rays?

The simplest supposition is that they represent the multiple reflection between earth and Heaviside layer, and the internal evidence for this seems fairly strong. This hypothesis is represented diagrammatically in the figure below (Fig. 7) where four rays are separately represented by the four lines $\theta_2 - \theta_5$.

If the upper layer were so well defined that the apex of each of the rays, or rather the equivalent height defined by the apex of the triangle (at $B$) is the same for all rays, then the height calculated by the triangulation of each of these rays should be constant.

This triangulation may be carried out as follows:

Suppose according to our hypothesis that the fifth signal is by the ray which cuts up the distance $d$ into five equal parts; then the elementary triangle to be calculated is shown in Fig. 7. $\theta_0$ is given by the delay time, $d_0$ is one fifth $d$, the distance between receiver and transmitter, the height $h$ is then determinate, and is given by the relation

$$x = \frac{h}{R} = -\frac{\sin^2\theta}{2} - \tfrac{1}{2}\sqrt{\sin^4\theta + 4y^2 - 4\sin^2\theta}.$$

where $\theta$ is the angle which $d_0$ subtends at the center of the earth, and $y = d_1/R$ where $d_1$ is the length of the ray to the apex $B$.

$d_1$ is directly known from the difference in time of the main signal and echo signal. Taking echoes 3, 4, and 5 we get the corresponding values of $h'$

| *No. of Echo* | *h* |
|---|---|
| 3 | 343 |
| 4 | 338 |
| 5 | 340 |

No. 2 echo is less accurately determined and as it will be more affected by its passage through the lower layer (100 km) it is therefore not included.

The extraordinary consistency of the three independent measures of $h$ in this table makes it very probable that the hypothesis of multi-reflection between the earth and Heaviside layer is correct, the rays at any angle reaching the same virtual height.

In confirmation of this we have the results of Kenrick and Jen,[1] who in a 67-m transmission with the pulse method have indicated a layer at this height. (344 km) (This layer appears to be exposed to a 67-m wave only at night.) See Fig. 3.

This of course is not conclusive, but the agreement seems more than accidental.

These facts would seem to imply that the bending of the ray takes place in a very limited height (of the order of the discordance of the values obtained for $h$).

It follows therefore that for waves of frequency large compared with $3\times 10^6$ (the critical frequency for the lower layer) the ray paths are very approximately the triangles exhibited in Fig. 7 with only a slight deviation near the apex.

The triangle will be approximately the same for all waves in the range which satisfy this condition, i.e., $\lambda \ll \lambda_0 = 100$ m, and therefore the time lags of individual echoes should be practically independent of the wavelength. The effect of a reduction of wavelength is therefore to reduce the number of echoes but not materially to alter the time between individual echoes.

## Short-Wave Attenuation

The information disclosed by the facsimile echo measurements has a great significance with regard to short-wave attenuation in the range between 14 and 50 m.

Take the typical example of the 22-m wave. A typical ray has an angle of elevation between 0 and 30 deg. Consider one with an angle of about 20 deg. It passes along nearly a straight line at 20-deg. elevation, through the lower layer at 100-km height and is bent back at $B$ (Fig. 7) to earth again. It is the region near the 100-km layer which is attenuating. It passes almost straight through this attenuating layer. (It can only suffer 5-deg. deviation in the layer $N$ being so small.) The total attenuation that the wave will suffer is

$$x = \lambda^2 \int \frac{1}{{}_s\lambda_0 c \tau_s} ds$$

where ${}_s\lambda_0$ and $\tau_s$ are the values of $\lambda_0$ and $\tau$ (the time between collisions at a distance $S$ measured along the ray.) ${}_S\lambda_0$ and $\tau_s$ are only functions of the state of the Heaviside layer.

So that $\alpha = K_s \lambda^2$.

Now it is obvious from the geometry of the system that the ray will be the same for any other wavelength (within the range considered).

*The attenuation will therefore be proportional to* $\lambda^2$. This is precisely what is found on analyzing the results obtained in the year's interception at Broomfield (for daytime transmission). We have in these results a rational explanation of the behavior of short-wave day attenuation.

## Night Transmission

The information as regards night transmission is less definite.

The following conclusions, however, appear to be pretty certain.

We have found that the echo delay time decreases throughout the night, being a minimum just before sunrise. This may be definitely interpreted as a gradual decrease of the maximum ionic density $N_{\max}$ in the upper layer as the night progresses. But $N_{\max}$ determines the short-wave limit for transmission according to the relation

$$\frac{\mu_{1\min}}{\mu_0} = \frac{R}{R+h}$$

or

$$1 - \frac{N_{\max} e^2 c^2}{\pi m n^2} = \left[\frac{R}{R+h}\right]^2$$

or

$$\frac{N_{\max} e^2 \lambda^2}{\pi m n^2} = 1 - \left[\frac{R}{R+h}\right]^2$$

$$\simeq 2x$$

or
$$\lambda^2_{\min}=\frac{2\times\pi m}{N_{\max}e^2}$$

and the smaller the value of $N_{\max}$ the greater $\lambda_{\min}$.

We should therefore expect a progressive increase in the short-wave limit as the night progresses. Thus in the early evening shorter waves can be used for long distance transmission than in the later hours of darkness. This is a well-known fact derived from the analysis of the results obtained in the year's interception.

We can give approximately the values of $\lambda$ limit as a function of the time elapsed from sunset. Thus see Fig. 5.

| | Hours after sunset. | $\lambda$ | $N$ |
|---|---|---|---|
| approx. | 0 | 8.6 | $9.3\times10^5$ |
| | 2.5 | 11 | $6.5\times10^5$ |
| | 7 | 14 | $4.2\times10^5$ |
| | 10.5 | 20 | $2\ \ \times10^5$ |

from the echo delays we have the values

| | |
|---|---|
| 6 | $4.8\times10^5$ |
| 7 | $4.2\times10^5$ |

in rough agreement with the above.

We may picture the effect somewhat as follows:

As the time after sunset increases $N$ decreases, the maximum ray angle consequently decreases and the useful fraction of the energy radiated, i.e., from $\theta=0$ up to $\theta=\theta_1$, decreases and consequently the signal strength decreases.

The relation between signal strength and echo delay and consequently $\theta_{\min}$ is strikingly given in Fig. 1.

The controlling factor determining signal strength at night is electron limitation and not attenuation, and to complete the theoretical aspect we should give reason why the attenuation appears to be negligible at night. The reason appears to be that the attenuating layer (100 km) rises at night, effectively increasing $\tau$ perhaps tenfold; also $N$ decreases, both of which factors decrease attenuation.

Summarizing we may say that the picture transmissions indicate the presence of four or five or even, in extreme cases, six separate rays between New York and Somerton (on 22 m).

From the measured delay time between the echoes we may state that the maximum angle of elevation of the rays is 35 deg. in all but exceptional cases, and that the main energy is transmitted along rays less than 20-deg. elevation, i.e., practically glancing incidence.

The maximum daylight density in the upper layer is approximately 8.3 to $9.3 \times 10^5$ electrons per cu. cm, which corresponds to a daylight minimum wave of 8.6 m.

The total echo delay (between first and last signal) decreases rapidly with the wavelength being 1 millisec. at 16 m and decreasing to zero at 8.6 m.

With less certainty we may conclude that

(1) Transmission takes place by multiple reflections between earth and Heaviside layer, the latter being fairly sharply defined—for this wavelength—at a height of 340 km.

(2) In the daytime attenuation takes place in the lower layer (100 km height) and is practically proportional to $\lambda^2$.

(3) Throughout the night the attenuation proper ceases to play an important part, but the limiting wavelength increases (on account of recombination) from 8.6 to nearly 20 m in extreme cases of long winter nights.

## List of Symbols Employed

$R$ = radius of earth
$n$ = frequency
$n_0$ = critical frequency of medium
$e$ = charge on electron
$m$ = mass of electron
$N$ = number of electrons per cu. cm
$c$ = velocity of light
$\mu$ = refractive index